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Research Article

Forccheimer Flow of Williamson Nano Fluid Over a Stretching Sheet With Cattaneo-Christov Heat Flux in Saturated Porous Media

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Abstract. The present study deals with heat transmission of a Williamson nanofluid on a porous plate in a Darcy-Forccheimer flow through Cattaneo-Christov heat flux, velocity, temperature and concentration slips. The basic leading equations were converted by means of similarity transformations. Later, obtained equations were resolved by "Runge-Kutta-Felhberg Method". The velocity, temperature and the concentration profiles were driven clearly and discussed thoroughly. The values of Nusselt number and reduced Sherwood number were given in tabulated form.

Keywords. Forccheimer flow, Williamson nanofluid, Cattaneo-Christov heat flux, Porous medium

Mathematics Subject Classification (2020). 35A09, 65L10, 76M20

1. Introduction

Heat transfer is necessary in our day-to-day life and often not even recognized as we go about with our lives. These heat characteristics were explored by Fourier law [8] in 18th century, but this was not adequate for any initial troubles is felt promptly all over the entire material, to overawed this problem. Cattaneo [3] introduced thermal relaxation time in the traditional Fourier's of heat conduction. This model helps in transport of heat through circulation of thermal waves along through determinate speed.

In order to obtain the material invariant formulation Christov [5] changed the Catteneo law with thermal relaxation time together with Oldroyd's upper-convected derivatives Cattaneo-Christov model along with thermal convection was studied by Straughan [21]. Later, this term was used by many of the researchers [10, 11, 15].

The applications of heat transfer study on nanofluids have novel properties fuel cells pharmaceutical processes, hybrid-powered engines microelectronics fuel, and polymer dyes etc. When compared with base fluids the nano fluids exhibit the enhancement in coefficient of convective heat transfer along with thermal conductivity. Various experimental studies proved that the heat transfer coefficient become improved up to 40% and enhancement in thermal conductivity with a range of fifteen to forty percent when compared to the standard fluid. There must be some other mechanism that plays a vital role in increasing thermal conductivity viz., volume fraction, particle shape/surface area, nanoparticle size particle accumulation, liquid layering on the interface of the nanoparticle-liquid and dimensions fraction.

The updated cooling techniques are essential in industrial sectors majorally in transport, manufacture, electricity, and electronic industries etc. This technology is essentially required in the forthcoming world of thin-film solar energy collector devices. Metals have three times higher thermal conductivity properties when compared to the aimed fluids so it is advantages to cartel these two materials to produce a heat transmission middling that acts as a fluid but has metal thermal properties. The term nanofluid was first coined by Choi and Eastman [4] which indicates engineered colloids that compose nanoparticles disbursed in the base fluid. Masuda *et al*. [13] studied the thermal enhancement characteristic of nanofluids.

The investigation of flow in porous media attracts many researchers since its vast industrial applications. The Darcy's law was one of the popular model in flow in porous media. This law was recognised because it over predicts the convective flow when vorticity diffusion coefficient and inertial drag coefficient was considered. Extending this work, Forchhimer [7] included the square velocity factor. Later, Forchheimer term was introduced by Mustak and Wyckoff [14] and he concluded that this term is valid for high Renold's numbers also.

Forchheimer effect has importance in non-Newtonian like processing of ceramic and improving the oil repossession. Later, the Darcy-Forchheimer flow of free, forced and combined convection over non-isothermal structures of arbitrary shaped embedded in non-Newtonian power-law fluid-drenched porous medium has been investigated by Shenoy [20].

Saddeek [19] analyzed the Darcy-Forchheimer mixed convection flow of mass and heat transfer towards an isothermal vertical flat plate. He conducted his study in two cases, one is with viscous dissipation and the other is without it. Later, Younghae *et al*. [6] conducted a numerical investigation for Darcy-Forchheimer nanoliquid flows over an unsteady extending sheet with convective thermal boundary conditions and Navier's slip condition with spectral relaxation method. Pal and Mondal [17] conducted an numerical investigation on Darcy-Forchheimer porous medium with effects of convective diffusion, hydromagnetic, flexible viscosity filed and non-uniform heat source/sink. After this, the Darcy-Forchheimer flow of Maxwell fluid was studied by Sadiq and Hayat [18]. Due to the vast application porous medium over shrinking sheet, Bakar *et al*. [1, 2] did her investigation on the stagnation point flow of Darcy-Forchheimer porous medium past a shrinking sheet.

