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Research Article

New Shrinkage Entropy Estimator for Mean of Exponential Distribution under Different Loss Functions

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Abstract. In this paper, a new shrinkage estimator of entropy function for mean of an exponential distribution is proposed. A progressive type censored sample is taken to obtain the estimator. For the new estimator, risk functions and relative risk functions are developed under symmetric and asymmetric loss functions, viz. squared error loss function and LINEX loss function, and new estimator is shown to have better performance than a classical estimator in terms of relative risk.

Keywords. Exponential distribution, Entropy function, Shrinkage estimation, Progressive type-II censored sample, Squared error loss function and LINEX loss function

Mathematics Subject Classification (2020). 62F03, 62N02, 94A15, 94A17

1. Introduction

Some uncertainty is always there for outcomes in each and every random experiment. The entropy provides a quantitative measure of this uncertainty. Entropy was originally defined in physics, especially in second law of thermodynamics but in information theory it was firstly defined by Shannon [10] as under:

If f be probability density function and F be distribution function of random variable X, then entropy function is given by

$$H(f) = E[-\log(f(X))]. \tag{1}$$

For the very sharply peaked distribution, entropy is very low and is much higher when the probability is spread out. In other words, entropy measures the uniformity of distribution.

On estimation of entropies for different life distributions, many researchers have worked. Noteworthy work in this direction may be refereed from Lazo and Rathie [7], Misra *et al*. [9], Jeevanand and Abdul-Sathar [3], and Kayal and Kumar [6] etc.

Suppose the random variable X has the probability distribution $f(x,\theta)$ where interest is to estimate entropy function as function of θ. When we estimate θ, some prior information about unknown parameter θ as an initial guess value θ_0 (based on the past experience) is given. Thompson [11] recommended that by modifying usual estimator of unknown parameter θ by moving it closer to θ_0, we can create shrinkage estimation.

When initial value is near to true value of the parameter θ then shrinkage estimator gives better results than usual estimator. Various authors have discussed the concept of shrinkage estimators for different parameters or parametric functions under a variety of distributions.

In this paper, we shall concentrate on obtaining shrinkage estimation of entropy function, under symmetric/asymmetric loss functions using progressive type II censored sample, when the underlying distribution is assumed to exponential distribution. The form of density we consider is

$$f(x,\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, \quad x \ge 0,\ \theta > 0. \tag{2}$$

Progressive censoring is a useful scheme for reliability and life time studies and for a more detailed discussion about progressive censoring, one may refer to Balakrishnan and Aggarwala [1].

2. Shrinkage Estimators of $H(f)$

For exponential distribution with mean θ, the entropy function becomes

$$H(f) = 1 + \ln(\theta). \tag{3}$$

Since $H(f)$ is linear function of $\ln(\theta)$, estimating $H(f)$ is equivalent to estimating $\ln(\theta)$. We shall write $I(\theta) = \ln(\theta)$ so that $H(f) = 1 + I(\theta)$. Now, we shall discuss estimation of $I(\theta)$.

From the exponential distribution (given in (2)), let $X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n}$ be type II progressive censored sample. Then, for this sample (Balakrishnan and Aggarwala [1]), the joint density is

$$\begin{aligned} & f(x_{1:m:n}, x_{2:m:n}, \dots, x_{m:m:n}) \\ &\quad = C\prod_{i=1}^{m} f(x_{i:m:n})(1-F(x_{i:m:n}))^{R_i}, \quad 0 \le x_{1:m:n} \le x_{2:m:n} \le \dots \le x_{m:m:n}, \end{aligned} \tag{4}$$

where $(R_1, R_2, \dots, R_m)$ be the progressive censoring scheme and m, the number of observed failures and $(R_1, R_2, \dots, R_m)$ are all pre-fixed with

$$C = n(n-R_1-1)(n-R_1-R_2-2)\dots(n-R_1-R_2-\dots-R_{m-1}-m+1).$$

Now substituting the density function and survival function in (4) we get

$$\begin{aligned} & f(x_{1:m:n}, x_{2:m:n}, \dots, x_{m:m:n}) \\ &\quad = C\left(\frac{1}{\theta}\right)^m \exp\left(-\frac{\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}{\theta}\right), \quad 0 \le x_{1:m:n} \le x_{2:m:n} \le \dots \le x_{m:m:n}. \end{aligned} \tag{5}$$

