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Research Article

On Common Fixed Point Theorems for Hybrid Mappings Satisfying Contractive Condition of Higher Degree and (CLR) Properties

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Abstract. In this study, we adopt an inequality of higher degree to acquire common fixed points for single and multi valued mappings with the help of (CLR) and (JCLR) properties. All of our results are strengthened with suitable examples.

Keywords. Hybrid pair, Coincidence point, Common fixed point, (CLR) property, (JCLR) property, P-weakly commuting

Mathematics Subject Classification (2020). 47H10, 54H25

1. Introduction

We use the following notations, properties and definitions throughout this paper.

(i) X is a metric space with metric d.

(ii) $CL(X) = \{A : A$ is non-empty closed subset of $X\}$.

(iii) For $x \in X$ and $A \subseteq X$, $d(x,A) = \inf_{y \in A} d(x,y)$.

(iv) For every $A,B \in CL(X)$,

$$H(A,B) = \begin{cases} \max\left\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\right\} & \text{if the maximum exists,} \\ \infty & \text{otherwise.} \end{cases}$$

Note that H is a metric on $CL(X)$, which is known as Pompeiu-Hausdorff metric.

(v) If $x \in A$, then $d(x,B) \le H(A,B)$.

(vi) If $x \in A$, then $d(x,A) = 0$.

(vii) If $A \subseteq B$, then $d(x,B) \le d(x,A)$.

(viii) If $y \in A$, then $d(x,A) \le d(x,y)$.

(ix) If h is a self map of X and P be a multi valued map from X to $CL(X)$, then

- the pair (h,P) is known as a hybrid pair.
- a point x in X is called a fixed point of P, if $x \in Px$.
- a point x in X is called a coincidence point of h and P, if $hx \in Px$.
  We write $C(h,P) = \{x : hx \in Px\}$.
- A point x in X is called a common fixed point of h and P, if $hx = x \in Px$.

Sessa [16] introduced the weak commutative notion of two mappings. Jungck [9] discovered the compatibility of two maps and proved that if two maps are weakly commuting, then they are compatible, but the compatibility need not imply commutativity. Sastry and Krishnamurthy [15] introduced tangential property, which is rediscovered by Aamri and Moutawakil [1] and named as (E.A) property. The class of maps that satisfy (E.A) property has remarkable results because it consists both compatible and non-compatible mappings. After that, Kamran [12] expanded the property (E.A) to a hybrid pair. Jungck [10] proposed the property of weak compatibility of self maps and further Jungck and Rhoades [11] extended it to hybrid pairs. The property of occasional weak compatibility is proposed by Al. Thagafi and Shahzad [3], which is further extended to a hybrid pair by Abbas and Rhoades[2].

As a generalization of weak compatibility, Singh and Mishra [17] proposed the concept of (IT)-commutativity to a hybrid pair. On the other hand, Kamran [12] introduced T-weakly commuting property and proved that (IT)-commutative property at the coincidence point implies T-weakly commuting but the other way not around.

The idea of Common Limit Range (CLR) property for single-valued mappings was proposed by Sintunavarat and Kumam [18], which does not need the underlying maps' range to be closed. Imdad *et al*. [7] later expanded this property to hybrid pairs and derived some fixed point outcomes in symmetric spaces. Further, the joint common limit range property was extended to hybrid pairs by Imdad *et al*. [6] which was proposed by Sintunvarat *et al*. [5] for a pair of self maps and proved that by applying (JCLR) property the containment condition between ranges of underlying maps can be removed, in order to acquire fixed points.

Before we get to our main results, we will go through certain definitions that will be used later. P and Q are mappings from X to $CL(X)$, where as h and k are sellf maps of X throughout the definitions.

Definition 1.1. The hybrid pair (h,P) is said to

(i) be compatible [13], if $hPx \in CL(X)$ for all x in X and $\lim_{n\to\infty} H(hPx_n, Phx_n) = 0$ for every sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} hx_n = t \in A = \lim_{n\to\infty} Px_n$.