Later, this study was extended by many researchers by considering various effects. For example, Ganesh *et al*. [9] explored some important results on Darcy-Forchheimer flow.

Since the study on different fluid gives interesting results, Khan *et al*. [12] did their research on estimation of entropy optimization about the Darcy-Forchheimer flow of Carreau-Yasuda fluid.

Inspired by the above studies the present study was carried out to fulfill the gap of Forchheimer flow. Therefore, the study deals with the free convection flow and heat transmission of a Williamson nanofluid over a porous plate in a Darcy-Forccheimer flow with velocity, temperature, and concentration slips.

2. Mathematical Formulation

A steady two dimensional (x, y) boundary layer flow of a Williamson nanofluid over linearly stretching sheet with stretching velocity $u_w = ax$. The flow was imperiled to the effect of magnetic field of strength B_0 which is applied perpendicular to the sheet for which we considered.

The Williamson fluid model can be obtained Nadeem *et al*. [16]. For the fluid prototypical, Cauchy stress tensor S is given as

$$S = -pI + \tau,$$

$$\tau = \left(\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma\gamma}\right) A_1,$$

where p is the pressure term τ is the extra stress tensor, μ_0, I is the identity vector, the preventive viscosities at zero and immeasurable shear rates was denoted by μ_∞, A_1 is the first Rivlin-Erickson tensor, $\Gamma > 0$ is a time constant and γ is distinct as

$$\gamma = \sqrt{\frac{\pi}{2}}, \quad \pi = trace(A_1^2).$$

Herein π is the second invariant strain tensor. For this problem we have considered only the case for which

$$\mu_\infty = 0 \text{ and } < 1.$$

Therefore, we obtain

$$\tau = \left(\frac{\mu_0}{1 - \Gamma\gamma}\right) A_1 \text{ or } \tau = \mu_0(1 + \Gamma\gamma) A_1.$$

By observing the above, the above given governing equations over a stretching sheet was expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}u - \frac{\sigma B^2}{\rho}u - F_r u^2, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \lambda\Big(u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x} + v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x} + u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y} + v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y} + 2uv\frac{\partial^2 T}{\partial x \partial y} + u\frac{\partial^2 T}{\partial x^2} + v\frac{\partial^2 T}{\partial y^2}\Big) + \tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2, \tag{4}$$

where u,v represents the velocity components in x,y direction, respectively. The variables x and y indicate the Cartesian coordinates along and normal to the sheet. Similarly, T is temperature, C is nanoparticle volumetric fraction, k is nanofluid thermal conductivity, D_B is Brownian diffusion coefficient. D_T is thermophoretic diffusion coefficient and T_∞ is the ambient fluid temperature, ν is the kinematic viscosity, $\Gamma>0$ is the characteristic time, ρ was viscosity, α is the thermal diffusivity of the fluid, C_p the specific heat, $Fr=\frac{cb}{xk^{\frac{1}{2}}}$ be the inertia coefficient of porous medium and the permeability of the porous medium was denoted by K.

With respected boundary conditions are given by

$$\left.\begin{array}{ll} u=u_w,\ \ v=0,\ \ T=T_w,\ \ C=C_w & \text{at}\ \ y=0, \\ U\to U_\infty,\ \ T\to T_\infty,\ \ C\to C_\infty & \text{as}\ \ y\to\infty. \end{array}\right\} \tag{5}$$

The following are the similarity transformations used in the problem

$$\eta=y\left(\frac{a}{\nu}\right)^{\frac{1}{2}}\psi=(a\nu)^{\frac{1}{2}}xf(\eta)\ \text{ and }\ u=\partial\psi/\partial y\ \text{ and }\ u=-\partial\psi/\partial x$$

$$u=\ axf'(\eta),\ \ v=-(a\nu)^{\frac{1}{2}}f(\eta),$$

$$f'''+ff''-f'^2-Kf'+We f''f'''-Mf'-Fr(f')^2=0, \tag{6}$$

$$\frac{1}{Pr}\theta''+\gamma(f^2\theta''-ff'\theta')+f\theta'+Nb\phi'\theta'+Nt(\theta')^2=0, \tag{7}$$

$$\phi''+Le f\phi'+\frac{Nt}{Nb}\theta''=0. \tag{8}$$

Wessienberg number $We=\sqrt{2\Gamma}\frac{a^{\frac{3}{2}}x}{\vartheta}$, Magnetic parameter $M=\frac{\Gamma\beta_0{}^2}{\delta a}$,

