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Research Article

Domination Defect of Some Parameterized Families of Graphs

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Abstract. In this paper, we study the concept of k-domination defect of a graph G and investigate it for some parameterized families of graphs. We produce characterizations of the ζ_k-sets of the path, cycle, centipede graph, sunlet graph, bi-star graph, crown graph, and complete bipartite graph, and then from these characterizations, the corresponding k-domination defects of the aforesaid graphs are determined.

Keywords. k-Domination defect set, k-Domination defect of a graph

Mathematics Subject Classification (2020). 05C35, 05C38, 05C69, 05C75

1. Introduction

The concept of domination defect of a graph was first introduced by Das and Desormeaux [4] in 2018. We studied this concept in [6] and investigated it for graphs resulting from some binary operations such as the join and corona. In this paper, we further explore the concept and examine it for some known parameterized families of graphs.

In what follows are some of the concepts and notations which are used in this paper. Let $G = (V(G), E(G))$ be a graph. For each $x \in V(G)$, the set $N_G(x) = \{y \in V(G) \mid xy \in E(G)\}$ is the *open neighborhood* of x in G while the set $N_G[x] = N_G(x) \cup \{x\}$ is the *closed neighborhood* of x in G. For a nonempty set $S \subseteq V(G)$, the *open neighborhood* of S in G and the *closed neighborhood* of S in G are given by the sets $N_G(S) = \bigcup_{x\in S} N_G(x)$ and $N_G[S] = N_G(S) \cup S$, respectively.

A nonempty set $S \subseteq V(G)$ is a *dominating set* of G if $N_G[S] = V(G)$. The minimum cardinality of a dominating set in G is the domination number of G, denoted by $\gamma(G)$. The minimality of $\gamma(G)$ implies that if a particular set W of vertices of G has cardinality less than $\gamma(G)$, then there is at least one vertex of G that is not dominated by W. This leads then to the concept of k-domination defect of a graph introduced by Das and Desormeaux [4] and further studied by Miranda and Eballe [6].

Let G be a specific graph of order n with $\gamma(G) \geq 2$ and let $1 \leq k < \gamma(G)$. Let $S \subseteq V(G)$ with $|S| = \gamma(G) - k$. The k-defect of S is $\zeta_k(S) = |V(G) \setminus N_G[S]| = n - |N_G[S]|$. The *$k$-domination defect* of G, denoted by $\zeta_k(G)$, is the minimum cardinality of the set $V(G) \setminus N_G[W]$ for $W \subseteq V(G)$ with $|W| = \gamma(G) - k$. A set $S \subseteq V(G)$ of cardinality $\gamma(G) - k$ for which $|V(G) \setminus N_G[S]| = \zeta_k(G)$ is called a *ζ_k-set* of G. We emphasize that if G is a graph with $\gamma(G) \geq 2$ and $S \subseteq V(G)$ is a ζ_k-set of G, where $1 \leq k < \gamma(G)$, then $|S| = \gamma(G) - k$ such that $|N_G[S]| = \max\{|N_G[W]| : W \subseteq V(G), |W| = \gamma(G) - k\}$.

In this paper, we characterize the ζ_k-sets of the path P_n, cycle C_n, centipede graph G_n, sunlet graph S_n, bi-star graph $B(r,s)$, crown graph $G(n,n)$, and complete bipartite graph $K_{m,n}$. Our final goal here is to obtain the corresponding k-domination defect of the aforementioned graphs, similar to those done in Militante *et al.* [7], Balandra and Canoy [1], and Consistente and Cabahug [3], hoping that results generated in this study will be of use when one considers more complex graphs.

For basic graph theoretic terminologies not given here, please refer to Chartrand and Zhang [2]. Throughout this paper, all graphs are considered finite, undirected, and simple graphs. To avoid triviality, only graphs with domination number at least 2 will be considered here, equivalent to the condition that only graphs with no spanning stars shall be investigated.