(ii) satisfy (E.A) property [12], if there exists a sequence $\{x_n\}$ in X with $\lim_{n\to\infty} hx_n = t \in A = \lim_{n\to\infty} Px_n$ for some $t \in X$ and $A \in CL(X)$.

(iii) be weakly compatible [11], if $x \in X, hx \in Px$ implies $hPx = Phx$.

(iv) be (I.T)-commuting [8, 17] at $x \in X$, if $hPx \subseteq Phx$.

(v) be occasionally weakly compatible [2], if $hPx \subset Phx$ for some point $x \in X$ such that $hx \in Px$.

(vi) satisfy commom limit in the range of h (CLR_h)property [7], if we can find a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} hx_n = hz \in A = \lim_{n\to\infty} Px_n$ where $z \in X$ and $A \in CL(X)$.

Definition 1.2. The map h is said to be P-weakly commuting[12] at $x \in X$ if $hhx \in Phx$.

Definition 1.3 ([6])**.** The hybrid pairs (h,P) and (k,Q) are said to satisfy joint common limit in the range of h and k $(JCLR_{hk})$ property if we can find two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = A, \quad \lim_{n\to\infty} Qy_n = B$$

and

$$\lim_{n\to\infty} hx_n = \lim_{n\to\infty} ky_n = t \in A \cap B \cap h(X) \cap k(X).$$

In 2011, Babu and Alemuyehu [4] proved some fixed point outcomes for self mappings that satisfy occasional weak compatibility property together with (E.A) property/common property (E.A). In their results they used an inequality involving quadratic terms. Later in 2015, M.Samreen et al [14] extended these results to hybrid pairs. In this paper we generalize these results using an inequality of higher degree. We also apply (CLR)/(JCLR) properties together with P-weakly commuting for this purpose.

2. Main Results

We begin with the following proposition, in which we apply (CLR) property.

Proposition 2.1. *Let h, k be mappings from a metric space X to itself, and P, Q be mappings from X to $CL(X)$ such that*

$$\begin{aligned}[H(Px,Qy)]^p \le c_1 \max\{&[d(hx,Px)]^p, [d(ky,Qy)]^p, [d(hx,ky)]^p\} \\ &+ c_2 \max\{[d(hx,Px)]^r [d(hx,Qy)]^s, [d(ky,Qy)]^r [d(ky,Px)]^s\} \\ &+ c_3 [d(hx,Qy)]^r [d(ky,Px)]^s \end{aligned} \tag{2.1.1}$$

for all x,y in X, where $c_1,c_2,c_3 \ge 0$, $c_1 < 1$ and $p,r,s \in Z^+$, $p = r+s$. Suppose that either

(i) *The pair (h,P) satisfy (CLR_h) property and $\bigcup_{x\in X} Px \subseteq k(X)$; or*

(ii) *The pair (k,Q) satisfy (CLR_k) property and $\bigcup_{x\in X} Qx \subseteq h(X)$.*

Then, the maps h and P have a coincidence point u and the maps k and Q have a coincidence point v.

Proof. Let us assume that (i) holds.

Then there must be a sequence $\{x_n\}$ in X and some $A \in CL(X)$, $u \in X$ such that

$$\lim_{n\to\infty} hx_n = hu \in A = \lim_{n\to\infty} Px_n. \tag{2.1.2}$$

Since $\bigcup_{x\in X} Px \subseteq k(X)$, we have $Px_n \subseteq k(X)$ for all n.

This implies $d(hu,k(X)) \le d(hu,Px_n)$.

Now by the definition of Hausdorff metric and since $hu \in A$, we have

$$d(hu,k(X)) \le d(hu,Px_n) \le H(A,Px_n) \quad \text{for all } n.$$

On letting $n \to \infty$,

$$d(hu,k(X)) \le \lim_{n\to\infty} d(hu,Px_n) \le \lim_{n\to\infty} H(A,Px_n) = 0.$$

This implies $hu \in \overline{k(X)}$.

Therefore,

$$\lim_{n\to\infty} ky_n = hu \ \text{ for some sequence } \{y_n\} \text{ in } X. \tag{2.1.3}$$

Now, we prove that $\lim_{n\to\infty} Qy_n = A$.