Forchhiemer flow parameter $Fr=\frac{c\vartheta}{xk^{\frac{1}{2}}}$, Porosity parameter $K=\frac{\vartheta}{ka}$,

Prandtl number $Pr=\frac{\alpha}{\vartheta}$, Brownian motion parameter $Nb=\frac{\rho_p C_p D_B(C_w-C_\infty)}{Sc_f\nu}$,

Thermophoresis parameter $Nt=\frac{D_T(T_w-T_\infty)}{T_\infty\nu}$, Lewis number $Le=\frac{\nu}{D_B}$.

The parameters k, λ, M, Fr indicates the porosity parameter, Williamson parameter, Magnetic parameter, and Forchheimer flow parameters, respectively, and similarly Pr, γ, Nb, Nt, Le represents the Prandtl number, thermal relaxation parameter, Bromian motion parameter, thermophoresis parameter and Lu is number, respectively.

3. Numerical Discussion

The partial differential equations were transfigured into ODE later these equations were solved by using Runge-Kutta-Felberg method. To verify the methodology comparison was done with the previous results. The values of $-f''(0)$, $-\theta'(0)$, $\phi'(0)$ for several values of the constraints were presented in the tables.

Figure 1 depicts the properties of magnetic restriction on velocity profile. It was very clear that the velocity was decreased with the rise of magnetic parameter M. The electrically conducting fluid introduces the Lorentz force which drags the flow if the magnetic field is applied perpendicular to the sheet.

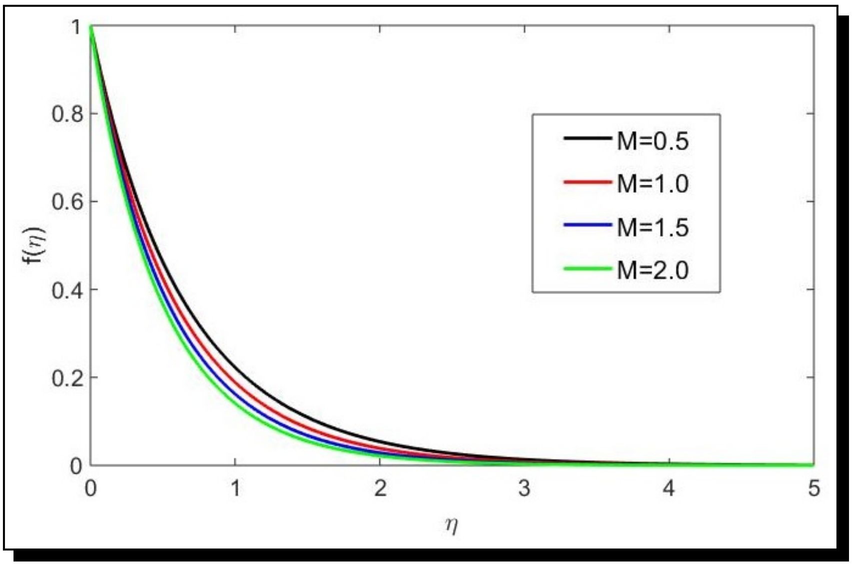

Figure 1. $f'(\eta)$ for various estimates of M (magnetic parameter)

Figure 2 indicates the effect of inertia parameter on velocity profile. The graphs indicates that effects of Forchheimer parameter Fr, i.e., inertial force on fluid velocity. From the figure, the increment of Forchheimer parameter decreases the fluid momentum. Physically, this is because of the stronger resistance power diminishes the velocity of the flow and also the edge layer thickness of flow momentum.