In [6], we presented and proved the following lemma:

Lemma 1.1 ([6]). *Let G be a nontrivial graph such that $\gamma(G) \geq 2$ and let $k = \gamma(G) - 1$. Then $S \subseteq V(G)$ is a ζ_k-set of G if and only if $S = \{v\}$ for some $v \in V(G)$ with* $\deg_G(v) = \Delta(G)$.

The above lemma is useful in proving some of the results in this paper.

2. Main Results

Recall that the *path* P_n of order n is the graph with n distinct vertices $v_1, v_2, \dots, v_n$ and $n-1$ distinct edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n$. The *cycle* C_n of order $n \geq 3$ is the graph that consists of n distinct vertices $v_1, v_2, \dots, v_n$ and n distinct edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1$. Figure 1 provides skeletal diagrams of these parameterized graphs.

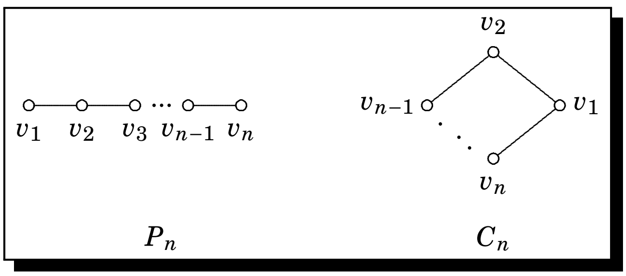

Figure 1. The path P_n and cycle C_n

Lemma 2.1. *Let G be a path or a cycle graph of order $n \geq 4$ and let $1 \leq k < \gamma(G)$. If $S \subseteq V(G)$ is a ζ_k-set of G, then $3|S| < n$.*

Proof. For a ζ_k*-set* $S \subseteq V(G)$ of a graph G, we have $|S| = \gamma(G) - k$, where $1 \leq k < \gamma(G)$. Using the fact that $\gamma(G) = \lceil \frac{n}{3} \rceil$ (see for instance, Frendrup *et al*. [5]) and some properties of the basic ceiling function, we obtain $|S| = \lceil \frac{n}{3} \rceil - k = \lceil \frac{n}{3} - k \rceil < \frac{n}{3} - k + 1$, implying that $3|S| < n - 3(k-1)$. Since $k \geq 1$, it follows that $3(k-1) \geq 0$; hence, $3|S| < n$. □

The ζ_k-sets of the path P_n and cycle C_n, where $n \geq 4$, are characterized below.

Theorem 2.2. *Let G be a path or a cycle graph of order $n \geq 4$ and let $k \in \{1, 2, \ldots, \gamma(G) - 1\}$. A set $S \subseteq V(G)$ of cardinality $\gamma(G) - k$ is a ζ_k-set of G if and only if $|N_G[S]| = 3|S|$.*

Proof. Assume $S \subseteq V(G)$ is a ζ_k*-set* of G. By definition, $|N_G[S]|$ is maximum among values $|N_G[W]$ such that $W \subseteq V(G)$, $|W| = \gamma(G) - k$, where $1 \leq k < \gamma(G)$. By the nature of the path and cycle graphs, 3 is always an upper bound for the maximum cardinality of the closed neighborhood of any vertex $v \in S$. Now, for notational convenience, denote the vertices of G by $\{v_1, v_2, \ldots, v_n\}$ with $v_i v_{i+1} \in E(G)$ for all $i = 1, 2, \ldots, n-1$ (with $v_n v_1 \in E(G)$ if G is a cycle) and choose $\lceil \frac{n}{3} - k \rceil$-element set $S = \{v_2, v_5, \ldots, v_{3t-1}\}$, where $t = 1, 2, 3, \ldots, \lceil \frac{n}{3} - k \rceil$. In this case, every $v \in S$ has exactly 2 unique neighbors in G. Thus, $|N_G[S]| = |S| + |N_G(S)| = |S| + |N_G(v_2)| + |N_G(v_5)| + \ldots + |N_G(3t-1) = |S| + 2|S| = 3|S|$.
Conversely, let $S \subseteq V(G)$ such that $|S| = \gamma(G) - k$, where $1 \leq k \leq \gamma(G)$, and that $|N_G[S]| = 3|S|$. Since $|N_G[v]| \leq 3$ for each $v \in S$, it follows that $|N_G[S]|$ is maximum for all $|N_G[W]$, where $W \subseteq V(G)$, $|W| = \gamma(G) - k$. As a consequence, S is a ζ_k-set of G. □