For this, we put $x = x_n$, $y = y_n$ in (2.1.1).

Then,

$$\begin{aligned}[H(Px_n,Qy_n)]^p \le c_1 &\max\{[d(hx_n,Px_n)]^p,[d(ky_n,Qy_n)]^p,[d(hx_n,ky_n)]^p\}\\ &+ c_2 \max\{[d(hx_n,Px_n)]^r[d(hx_n,Qy_n)]^s,[d(ky_n,Qy_n)]^r[d(ky_n,Px_n)]^s\}\\ &+ c_3[d(hx_n,Qy_n)]^r[d(ky_n,Px_n)]^s.\end{aligned}$$

On taking limit superior and using (2.1.2) and (2.1.3), we get

$$\limsup_{n\to\infty}[H(A,Qy_n)]^p \le c_1 \limsup_{n\to\infty}[d(hu,Qy_n)]^p \le c_1 \limsup_{n\to\infty}[H(A,Qy_n)]^p.$$

Since $c_1 < 1$, we will have $\lim_{n\to\infty}[H(A,Qy_n)]^p = 0$, which implies

$$\lim_{n\to\infty} Qy_n = A. \tag{2.1.4}$$

Now we show that $hu \in Pu$.

To do this, we take $x = u$, $y = y_n$ in (2.1.1).

Then,

$$\begin{aligned}[H(Pu,Qy_n)]^p \le c_1 &\max\{[d(hu,Pu)]^p,[d(ky_n,Qy_n)]^p,[d(hu,ky_n)]^p\}\\ &+ c_2 \max\{[d(hu,Pu)]^r[d(hu,Qy_n)]^s,[d(ky_n,Qy_n)]^r[d(ky_n,Pu)]^s\}\\ &+ c_3[d(hu,Qy_n)]^r[d(ky_n,Pu)]^s.\end{aligned}$$

On letting $n \to \infty$ and using (2.1.3) and (2.1.4), $[H(Pu,A)]^p \le c_1[d(hu,Pu)]^p$.

Since $hu \in A$, by the definition of Hausdorff metric,

$$d[hu,Pu)]^p \le [H(A,Pu)]^p \le c_1[d(hu,Pu)]^p.$$

This implies $d(hu,Pu) = 0$, since $c_1 < 1$.

Hence,

$$hu \in \overline{Pu} = Pu, \quad \text{since } Pu \text{ is closed.} \tag{2.1.5}$$

This implies that the hybrid pair (h,P) has a coincidence point u.
Since $\bigcup_{x\in X} Px \subseteq k(X)$, we have $hu \in k(X)$.
This implies

$$hu = kv \quad \text{for some } v \in X. \tag{2.1.6}$$

Now we will show that $kv \in Qv$.
This can be done by putting $x = u$, $y = v$ in (2.1.1).
Thus

$$\begin{aligned}[H(Pu,Qv)]^p \le\ & c_1 \max\{[d(hu,Pu)]^p,[d(kv,Qv)]^p,[d(hu,kv)]^p\} \\ & + c_2 \max\{[d(hu,Pu)]^r[d(hu,Qv)]^s,[d(kv,Qv)]^r[d(kv,Pu)]^s\} \\ & + c_3[d(hu,Qv)]^r[d(kv,Pu)]^s.\end{aligned}$$

On using (2.1.5) and (2.1.6), $[H(Pu,Qv)]^p \le c_1[d(kv,Qv)]^p$.
Then $[d(kv,Qv)]^p \le [H(Pu,Qv)]^p \le c_1[d(kv,Qv)]^p$.
Since $c_1 < 1$, we have $kv \in \overline{Qv} = Qv$, since Qv is closed.
Thus, the pair (k,Q) has a coincidence point v.
In similar manner, the proof follows under assumption (ii). □

Theorem 2.2. *If all the conditions of Proposition* 2.1 *on h, k, P and Q hold and in addition to that*

(i) *If h is P-weakly commuting at u and $hhu = hu$ then h and P have a common fixed point.*

(ii) *If k is Q-weakly commuting at v and $kkv = kv$ then k and Q have a common fixed point.*