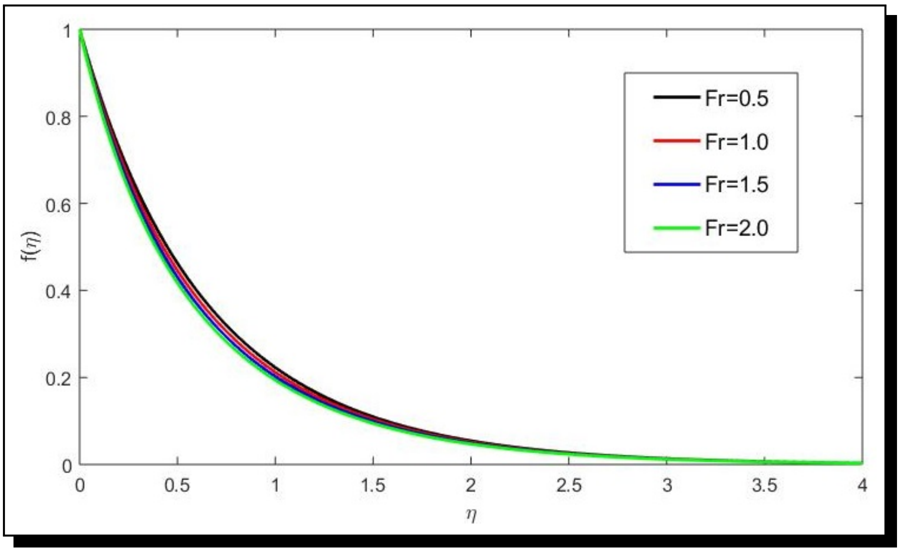

Figure 2. $f'(\eta)$ for various estimates of Fr (inertia parameter)

Figure 3 portray the impact of porous parameter on flow momentum, it is clear from the graph that the porosity parameter decreases the flow momentum this is because of the frictional forces that resists the flow fluid.

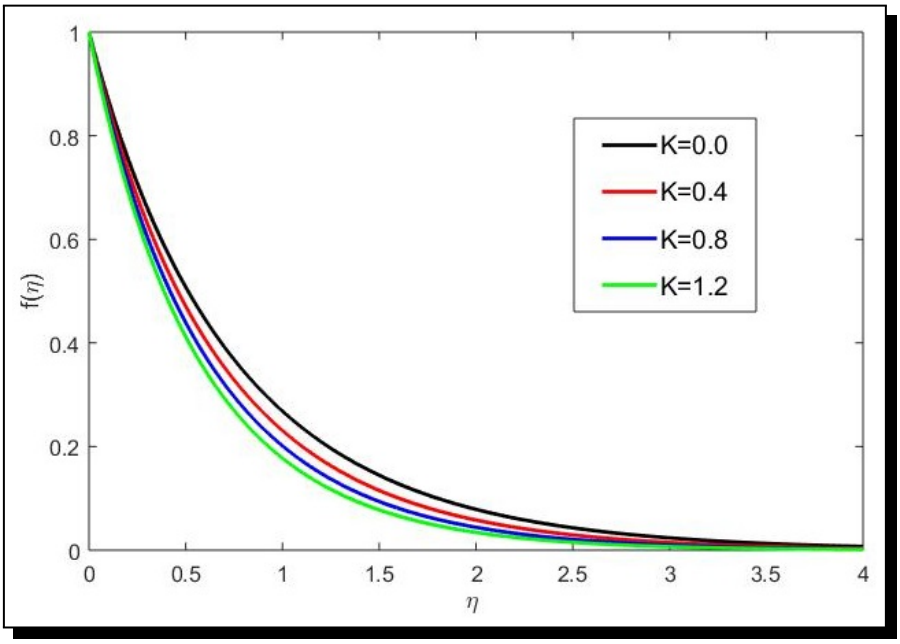

Figure 3. $f'(\eta)$ for various estimates of K (thermal relaxation parameter)

The progressive values of Weissenberg number on velocity profile was depicted on Figure 4. This is because of that relaxation time of the fluid increases for complex results of Weissenberg number *We* causes the decrement in velocity.