A slightly different but equivalent characterization of the ζ_k-sets of the path P_n, $n \geq 4$, is given next.

Theorem 2.3. *Let P_n be a path of order $n \geq 4$. Then a set $S \subseteq V(P_n)$ of cardinality $\gamma(P_n) - k$, where $1 \leq k < \gamma(P_n)$, is a ζ_k-set of P_n if and only if the following conditions hold:*

(i) *for each $v \in S$, $\deg_{P_n}(v) = 2$;*

(ii) *for all distinct vertices $v_i, v_j \in S$, $N_{P_n}[v_i] \cap N_{P_n}[v_j] = \emptyset$.*

Proof. Let $P_n = [v_1, v_2, \ldots, v_n]$ be a path of order $n \geq 4$ and let $S \subseteq V(P_n)$ be a ζ_k-set of P_n. We note that $N_{P_n}[S] = \bigcup\{N_{P_n}[v] : v \in S\}$ and $|N_{P_n}[v_i]| \leq 3$, for every $v_i \in V(P_n)$. Since $|N_{P_n}[S]| = 3|S|$ by Theorem 2.2, it follows that $|N_{P_n}[v]| = 3$ for each $v \in S$, implying $\deg_{P_n}(v) = 2$ for each $v \in S$, and that for all distinct vertices $v_i, v_j \in S$, $N_{P_n}[v_i] \cap N_{P_n}[v_j] = \emptyset$.
Conversely, conditions (i) and (ii) immediately imply that $|N_G[S]| = 3|S|$. By Theorem 2.2, S is a ζ_k-set of P_n. □

Corollary 2.4. *If P_n is a path of order $n \geq 4$ and $S \subseteq V(P_n)$ is a ζ_k-set of P_n, where $1 \leq k < \gamma(P_n)$, then $\zeta_k(P_n) = n - 3|S|$.*

Proof. Suppose that $S \subseteq V(P_n)$ is a ζ_k*-set* of P_n. By definition, $\zeta_k(P_n) = n - |N_{P_n}[S]|$. By Theorem 2.2, $\zeta_k(P_n) = n - 3|S|$. □

Corollary 2.5 (Das' Theorem 1.1, [4]). *If P_n is a path with n vertices, then*

$$\zeta_k(P_n) = \begin{cases} 3k-2, & \text{if } n = 3t+1; \\ 3k-1, & \text{if } n = 3t+2; \\ 3k, & \text{if } n = 3t. \end{cases}$$

Proof. Let S be a ζ_k-set of P_n. If $n = 3t+1$, then the equation $\zeta_k(P_n) = n - 3|S|$ in Corollary 2.4 becomes $\zeta_k(P_n) = (3t+1) - 3|S|$. From the identity $|S| = \gamma(P_n) - k$ and the fact that $\gamma(P_n) = \lceil \frac{n}{3} \rceil$ (Frendrup *et al.* [5]), we have $|S| = \lceil \frac{n}{3} \rceil - k$. By substitution, $\zeta_k(P_n) = (3t+1) - 3\left(\lceil \frac{n}{3} \rceil - k\right)$ $= (3t+1) - 3\left(\lceil \frac{(3t+1)}{3} \rceil - k\right)$. Simplifying further, $\zeta_k(P_n) = (3t+1) - 3\left(\lceil t + \frac{1}{3} \rceil - k\right) = (3t+1)$ $-3(t+1-k) = 3k-2$. In a similar fashion, $\zeta_k(P_n) = 3k-1$ and $\zeta_k(P_n) = 3k$ if $n = 3t+2$ and $n = 3t$, respectively, for some $t \in \mathbb{N}$. □

The next result is a slightly different characterization of the ζ_k-sets of the cycle C_n, $n \geq 4$.