(iii) *If both* (i) *and* (ii) *hold, then h, k, P and Q have a common fixed point.*

Proof. By (i), $hhu = hu$ and h is P-weakly commuting at u.
This implies $hu = hhu \in Phu$. Hence $z = hz \in Pz$, where $z = hu$.
Thus z is the common fixed point of h and P.
By (ii), $kkv = kv$ and k is Q-weakly commuting at v.
This imples $kv = kkv \in Qkv$. Hence $w = kw \in Qw$, where $w = kv$.
Thus w is the common fixed point of k and Q.
If both (i) and (ii) hold, (iii) immediately follows, since $hu = kv = z = w$. □

Example 2.3. The above result is backed up by this example.
Let (X,d) be a metric space with usual metric where $X = [0,1)$.
Let $h,k : X \to X$ and $P,Q : X \to CL(X)$ be defined by

$$h(x) = \begin{cases} \frac{x}{2}, & \text{if } x \in \left[0,\frac{1}{2}\right), \\ \frac{1}{4}+\frac{x}{2}, & \text{if } x \in \left[\frac{1}{2},1\right), \end{cases}$$

$$k(x) = x \quad \text{for all } x \in [0,1),$$

$$P(x)=\begin{cases}\{\frac{3}{4}\}, & \text{if } x\in\left[0,\frac{1}{2}\right),\\ \left[\frac{1}{2},\frac{2}{3}\right], & \text{if } x\in\left[\frac{1}{2},1\right),\end{cases}$$

$$Q(x)=\left[\frac{1}{2},\frac{2}{3}\right] \quad \text{for all } x\in[0,1).$$

Case I: If $x\in\left[0,\frac{1}{2}\right)$, then

$$H(Px,Qy)=\frac{1}{4},\ \ d(hx,Px)=\left|\frac{x}{2}-\frac{3}{4}\right|=\frac{(3-2x)}{4}\in\left(\frac{1}{2},\frac{3}{4}\right].$$

$$[H(Px,Qy)]^p=\left(\frac{1}{4}\right)^p<\frac{9}{10}\left(\frac{1}{2}\right)^p<\frac{9}{10}[d(hx,Px)]^p$$
$$<\frac{9}{10}\max\{[d(hx,Px)]^p,[d(ky,Qy)]^p,[d(hx,ky)]^p\}.$$

Hence inequality (2.1.1) holds for $c_1=\frac{9}{10}<1$, $c_2,c_3\ge 0$ and $p=r+s>1$.

Case II: If $x\in\left[\frac{1}{2},1\right)$, then $H(Px,Qy)=0$.
Hence inequality (2.1.1) holds for every $c_1,c_2,c_3\ge 0$, $c_1<1$ and $p=r+s>1$.
Thus inequality (2.1.1) holds in both the cases.
Clearly,

$$\bigcup_{x\in X}Px=\left[\frac{1}{2},\frac{2}{3}\right]\cup\left\{\frac{3}{4}\right\}\subseteq[0,1)=k(X).$$

We observe that neither $h(X)$ nor $k(X)$ are closed.
For the sequence $x_n=\dfrac{1}{2}+\dfrac{1}{4n}$, $n=1,2,3,\dots$ in X,

$$\lim_{n\to\infty}hx_n=\lim_{n\to\infty}\left\{\frac{1}{4}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{4n}\right)\right\}=\frac{1}{2}=h\left(\frac{1}{2}\right)\in\left[\frac{1}{2},\frac{2}{3}\right]=\lim_{n\to\infty}Px_n.$$

Then the pair (h,P) satisfy (CLR_h) property.
Here $\frac{1}{2}\in C(h,P)$ and $\frac{1}{2}\in C(k,Q)$.
Also,

$$hh\left(\frac{1}{2}\right)=\frac{1}{2}\in\left[\frac{1}{2},\frac{2}{3}\right]=Ph\left(\frac{1}{2}\right)\ \text{ and }\ hh\left(\frac{1}{2}\right)=h\left(\frac{1}{2}\right)$$

and

$$kk\left(\frac{1}{2}\right)=\frac{1}{2}\in\left[\frac{1}{2},\frac{2}{3}\right]=Qk\left(\frac{1}{2}\right)\ \text{ and }\ kk\left(\frac{1}{2}\right)=k\left(\frac{1}{2}\right).$$