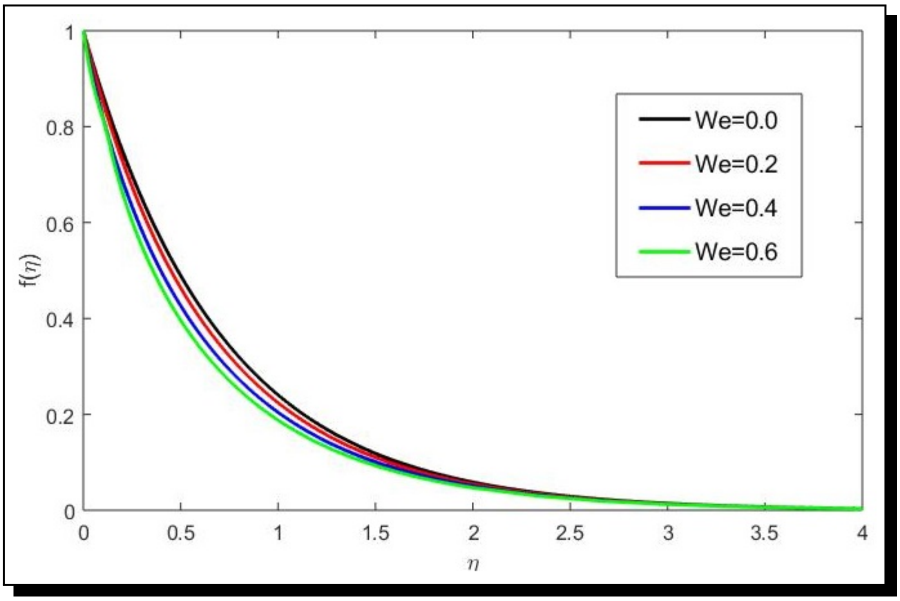

Figure 4. $f'(\eta)$ for various estimates of λ (Weissenberg number)

Figure 5 indicates the effects of Brownian motion parameter *Nb* on temperature profiles the hike of *Nb* up rises the temperature and increases the thermal frontier layer thickness. On the other hand the same result found on temperature profile i.e., in the case of Thermophoresis parameter *Nt* which was clearly presented in Figure 6.

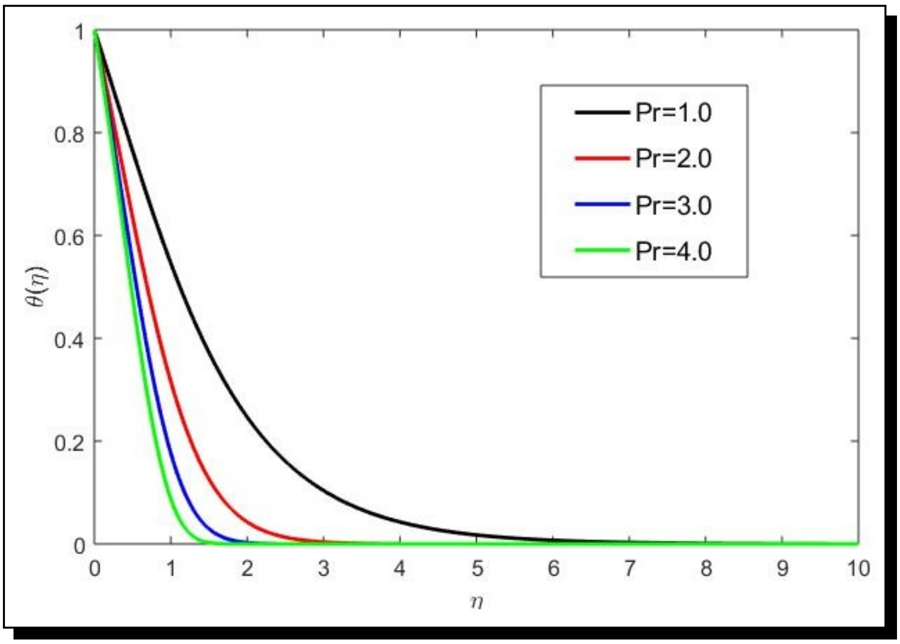

Figure 5. $\theta(\eta)$ for various estimates of *Pr* (Prandtl number)