Then *MLE* (*Maximum Likelihood Estimator*) of θ can easily be obtained as

$$\widehat{\theta} = \frac{\sum_{i=1}^{m} (R_i + 1)x_{i:m:n}}{m}. \tag{6}$$

Since $I(\theta)$ is continuous function of θ, so we replace θ by its *MLE* $\widehat{\theta}$ in $I(\theta)$, to obtain the *MLE* of $I(\theta)$. Therefore, *MLE* of entropy function for the exponential distribution is

$$\widehat{H}(f) = 1 + \ln(\widehat{\theta}). \tag{7}$$

It can easily be shown that the distribution of $\widehat{\theta}$ has distribution as

$$f(\widehat{\theta};\theta) = \left(\frac{m}{\theta}\right)^m \frac{\widehat{\theta}^{m-1} \exp\left(-\frac{m\widehat{\theta}}{\theta}\right)}{\Gamma(m)}, \quad \widehat{\theta} > 0. \tag{8}$$

Kambo *et al*. [5] modified a preliminary test single stage shrinkage estimator for exponential distribution mean which takes prior estimate θ_0 at both stages of the estimator when the data is right censored and Jiheel and Shanubhogue [4] considered two shrinkage entropy estimators for the same distribution. This motivates us to propose a new estimator.

We proposed the following shrinkage entropy estimators:

$$\widetilde{I}_1(\theta) = \begin{cases} k_1 \ln(\widehat{\theta}) + (1-k_1)\ln(\theta_0) H_0 : \theta = \theta_0, & \text{accepted}, \\ (1-k_1)\ln(\widehat{\theta}) + k_1 \ln(\theta_0), & \text{otherwise}, \end{cases} \tag{9}$$

where k_1 is constant such that $0 \le k_1 \le 1$ and χ_1^2 and χ_2^2 are lower and upper αth percentile values of chi-square distribution with $2m$ degrees of freedom, respectively. A second choice for shrinkage factor depends on minimization of mean square error function, for the estimator given in (9), with respect to k_1. The value is denoted by k_2 and the corresponding shrinkage estimator is given by

$$\widetilde{I}_2(\theta) = \begin{cases} k_2 \ln(\widehat{\theta}) + (1-k_2)\ln(\theta_0) H_0 : \theta = \theta_0, & \text{accepted}, \\ (1-k_2)\ln(\widehat{\theta}) + k_2 \ln(\theta_0), & \text{otherwise}. \end{cases} \tag{10}$$

In estimation, various loss functions are taken in literature. These can be mainly divided into two categories, viz. symmetric and asymmetric. Broadly both types of loss functions have been taken in these problems. Among various symmetric loss functions (Berger [2], Martz and Waller [8]) *Square Error Loss Function* (*SELF*) is well known and extensively used in estimation problems. Several circumstances may happen in practice where 'SELF' may be appropriately used, especially when underestimation and overestimation are of similar importance.

In spite of above considered justification for '*SELF*', there may be some practical cases where overestimation and underestimation are not equally penalized and thus for such cases, a fruitful asymmetric loss function, that is LINEX loss function was introduced by Varian [12]. It is defined as

$$L(\Delta) = b(e^{a\Delta} - a\Delta - 1), \quad b > 0,\ a \ne 0, \tag{11}$$

where $\Delta = \widehat{\theta} - \theta$, b is scale parameter and a is shape parameter. When overestimation is more critical than underestimation then the positive value of a is used and for other cases, its negative value is used.

In next section, we calculate the risk of above estimator under both type of loss functions (symmetric and asymmetric) described above.

3. Risk of Estimators

3.1 Risk of *MLE* $\widehat{I}(\theta)$

Under *LLF* (*LINEX Loss Function*), the risk of the estimator $\widehat{I}(\theta)$ is given by

$$\begin{aligned}R_{LLF}(\widehat{I}(\theta)) &= E(\widehat{I}(\theta)/LLF)\\ &= \int_0^\infty (\exp(a(\ln(\widehat{\theta})-\ln(\theta)))-a(\ln(\widehat{\theta})-\ln(\theta))-1)f(\widehat{\theta};\theta)d(\widehat{\theta})\\ &= \int_0^\infty \left(\exp\left(a\left(\ln\left(\frac{\widehat{\theta}}{\theta}\right)\right)\right)-a\left(\ln\left(\frac{\widehat{\theta}}{\theta}\right)\right)-1\right)f(\widehat{\theta};\theta)d(\widehat{\theta}).\end{aligned}$$