Theorem 2.6. *Let C_n be a cycle of order $n \geq 4$. Then a set $S \subseteq V(C_n)$ of cardinality $\gamma(C_n) - k$, where $1 \leq k < \gamma(C_n)$, is a ζ_k-set of C_n if and only if $N_{C_n}[v_i] \cap N_{C_n}[vj] = \emptyset$, for all distinct vertices $v_i, v_j \in S$.*

Proof. The reasoning here runs almost the same as in Theorem 2.3, except that $\deg_{v \in C_n}(v) = 2$ is already a fact rather than an assumption for the converse. □

Corollary 2.7. *If C_n is a cycle of order $n \geq 4$ and $S \subseteq V(C_n)$ is a ζ_k-set of C_n, where $1 \leq k < \gamma(C_n)$, then $\zeta_k(C_n) = n - 3|S|$.*

Proof. This is very similar to the process in which Corollary 2.4 was argued. □

For our second group of special parameterized graphs, we consider the centipede G_n and sunlet S_n.

Recall that a *centipede graph* G_n is a tree with $2n$ vertices obtained by joining the bottoms of n copies of the path graph P_2 laid in a row with edges forming a path P_n as its spine. Graph G_n has n pendant vertices $\{u_1, u_2, \ldots, u_n\}$ and n spine vertices $\{v_1, v_2, \ldots, v_n\}$. On the other hand, the *n-sunlet graph* on $2n$ vertices is obtained by attaching n-pendant vertices to the cycle C_n and is denoted by S_n. Graph S_n has n pendant vertices $\{u_1, u_2, \ldots, u_n\}$ and n cycle vertices $\{v_1, v_2, \ldots, v_n\}$. These graphs are illustrated in Figure 2.

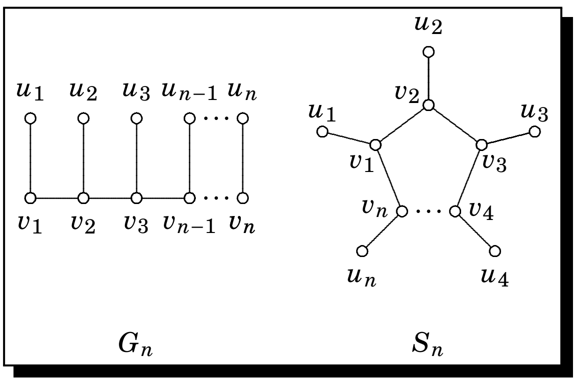

Figure 2. The centipede G_n and sunlet S_n

From the graph illustrations above, we note that the domination number γ of G_n and S_n is equal to n. Further, the following observations are straightforward.

Remark 2.1. *Let G be a centipede G_n or sunlet graph S_n of order $2n$, $n \geq 3$. Let $S \subseteq V(G)$, $|S| = n-k$, where $k = 1,2,\dots,n-1$. To maximize $|N_G[S]|$, it is necessary that $|\{u_i,v_i\} \cap S| \leq 1$, for all $i = 1,2,\dots,n$.*

Remark 2.2. *Let G be a centipede G_n or sunlet graph S_n of order $2n$, $n \geq 3$. Let $S \subseteq V(G)$, $|S| = n-k$, where $k = 1,2,\dots,n-1$. Let T be the set of pendant vertices of G and let $T' = T \cap N_{G_n}[S]$. If $|\{u_i,v_i\} \cap S| \leq 1$, for all $i = 1,2,\dots,n$, then $|T'| = |S|$.*