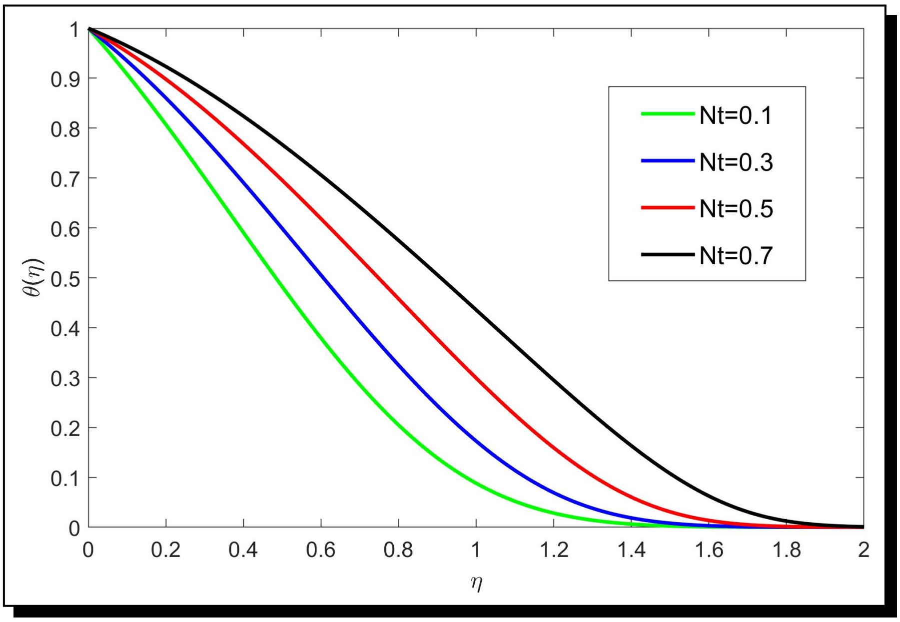

Figure 6. $\theta(\eta)$ for various estimates of *Nt* (Brownian motion number)

Figure 7 explores the actions of Prandtl number on temperature profile $\theta(\eta)$. It was noted that the augmentation of Prandtl number decreases the temperature profile. It is due to the Prandtl number being the percentage of kinematic viscosity to thermal diffusivity. Increment of *Pr* means the lower thermal diffusivity, therefore whenever the thermal diffusivity is low the temperature outline and the thermal edge layer depth decreases.

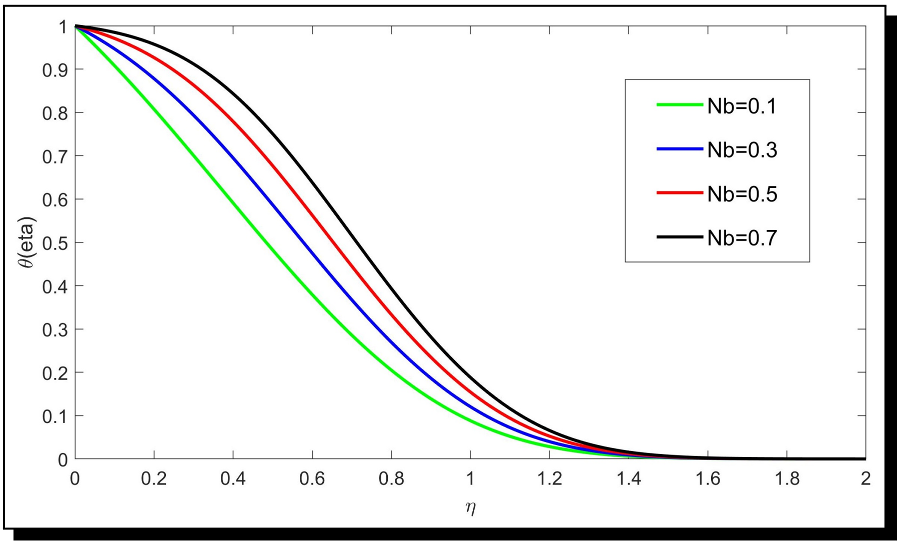

Figure 7. $\theta(\eta)$ for various estimates of Nb (thermophoresis)

In Figure 8 it was noticed that the hike in thermal relaxation parameter γ decrease both the temperature profile and the thermal edge layer thickness.