Thus h is P-weakly commuting and k is Q-weakly commuting.
All the required conditions of Theorem 2.2 hold.
It can be noted that '$\frac{1}{2}$' is the common fixed point of h,k,P and Q, since

$$h\left(\frac{1}{2}\right)=k\left(\frac{1}{2}\right)=\frac{1}{2}\in P\left(\frac{1}{2}\right)\cap Q\left(\frac{1}{2}\right).$$

In the following proposition, we use $(JCLR)$ property so that containment conditions can be removed.

Proposition 2.4. *Let h, k be mappings from a metric space X to itself, and P, Q be mappings from X to $CL(X)$ such that*

$$\begin{aligned}[H(Px,Qy)]^p \le c_1 \max\{[d(hx,Px)]^2,[d(ky,Qy)]^p,[d(hx,ky)]^p\} \\ + c_2 \max\{[d(hx,Px)]^r[d(hx,Qy)]^s,[d(ky,Qy)]^r[d(ky,Px)]^s\} \\ + c_3[d(hx,Qy)]^r[d(ky,Px)]^s \end{aligned} \tag{2.4.1}$$

for all x,y in X, where $c_1,c_2,c_3 \ge 0$, $c_1 < 1$ and $p,r,s \in Z^+$, $p = r+s$.
Suppose that the pairs (h,P) and (k,Q) satisfy $(JCLR_{hk})$ property.
Then the maps h and P have a coincidence point u and the maps k and Q have a coincidence point v.

Proof. Since the pairs (h,P) and (k,Q) satisfy $(JCLR_{hk})$ property, there exist sequences $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} Px_n = A$, $\lim_{n\to\infty} Qy_n = B$,

$$\lim_{n\to\infty} hx_n = \lim_{n\to\infty} ky_n = t \in A \cap B \cap h(X) \cap k(X) \tag{2.4.2}$$

Hence

$$t = hu = kv \quad \text{for some } u,v \in X. \tag{2.4.3}$$

Now, we will show that $hu \in Pu$.
This can be done by taking $x = u, y = y_n$ in (2.4.1).
Then,

$$\begin{aligned}[H(Pu,Qy_n)]^p \le c_1 \max\{[d(hu,Pu)]^p,[d(ky_n,Qy_n)]^p,[d(hu,ky_n)]^p\} \\ + c_2 \max\{[d(hu,Pu)]^r[d(hu,Qy_n)]^s,[d(ky_n,Qy_n)]^r[d(ky_n,Pu)]^s\} \\ + c_3[d(hu,Qy_n)]^r[d(ky_n,Pu)]^s. \end{aligned}$$

On letting $n \to \infty$ and using (2.4.2) and (2.4.3), $[H(Pu,B)]^p \le c_1[d(hu,Pu)]^p$.
Since $hu = t \in B$, $[d(hu,Pu)]^p \le [H(B,Pu)]^p \le c_1[d(hu,Pu)]^p$.
Since $c_1 < 1$, it follows that $d(hu,Pu) = 0$, which implies

$$hu \in \overline{Pu} = Pu, \text{ as } Pu \text{ is closed.} \tag{2.4.4}$$

Therefore the pair (h,P) has the coincidence point u.
Now, we prove that $kv \in Qv$.
For this purpose, we take $x = u$ and $y = v$ in (2.4.1). Then,

$$\begin{aligned}[H(Pu,Qv)]^p \le c_1 \max\{[d(hu,Pu)]^p,[d(kv,Qv)]^p,[d(hu,kv)]^p\} \\ + c_2 \max\{[d(hu,Pu)]^r[d(hu,Qv)]^s,[d(kv,Qv)]^r[d(kv,Pu)]^s\} \\ + c_3[d(hu,Qv)]^r[d(kv,Pu)]^s. \end{aligned}$$