Theorem 2.8. *Let G be a centipede G_n or sunlet graph S_n of order $2n$, $n \geq 3$. Let H be the spine of G_n or the cycle of S_n. If $S \subseteq V(G)$ with $|S| = n-k$, where $k = 1,2,\dots,n-1$, is a ζ_k-set of G, then the maximum cardinality of the closed neighborhood of S in G is given by:*

$$|N_G[S]| = \begin{cases} 4(n-k), & \text{if } |S| \in \{1,2,\dots,\gamma(H)-1\}, \\ 2n-k, & \text{if } |S| \in \{\gamma(H),\gamma(H)+1,\dots,n-1\}. \end{cases}$$

Proof. Let G and H be defined as given above, and let T be the set of all pendant vertices of G. Let $H' = H \cap N_{G_n}[S]$ and let $T' = T \cap N_{G_n}[S]$. Note that $H' \cap T' = \emptyset$. Hence, $|N_G[S]| = |H'| + |T'|$. We consider the following two cases for the value of $|S|$:

Case 1. Suppose $|S| \in \{1,2,\dots,\gamma(H)-1\}$. Then $|H'| < n$ and by Remark 2.2, $|T'| = |S|$. Since $|S| < \gamma(H)$, $S \subseteq V(H)$. By Theorem 2.2, $|H'| = 3|S|$. Hence, $|N_G[S]| = |H'| + |T'| = 3|S| + |S| = 4|S| = 4(n-k)$.

Case 2. Suppose $|S| \in \{\gamma(H),\gamma(H)+1,\dots,n-1\}$. Since $|S| > \gamma(H)$, we can distribute the vertices of S in a manner that will dominate the entire vertices of H, giving $|H'| = n$. Therefore, $|N_G[S]| = |H'| + |T'| = n + |S| = 2n-k$. □

Theorem 2.9. *Let G_n be a centipede graph of order $2n$, $n \geq 3$ with spine P_n. Let $S \subseteq V(G_n)$ with $|S| = n-k$, where $k = 1,2,\dots,n-1$, such that $|\{u_i,v_i\} \cap S| \leq 1$, for all $i = 1,2,\dots,n$. Then S is a ζ_k-set of G_n if and only if any of the following holds:*

(i) *$|S| \in \{1,2,\dots,\gamma(P_n)-1\}$, where $S \subseteq V(P_n)$ and $|N_{G_n}[S]| = 4|S|$;*

(ii) *$|S| \in \{\gamma(P_n),\gamma(P_n)+1,\dots,n-1\}$, where $S \cap V(P_n)$ is a dominating set in P_n.*

Proof. Let G_n be a centipede graph of order $2n$, $n \geq 3$ with spine P_n. Let T be the set of pendant vertices of G_n, let $H' = P_n \cap N_{G_n}[S]$ and let $T' = T \cap N_{G_n}[S]$. Suppose $S \subseteq V(G)$, with $|S| = n-k$, is a ζ_k*-set* of G_n. We consider the following cases:

Case 1. Suppose $|S| \in \{1,2,\dots,\gamma(P_n)-1\}$. Since $|S| < \gamma(P_n)$, consequently $S \subseteq V(P_n)$. Set S being a ζ_k-set of G_n implies that $|N_{G_n}[S]|$ is maximum among sets $W \subseteq V(G)$, $|W| = n-k$, where $k = 1,2,\dots,n-1$. By Theorem 2.8, $|N_{G_n}[S]| = 4(n-k) = 4|S|$. Conversely, suppose $S \subseteq V(P_n)$ and $|N_{G_n}| = 4|S|$. Then $|N_{G_n}[v]| = 4$, for all $v \in S \subseteq V(P_n)$. With the nature of the centipede, 4 is the upper bound of $|N_{G_n}[v]$, for any $v \in G_n$. It follows that $|N_{G_n}[S] = 4|S|$ is maximum among the subsets $W \subseteq V(G_n)$. Hence, S is a ζ_k-set of G_n.