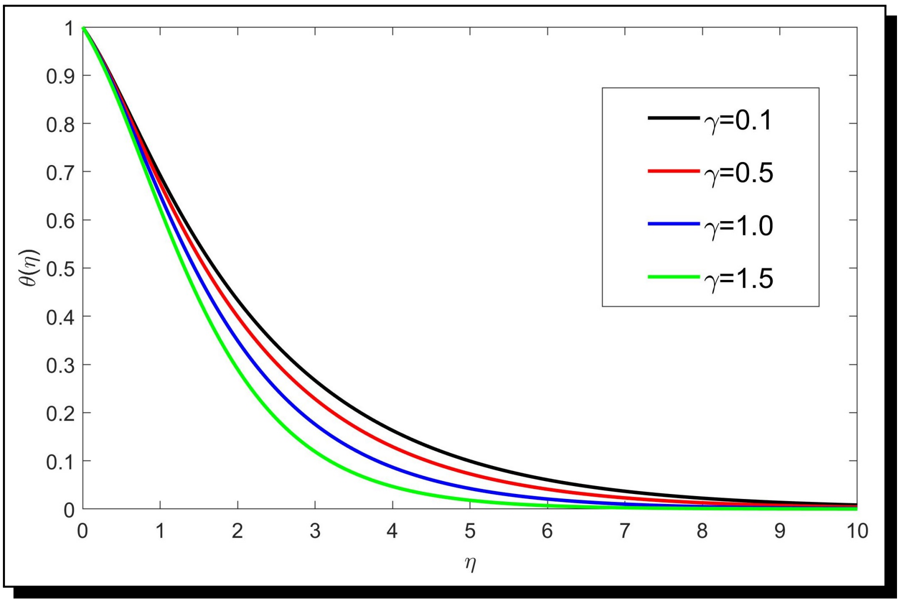

Figure 8. $\theta(\eta)$ for various estimates of γ (relaxation parameter)

The properties of Lewis number on concentration profile were depicted on Figure 9. Generally, the Brownian diffusion coefficient varies inversely with respect to Lewis number. Therefore, Brownian diffusion coefficient converts smaller with respect to the larger values of Lewis number. The small amount of Brownian diffusion coefficient depicts the decrement in the absorption profile and thickness of edge layer too.

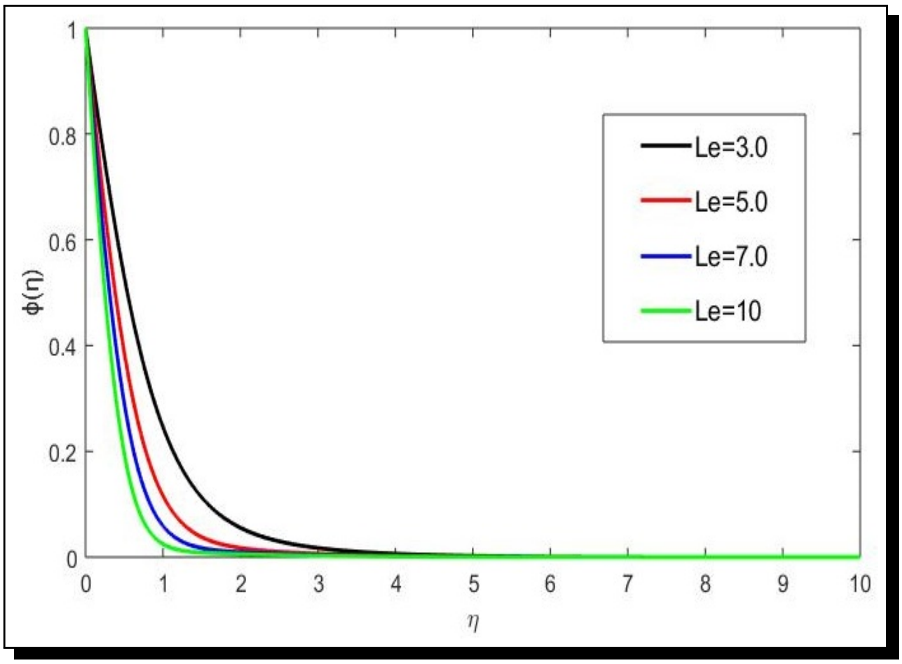

Figure 9. $\phi(\eta)$ for various estimates of Le (Lewis number)

Figure 10 and Figure 11 indicates the effects of Brownian motion parameter Nb and thermophoresis parameter Nt on the concentration profile. It was noticed that increase in Nb decreases the concentration profile whereas an increment in Nt gives the increment in concentration profile as well as concentration edge layer also decreases.