Taking the transformation $x=\frac{m\widehat{\theta}}{\theta}$ and solving the integral, we get

$$R_{LLF}(\widehat{I}(\theta)) = \frac{\Gamma(m+a)}{m^a\Gamma(m)} - a(\Psi(m)-\ln(m)-1), \tag{12}$$

where

$$\Psi(n) = \frac{\frac{d}{dn}\Gamma(n)}{\Gamma(n)}.$$

Also, under *SELF*, the risk of estimator $\widehat{I}(\theta)$ is obtained as

$$\begin{aligned}R_{SELF}(\widehat{I}(\theta)) &= E\left(\ln(\widehat{\theta})-\ln(\theta)\right)^2\\ &= \int_0^\infty (\ln(\widehat{\theta})-\ln(\theta))^2 f(\widehat{\theta};\theta)d(\widehat{\theta})\\ &= \int_0^\infty \left(\ln\left(\frac{\widehat{\theta}}{\theta}\right)\right)^2 f(\widehat{\theta};\theta)d(\widehat{\theta})\\ &= G(0,\infty,(\log(x))^2)-2\ln(m)\psi(m)+(\ln(m))^2,\end{aligned} \tag{13}$$

where

$$G(t_1,t_2,W) = \frac{\int_{t_1}^{t_2} W x^{n-1}e^{-x}dx}{\Gamma(n)} \text{ and } W \text{ is a function of } x.$$

3.2 Risk of Shrinkage Estimator $\widetilde{I}_1(\theta)$

Risk of estimator $\widetilde{I}_1(\theta)$ under *LLF* is obtained as under:

$$\begin{aligned}R_{LLF}(\widetilde{I}_1(\theta)) &= E(\widetilde{I}_1(\theta)/LLF)\\ &= \int_{r_1}^{r_2} (\exp(a(k_1\ln(\widehat{\theta})+(1-k_1)\ln(\theta_0)-\ln(\theta)-a(k_1\ln(\widehat{\theta})\\ &\qquad +(1-k_1)\ln(\theta_0)-\ln(\theta))-1)f(\widehat{\theta};\theta)d\widehat{\theta}\\ &\quad + \int_0^\infty (\exp(a(1-k_1)\ln(\widehat{\theta})+k_1\ln(\theta_0)-\ln(\theta))\\ &\qquad -a((1-k_1)\ln(\widehat{\theta})+k_1\ln(\theta_0)-\ln(\theta))-1)f(\widehat{\theta};\theta)d\widehat{\theta}\\ &\quad - \int_{r_1}^{r_2} (\exp(a(1-k_1)\ln(\widehat{\theta})+k_1\ln(\theta_0)-\ln(\theta))\\ &\qquad -a((1-k_1)\ln(\widehat{\theta})+k_1\ln(\theta_0)-\ln(\theta))-1)f(\widehat{\theta};\theta)d\widehat{\theta},\end{aligned}$$

where r_1 and r_2 are the boundaries of acceptance region of a test of hypothesis $H_0:\theta=\theta_0$ against alternative $H_1:\theta\neq\theta_0$. Letting $r_1=\frac{\theta_0\chi_1^2}{2m}$, $r_2=\frac{\theta_0\chi_2^2}{2m}$, $x=\frac{m\widehat{\theta}}{\theta}$ and solving the integrals, we get

$$
\begin{aligned}
R_{LLF}(\widehat{I}_1(\theta)) = & \frac{\lambda^a \Gamma(ak_1+m)}{\Gamma(m)(\lambda m)^{ak_1}}[I(r'_2, ak_1+m) - I(r'_1, ak_1+m)] + (1-2k_1)aG(r'_1, r'_2, \ln(x)) \\
& + \frac{(m\lambda)^{ak_1}\Gamma(a(1-k_1)+m)}{\Gamma(m)m^a} - \frac{(m\lambda)^{ak_1}\Gamma(a(1-k_1)+m)}{\Gamma(m)m^a} \\
& \cdot [I(r'_2, a(1-k_1)+m) - I(r'_1, a(1-k_1)+m)] \\
& + (2k_1-1)a\ln(m\lambda)[I(r'_2, m) - I(r'_1, m)] + a(k_1-1)G(0,\infty,\ln(x)) \\
& - ak_1\ln(m\lambda) + a\ln m - 1,
\end{aligned} \tag{14}
$$

where

$$
r'_1 = \frac{2\chi_1^2}{\lambda}, \quad r'_2 = \frac{2\chi_2^2}{\lambda}, \quad \lambda = \frac{\theta_0}{\theta}
$$

and $I(x,n)$ is cumulative distribution function of gamma distribution given as

$$
I(x,n) = \frac{\int_0^\infty t^{n-1}e^{-t}dt}{\Gamma(n)}.
$$

Now, we can obtained risk of estimator $\widetilde{I}_1(\theta)$ under $SELF$ as under:

$$
\begin{aligned}
R_{SELF}(\widetilde{I}_1(\theta)) &= E(\widetilde{I}_1(\theta)/SELF) \\
&= \int_{r_1}^{r_2} (k_1\ln(\widehat{\theta}) + (1-k_1)\ln(\theta_0) - \ln(\theta))^2 f(\widehat{\theta};\theta)d\widehat{\theta} \\
&\quad + \int_0^\infty ((1-k_1)\ln(\widehat{\theta}) + k_1\ln(\theta_0) - \ln(\theta))^2 f(\widehat{\theta};\theta)d\widehat{\theta} \\
&\quad - \int_{r_1}^{r_2} ((1-k_1)\ln(\widehat{\theta}) + k_1\ln(\theta_0) - \ln(\theta))^2 f(\widehat{\theta};\theta)d\widehat{\theta}.
\end{aligned}
$$