Case 2. Suppose $|S| \in \{\gamma(P_n),\gamma(P_n)+1,\dots,n-1\}$. Set S being a ζ_k-set of G_n implies that $|N_{G_n}[S]|$ is maximum among the subsets $W \subseteq V(G_n)$, $|W| = n-k$, where $k = 1,2,\dots,n-1$. By Theorem 2.8, $|N_{G_n}[S]| = 2n-k = n+(n-k) = n+|S|$. Since $|N_G[S]| = |H'|+|T'|$, it follows that $|H'| = n$ implying that $S\cap V(P_n)$ dominates P_n. For the converse, if $S\cap V(P_n)$ is a dominating set of P_n, it follows that $H'=n$. From the fact that $1\le |H'|\le n$ and with Remark 2.2, $|S|=|T'|$, implying further that $|N_{G_n}[S]|=|H'|+|T'|=n+|S|$ is maximum among the subsets $W\subseteq V(G_n)$ with $|W|=n-k$. Hence, S is a ζ_k-set of G_n. □

Corollary 2.10. *Let G_n be a centipede graph of order $2n$, with spine P_n, $n\ge 3$. Then,*

$$\zeta_k(G_n)=\begin{cases} k, & \text{if } k=n-\gamma(P_n),n-(\gamma(P_n)-1),\dots,n-1;\\ 4k-2n, & \text{if } k=1,2,\dots,n-\gamma(P_n).\end{cases}$$

Proof. Suppose $S\subseteq V(G_n)$ is a ζ_k-set of G_n. By definition, $\zeta_k(G_n)=2n-|N_{G_n}[S]|$. We consider the following two cases for the values of $|S|$:

Case 1. Suppose $|S|\in\{\gamma(P_n),\gamma(P_n)+1,\dots,n-1\}$. By Theorem 2.8, $|N_{G_n}[S]|=2n-k$. Hence, $\zeta_k(G_n)=2n-|N_{G_n}[S]|=2n-(2n-k)=k$.

Case 2. Suppose $|S|\in\{1,2,\dots,\gamma(H)-1\}$. By Theorem 2.8, $|N_{G_n}[S]|=4(n-k)$. Hence, $\zeta_k(G_n)=2n-|N_{G_n}[S]|=2n-4(n-k)=4k-2n$. □

The next theorem and corollary are derived in a similar fashion as that of Theorem 2.9 and Corollary 2.10, respectively.

Theorem 2.11. *Let S_n be a sunlet graph of order $2n$, with its cycle C_n, $n\ge 3$. Let $S\subseteq V(S_n)$ with $|S|=n-k$, where $k=1,2,\dots,n-1$, such that $|\{u_i,v_i\}\cap S|\le 1$, for all $i=1,2,\dots,n$. Then S is a ζ_k-set of S_n if and only if any of the following holds:*

(i) $|S|\in\{1,2,\dots,\gamma(C_n)-1\}$, *where $S\subseteq V(C_n)$ and $|N_{S_n}[S]|=4|S|$, for all $v\in S$;*

(ii) $|S|\in\{\gamma(C_n),\gamma(C_n)+1,\dots,n-1\}$, *where $S\cap V(C_n)$ is dominating set in C_n.*

Corollary 2.12. *Let S_n be a sunlet graph of order $2n$, with its cycle C_n, $n\ge 3$. Then*

$$\zeta_k(S_n)=\begin{cases} k, & \text{if } k=1,2,\dots,n-\gamma(S_n),\\ 4k-2n, & \text{if } k=n-\gamma(S_n),n-(\gamma(S_n)-1),\dots,n-1.\end{cases}$$

For our third and last group of special parameterized graphs, we consider the bi-star graph $B(r,s)$, crown graph $G_{n,n}$, and complete bipartite graph $K_{m,n}$.