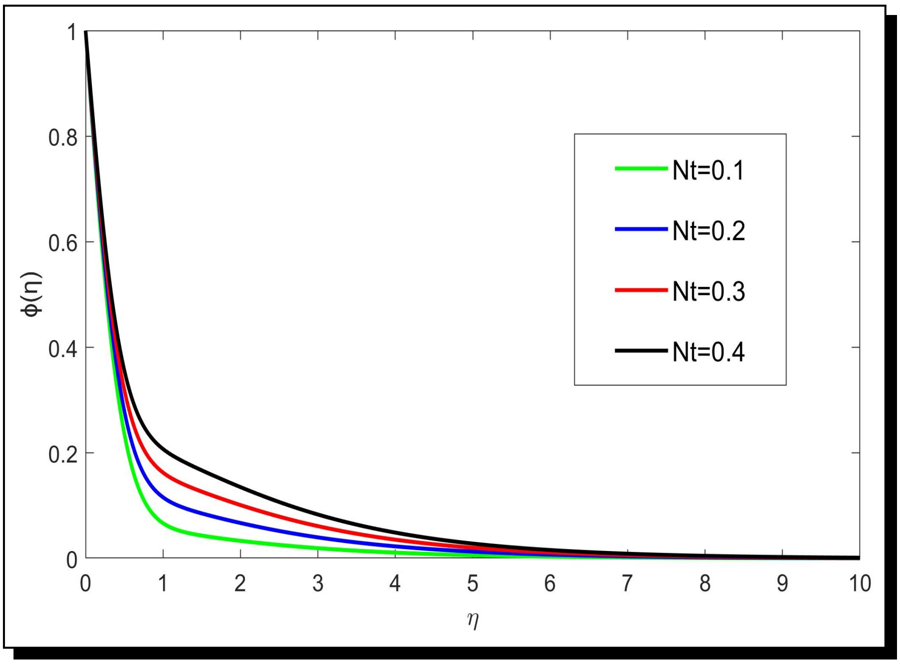

Figure 10. $\phi(\eta)$ for various estimates of Nt (Brownian motion number)

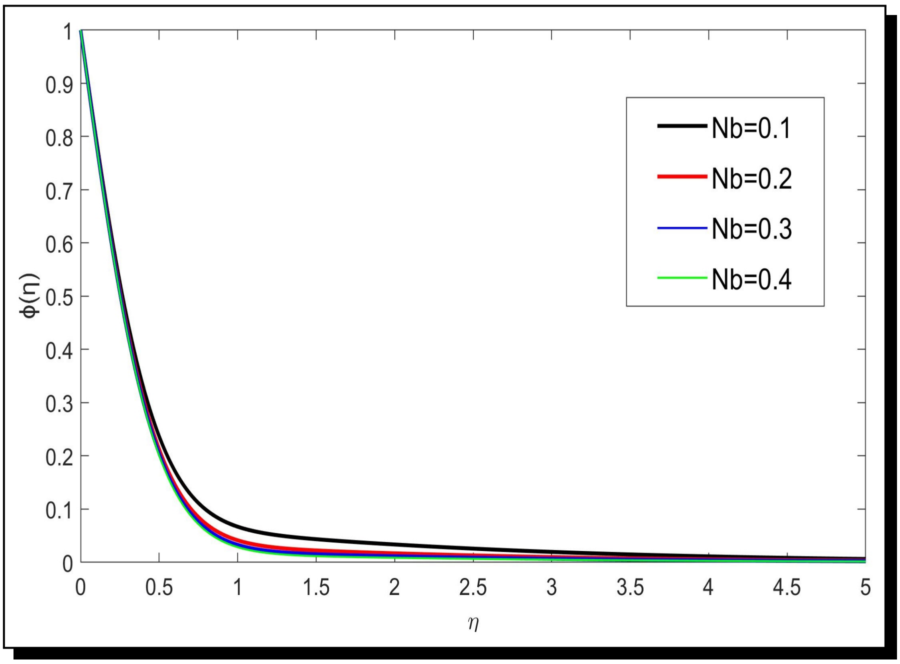

Figure 11. $\phi(\eta)$ for various estimates of Nb (thermophoresis)

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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