By letting $x = \frac{m\widehat{\theta}}{\theta}$ and solving the integrals, we obtain

$$
\begin{aligned}
& R_{SELF}(\widetilde{I}_1(\theta)) \\
&= ((\ln\lambda)^2 - (\ln m)^2 - 2k_1(\ln m\lambda)(\ln\lambda) + 2k_1(\ln m\lambda)(\ln m)) \\
&\quad \cdot [I(r'_2, m) - I(r'_1, m)] + (2k_1\ln\lambda - 2k_1\ln m - 2k_1\ln(m\lambda) + 2\ln m)G(r'_1, r_2, (\ln x)) \\
&\quad + (k_1-1)^2 G(0,\infty,(\ln x)^2) + (2k_1\ln(m\lambda) + 2k_1\ln m - 2k_1^2\ln(m\lambda) - 2\ln m)G(0,\infty,(\ln x)) \\
&\quad + (2k_1-1)G(r'_1, r'_2, (\ln x)^2) + (\ln m)^2 + k_1^2(\ln m\lambda)^2 - 2k_1(\ln m\lambda)(\ln m).
\end{aligned} \tag{15}
$$

3.3 Risk of Shrinkage Estimator $\widetilde{I}_2(\theta)$

After minimizing risk function under SELF with respect k_1 define in (15) we get

$$
\begin{aligned}
k^* = & \frac{2(\ln m\lambda)(\ln m) - 2(\ln m\lambda + \ln m)G(0,\infty,\ln x) + 2G(0,\infty,(\ln x)^2)}{2G(0,\infty,(\ln x)^2) - 4(\ln m\lambda)G(0,\infty,\ln x) + 2(\ln m\lambda)^2} \\
& - \frac{2(\ln\lambda - \ln m - \ln m\lambda)G(r'_1, r'_2, \ln x) + 2G(r'_1, r'_2, (\ln x)^2)}{2G(0,\infty,(\ln x)^2) - 4(\ln m\lambda)G(0,\infty,\ln x) + 2(\ln m\lambda)^2} \\
& - \frac{2((\ln m\lambda)(\ln m) - (\ln m\lambda)(\ln\lambda))[I(r'_2, m) - I(r'_1, m)]}{2G(0,\infty,(\ln x)^2) - 4(\ln m\lambda)G(0,\infty,\ln x) + 2(\ln m\lambda)^2}.
\end{aligned} \tag{16}
$$

Theoretically, we could not show that k^* lies between 0 and 1. Therefore, we define the value of k^* as

$$k_2 = \left\{ \begin{array}{ll} 0 & k^* < 0 \\ k^* & 0 \le k^* \le 1 \\ 1 & k^* > 1 \end{array} \right\}. \tag{17}$$

Now, by replacement k_1 by k_2 we obtain estimator $\widetilde{I}_2(\theta)$ defined in (10), also by replacement k_1 by k_2 in (15) we obtain risk function under *SELF* for estimator $\widetilde{I}_2(\theta)$.

4. Relative Risk

To examine performance of the estimators $\widetilde{I}_1(\theta)$ and $\widetilde{I}_2(\theta)$ under *SELF* and *LLF* we compare relative risks of these estimators with respect to the best possible estimator $\widehat{I}(\theta)$ in this case. Relative risk of $\widetilde{I}_1(\theta)$ under *LLF* is

$$RR_{LLF}(\widetilde{I}_1(\theta)) = \frac{R_{LLF}(\widehat{I}(\theta))}{R_{LLF}(\widetilde{I}_1(\theta))}. \tag{18}$$

Also, relative risk of $\widetilde{I}_1(\theta)$ and $\widetilde{I}_2(\theta)$ under *SELF* is

$$RR_{SELF}(\widetilde{I}_1(\theta)) = \frac{R_{SELF}(\widehat{I}(\theta))}{R_{SELF}(\widetilde{I}_1(\theta))} \tag{19}$$

and

$$RR_{SELF}(\widetilde{I}_2(\theta)) = \frac{R_{SELF}(\widehat{I}(\theta))}{R_{SELF}(\widetilde{I}_2(\theta))}. \tag{20}$$

5. Numerical Computations and Graphical Analysis

It is noticed that $RR_{LLF}(\widetilde{I}_1(\theta))$, $RR_{SELF}(\widetilde{I}_1(\theta))$ and $RR_{SELF}(\widetilde{I}_2(\theta))$ are functions of m, k_1, a, α and λ. To manifest performance of proposed estimators under *LLF* and *SELF*, we have taken following values:

$$m = 6, 9, 12,\ \ k_1 = 0.2, 0.4, 0.6,\ \ a = 1, 2, 3,\ \ \alpha = 0.01, 0.05,\ \ \lambda = 0.25(0.25)1.75$$

Tables 1-3 and Figures 1-8 present the behaviour of relative risks of the estimators with respect to a for varying values of k_1, m and α.