On using (2.4.3) and (2.4.4), we have $[H(Pu,Qv)]^p \le c_1[d(kv,Qv)]^p$.
Since $t = kv = hu \in Pu$, we have

$$[d(kv,Qv)]^p \le [H(Pu,Qv)]^p \le c_1[d(kv,Qv)]^p.$$

Since $c_1 < 1$, it follows that $d(kv,Qv) = 0$, which implies $kv \in \overline{Qv} = Qv$, as Qv is closed.
Hence the the pair (k,Q) has the the coincidence point v. □

Theorem 2.5. *If all the conditions of Proposition* 2.4 *on* h,k,P *and* Q *hold and in addition to that*

(i) *If* h *is* P*-weakly commuting at* u *and* $hhu = hu$ *then* h *and* P *have a common fixed point.*

(ii) *If* k *is* Q*-weakly commuting at* v *and* $kkv = kv$ *then* k *and* Q *have a common fixed point.*

(iii) *If both* (i) *and* (ii) *hold, then* h,k,P *and* Q *have a common fixed point.*

Proof. The proof follows in the same lines of Theorem 2.2. □

Example 2.6. Let (X,d) be a metric space with usual metric where $X = [0,1)$.
Let $h,k : X \to X$ and $P,Q : X \to CB(X)$ be defined by

$$h(x) = \begin{cases} \frac{x}{2}, & \text{if } x \in \left[0,\frac{1}{2}\right), \\ \frac{3}{4} - \frac{x}{2}, & \text{if } x \in \left[\frac{1}{2},1\right), \end{cases}$$

$$k(x) = \begin{cases} \frac{x}{3}, & \text{if } x \in \left[0,\frac{1}{2}\right), \\ \frac{1}{4} + \frac{x}{2}, & \text{if } x \in \left[\frac{1}{2},1\right), \end{cases}$$

$$P(x) = \begin{cases} \left\{\frac{3}{4}\right\}, & \text{if } x \in \left[0,\frac{1}{2}\right), \\ \left[\frac{1}{2},\frac{2}{3}\right], & \text{if } x \in \left[\frac{1}{2},1\right), \end{cases}$$

$$Q(x) = \left[\frac{1}{2},\frac{2}{3}\right] \quad \text{for all } x \in [0,1).$$

We can easily prove that inequality (2.4.1) holds, same in the lines of Example 2.3.
We observe that $\bigcup_{x\in X} Px \not\subseteq k(X)$ and $\bigcup_{x\in X} Qx \not\subseteq h(X)$.
Also neither $h(X)$ nor $k(X)$ is closed.
For the sequences $x_n = \frac{1}{2} + \frac{1}{4n}$ and $y_n = \frac{1}{2} + \frac{1}{3n^2}$, $n = 1,2,\dots$.

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = \left[\frac{1}{2},\frac{2}{3}\right],$$

$$\lim_{n\to\infty} hx_n = \lim_{n\to\infty} ky_n = \frac{1}{2} = h\left(\frac{1}{2}\right) = k\left(\frac{1}{2}\right) \in \left[\frac{1}{2},\frac{2}{3}\right].$$

Therefore, the pairs (h,P) and (k,Q) satisfy $(JCLR_{hk})$ property.
Also,

$$hh\left(\frac{1}{2}\right) = \frac{1}{2} \in \left[\frac{1}{2},\frac{2}{3}\right] = Ph\left(\frac{1}{2}\right) \text{ and } hh\left(\frac{1}{2}\right) = h\left(\frac{1}{2}\right)$$

and

$$kk\left(\frac{1}{2}\right) = \frac{1}{2} \in \left[\frac{1}{2},\frac{2}{3}\right] = Qk\left(\frac{1}{2}\right) \text{ and } kk\left(\frac{1}{2}\right) = k\left(\frac{1}{2}\right).$$

Thus h is P-weakly commuting and k is Q-weakly commuting.
All the required conditions of Theorem 2.5 hold.
It can be seen that '$\frac{1}{2}$' is the common fixed point of h,k,P and Q, since

$$h\left(\frac{1}{2}\right) = k\left(\frac{1}{2}\right) = \frac{1}{2} \in P\left(\frac{1}{2}\right) \cap Q\left(\frac{1}{2}\right).$$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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