Recall that the *bi-star* graph $B(r,s)$ for $r,s\ge 2$ is the graph obtained by joining thru an edge the centers of two stars $K_{1,r}$ and $K_{1,s}$. The *crown* graph $G_{n,n}$ is the graph on $2n$ vertices with two sets of vertices u_i and v_j where $u_iv_j\in E(G(n,n))$ whenever $i\neq j$, for $i,j=1,2,\dots,n$. On the other hand, a graph G is called *bipartite* if its vertex-set $V(G)$ can be partitioned into two nonempty subsets V_1 and V_2 such that every edge of G has one end in V_1 and one end in V_2. The sets V_1 and V_2 are called the *partite* sets of G. If, in addition, each vertex in V_1 is adjacent to every vertex in V_2, then G is called a *complete bipartite graph*. If $|V_1|=m$ and $|V_2|=n$, then the complete bipartite graph is denoted by $K_{m,n}$. Figure 3 provides skeletal diagrams of these graphs.

Note that these graphs have domination number $\gamma(B(r,s))=\gamma(G_{n,n})=\gamma(K_{m,n})=2$. If G is any of these graphs, then every ζ_1-set S of G is such that $|S|=\gamma(G)-k=1$.

With Lemma 1.1, the ζ_1-*sets* of the bi-star graph $B(r,s)$, crown graph $G_{n,n}$, and complete bipartite graph $K_{m,n}$ can easily be determined. Consequently, the corresponding 1-domination defect numbers of these graphs immediately follow. These are given in the theorems below whose proofs are omitted since they are straightforward.

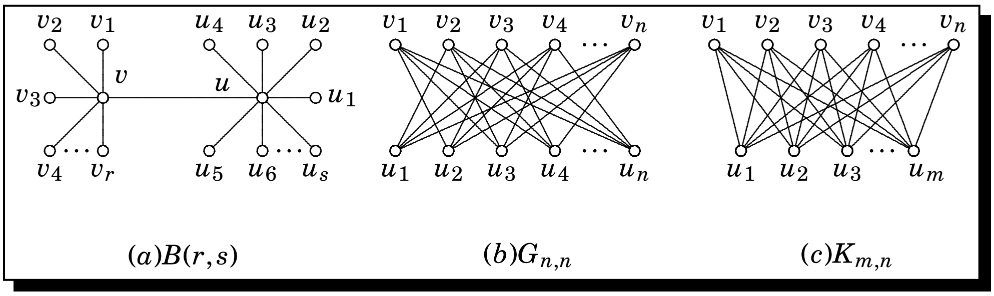

Figure 3. The bi-star $B(r,s)$, crown $G_{n,n}$, and complete bipartite $K_{m,n}$

Theorem 2.13. *The* 1*-domination-defect number of the bi-star graph* $B(r,s)$ *with* $r,s \geq 2$ *is given by* $\zeta_1(B(r,s)) = \min(r,s)$.

Theorem 2.14. *The* 1*-domination-defect number of the crown graph* $G_{n,n}$ *of order* $2n$ *with* $n \geq 2$ *is given by* $\zeta_1(G_{n,n}) = n$.

Theorem 2.15. *The* 1*-domination-defect number of the complete bipartite graph* $K_{m,n}$ *with partite sets* V_1 *of order* $m \geq 2$ *and* V_2 *of order* $n \geq 2$ *is given by* $\zeta_1(K_{m,n}) = \min(m-1, n-1)$.

3. Final Remarks

The concept of k-domination defect of a graph allows us to study the vulnerability of a facility if it would be guarded with fewer than the minimum number of necessary guards. In this paper, we produced characterizations of the k-domination defect sets of some known parameterized families of graphs such as path, cycle, centipede graph, sunlet graph, bi-star graph, crown graph, and complete bipartite graph. Further, the corresponding k-domination defects of said graphs were successfully obtained.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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