(i) Relative risks of both proposed estimators are high in and around $\lambda = 1$, i.e. true value of θ is closer to θ_0, under *SELF* as well as *LLF*. Further, as m increases range of relative risk greater than one become small.

(ii) $RR_{LLF}(\widetilde{I}_1(\theta))$ is higher than $RR_{SELF}(\widetilde{I}_1(\theta))$ and $RR_{SELF}(\widetilde{I}_2(\theta))$ is higher than $RR_{SELF}(\widetilde{I}_1(\theta))$.

(iii) $RR_{LLF}(\widetilde{I}_1(\theta))$ is increasing function of a if $0.25 \le \lambda \le 0.75$ and decreasing function of a if $\lambda = 1.5$, 1.75. Further, if $\alpha = 0.01$ and $\lambda = 1.25$, it is decreasing with a except $m = 6$, $k_1 = 0.2$. If $\alpha = 0.05$ it is decreasing except $m = 6$, $k_1 = 0.4$ and for $m = 6, 9, 12$, $k_1 = 0.2$. For $\lambda = 1$, $\alpha = 0.01$, $RR_{LLF}(\widetilde{I}_1(\theta))$ is decreasing when $k = 0.2$, otherwise it is increasing. For $\alpha = 0.05$ and $\lambda = 1$, it is increasing when $k_1 = 0.2$ and $k_1 = 0.4$.

(iv) For $\lambda = 0.25$, $RR_{LLF}(\widetilde{I}_1(\theta))$, $RR_{SELF}(\widetilde{I}_1(\theta))$ are decreasing with both m and k_1 and if $\lambda = 1.5, 1.75$ then the relative risks are decreasing with m but increasing with k_1.

(v) $RR_{SELF}(\widetilde{I}_2(\theta))$ is increasing with m if $\lambda = 0.25$ and $\lambda = 1$ and otherwise it is decreasing.

(vi) $RR_{LLF}(\widetilde{I}_1(\theta))$, $RR_{SELF}(\widetilde{I}_1(\theta))$ are decreasing functions of α, if $0.75 \le \lambda \le 1.25$ except $k = 0.6$ and $RR_{SELF}(\widetilde{I}_2(\theta))$ is decreasing with α, if $0.75 \le \lambda \le 1.25$.

Table 1. Relative risk of estimator $\widetilde{I}_1(\theta)$ under LLF

$\alpha = 0.01$			λ						
m	k_1	a	0.25	0.50	0.75	1	1.25	1.50	1.75
6	0.2	1	0.6724	0.7235	2.1920	10.7343	3.6235	1.2976	0.6950
		2	0.8411	0.8303	2.3113	10.7027	3.5268	1.1741	0.5938
		3	1.0868	1.0048	2.3866	10.6109	3.6033	1.1248	0.5358
	0.4	1	0.4752	0.9058	2.5094	5.4178	3.7452	1.9394	1.1643
		2	0.5841	1.1167	2.7318	5.5340	3.4797	1.7085	0.9878
		3	0.7526	1.3812	3.1701	5.9805	3.4331	1.6006	0.8897
	0.6	1	0.3043	1.0811	2.2719	2.8287	2.6753	2.1946	1.7150
		2	0.3802	1.2513	2.5671	2.8975	2.4743	1.8972	1.4270
		3	0.4977	1.5394	3.0881	3.1359	2.4165	1.7375	1.2584
9	0.2	1	0.6735	0.5949	1.6399	10.8305	2.5778	0.8792	0.4787
		2	0.8096	0.6907	1.6943	10.8093	2.4873	0.7921	0.4075
		3	0.9947	0.8279	1.8145	10.7355	2.4867	0.7411	0.3587
	0.4	1	0.3566	0.7514	2.0717	5.4938	3.0836	1.4058	0.8188
		2	0.4310	0.8691	2.2414	5.5729	2.8653	1.2479	0.7003
		3	0.5359	1.0448	2.5335	5.8729	2.7751	1.1547	0.6228
	0.6	1	0.2017	0.7721	2.1125	2.8567	2.5599	1.8915	1.3259
		2	0.2527	0.8804	2.3570	2.9031	2.3534	1.6461	1.1270
		3	0.3234	1.0455	2.7438	3.0626	2.2480	1.4909	0.9954
12	0.2	1	0.6324	0.5466	1.3217	10.8784	2.0029	0.6738	0.3803
		2	0.7399	0.6372	1.3779	10.8631	1.9229	0.6052	0.3225
		3	0.8822	0.7582	1.4773	10.8026	1.8981	0.5589	0.2797
	0.4	1	0.2826	0.6364	1.7680	5.5327	2.6131	1.1087	0.6453
		2	0.3387	0.7322	1.9066	5.5926	2.4330	0.9888	0.5540
		3	0.4135	0.8653	2.1256	5.8186	2.3382	0.9096	0.4892
	0.6	1	0.1516	0.5824	1.9589	2.8709	2.4404	1.6378	1.0379
		2	0.1900	0.6607	2.1658	2.9055	2.2409	1.4389	0.8982
		3	0.2403	0.7719	2.4724	3.0258	2.1197	1.3028	0.7994

(Table *Contd.*)

$\alpha = 0.01$			λ						
m	k_1	a	0.25	0.50	0.75	1	1.25	1.50	1.75
$\alpha = 0.05$									
6	0.2	1	0.8034	0.8811	1.8931	4.7812	2.5208	1.1466	0.6948
		2	0.9797	1.0097	1.9198	4.7926	2.5456	1.0658	0.6020
		3	1.2427	1.2163	2.0372	4.7696	2.6588	1.0424	0.5485
	0.4	1	0.4943	1.0451	2.3731	4.4762	3.2389	1.8084	1.1554
		2	0.6040	1.2175	2.5800	4.5746	3.0692	1.6123	0.9824
		3	0.7752	1.5074	2.9844	4.9056	3.0698	1.5255	0.8864
	0.6	1	0.2969	0.9908	2.3965	3.1777	3.0116	2.3906	1.7347
		2	0.3723	1.1451	2.7173	3.2550	2.7343	2.0318	1.4383
		3	0.4883	1.4080	3.2873	3.5430	2.6361	1.8355	1.2650
9	0.2	1	0.7353	0.7776	1.5278	4.7936	1.9780	0.8500	0.5386
		2	0.8692	0.8979	1.5726	4.8022	1.9616	0.7764	0.4599
		3	1.0566	1.0681	1.6738	4.7839	2.0011	0.7344	0.4052
	0.4	1	0.3619	0.8351	2.0146	4.5271	2.7392	1.3729	0.8684
		2	0.4363	0.9644	2.1801	4.5940	2.5814	1.2230	0.7397
		3	0.5416	1.1571	2.4627	4.8162	2.5282	1.1351	0.6549
	0.6	1	0.2000	0.7	2.1753	3.2135	2.8582	1.9545	1.2136
		2	0.2509	0.8003	2.4288	3.2656	2.5871	1.6915	1.0381
		3	0.3213	0.9529	2.8320	3.4583	2.4411	1.5249	0.9231
12	0.2	1	0.6532	0.7444	1.2998	4.8000	1.6394	0.6995	0.4748
		2	0.7589	0.8592	1.3511	4.8069	1.6072	0.6327	0.4024
		3	0.9010	1.0109	1.4420	4.7919	1.6133	0.5873	0.3481
	0.4	1	0.2839	0.7103	1.7588	4.5532	2.3741	1.1259	0.7230
		2	0.34	0.8138	1.8990	4.6038	2.2330	1.0035	0.6175
		3	0.4149	0.9580	2.1185	4.7711	2.1643	0.9223	0.5421
	0.6	1	0.1512	0.5318	1.9702	3.2316	2.6937	1.6016	0.8850
		2	0.1896	0.6058	2.1758	3.2709	2.4424	1.4089	0.7698
		3	0.2399	0.7106	2.4820	3.4157	2.2863	1.2776	0.6894

Table 2. Relative risk of estimator $\widetilde{I}_1(\theta)$ under *SELF*

$\alpha = 0.01$		λ						
m	k_1	0.25	0.50	0.75	1	1.25	1.50	1.75
6	0.2	0.5479	0.6668	2.2757	10.6353	3.8672	1.5107	0.8573
	0.4	0.4025	0.8791	2.4616	5.6176	4.2780	2.3452	1.4617
	0.6	0.2539	0.9912	2.1401	2.9240	3.0651	2.7023	2.1967
9	0.2	0.5667	0.5290	1.6504	10.7628	2.7552	1.0116	0.5816
	0.4	0.3018	0.6748	2.0018	5.6294	3.4590	1.6530	0.9977
	0.6	0.1636	0.7043	1.9745	2.9208	2.8986	2.2686	1.6261
12	0.2	0.5476	0.4781	1.3063	10.8277	2.1394	0.7706	0.4589
	0.4	0.2392	0.5676	1.6953	5.6353	2.8974	1.2837	0.7743
	0.6	0.1213	0.5284	1.8283	2.9192	2.7413	1.9246	1.2361
$\alpha = 0.05$								
6	0.2	0.6780	0.8102	1.9557	4.7096	2.5718	1.2923	0.8416
	0.4	0.4224	0.9532	2.3244	4.5989	3.6078	2.1566	1.4465
	0.6	0.2466	0.9113	2.2558	3.3051	3.5356	3.0051	2.2321
9	0.2	0.6352	0.6937	1.5372	4.7461	2.0470	0.9621	0.6506
	0.4	0.3077	0.7506	1.9439	4.6104	3.0215	1.6079	1.0621
	0.6	0.1619	0.6371	2.0343	3.2992	3.2988	2.3594	1.4799
12	0.2	0.5724	0.6569	1.2854	4.7459	1.7096	0.7931	0.5721
	0.4	0.2407	0.6361	1.6834	4.6162	2.6021	1.3042	0.8713
	0.6	0.1209	0.4802	1.8423	3.2962	3.0711	1.8803	1.0496

Table 3. Relative risk of estimator $\widetilde{I}_2(\theta)$ under *SELF*

$\alpha = 0.01$	λ						
m	0.25	0.50	0.75	1	1.25	1.5	1.75
6	0.5101	0.4775	1.7642	11.1057	2.4954	0.9288	0.5256
9	0.6822	0.3780	1.2113	11.3394	1.7651	0.6283	0.3615
12	0.8476	0.3464	0.9311	11.4601	1.3668	0.4818	0.2880
$\alpha = 0.05$							
6	0.7263	0.6050	1.4563	3.4817	1.6160	0.7917	0.5159
9	0.8702	0.5271	1.1112	3.5150	1.2890	0.5966	0.4061
12	0.9544	0.5173	0.9135	3.5318	1.0782	0.4964	0.3632

Graph of relative risk of estimator $\widetilde{I}_1(\theta)$ under LLF

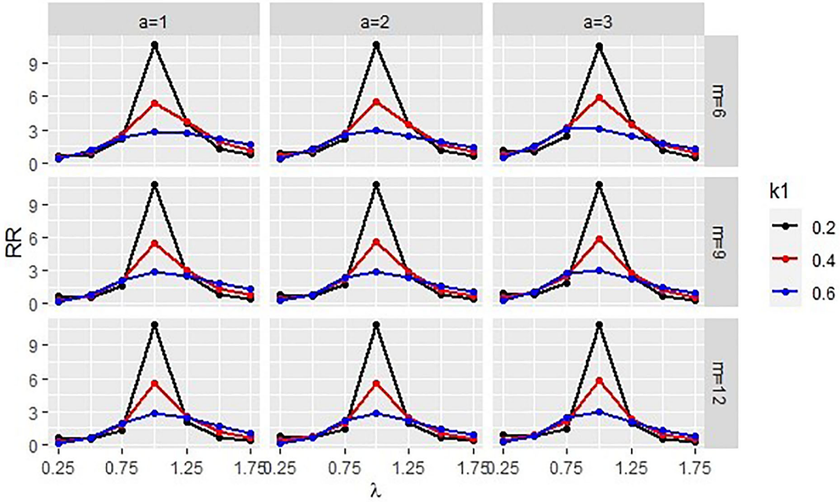

Figure 1. For $\alpha = 0.01$

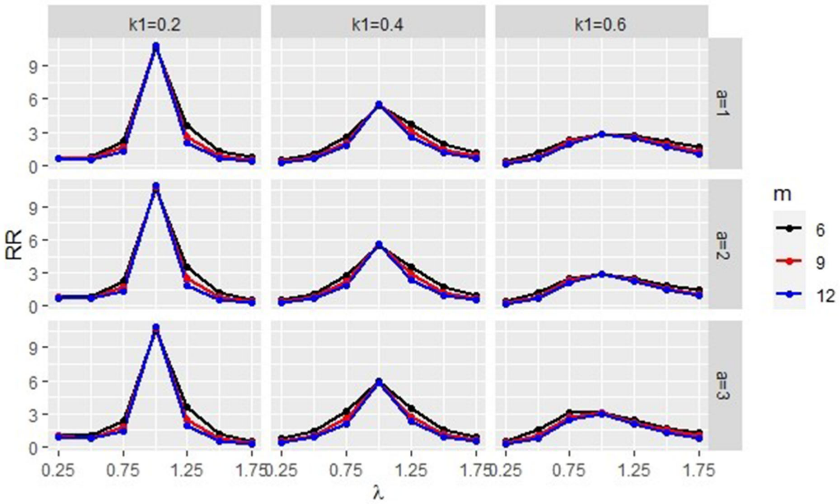

Figure 2. For $\alpha = 0.01$

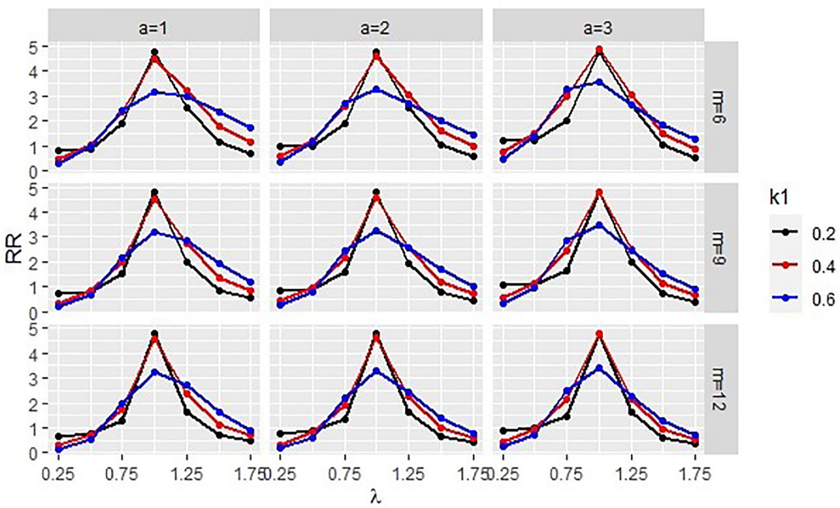

Figure 3. For $\alpha = 0.05$

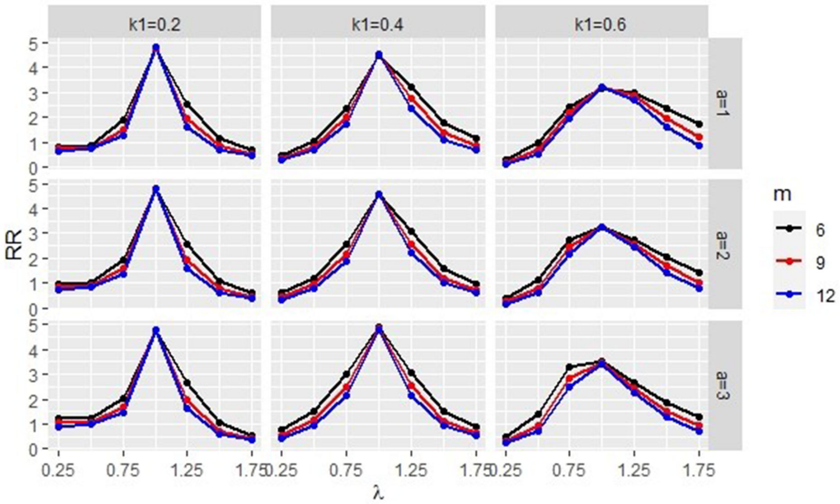

Figure 4. For $\alpha = 0.05$

Relative risk of estimator $\widetilde{I}_1(\theta)$ with respect to $\boldsymbol{\alpha}$ under *SELF*

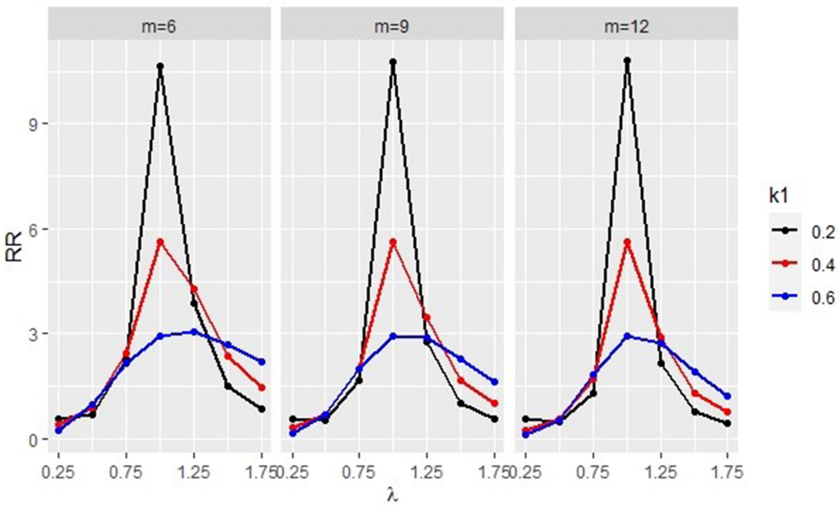

Figure 5. For $\alpha = 0.01$

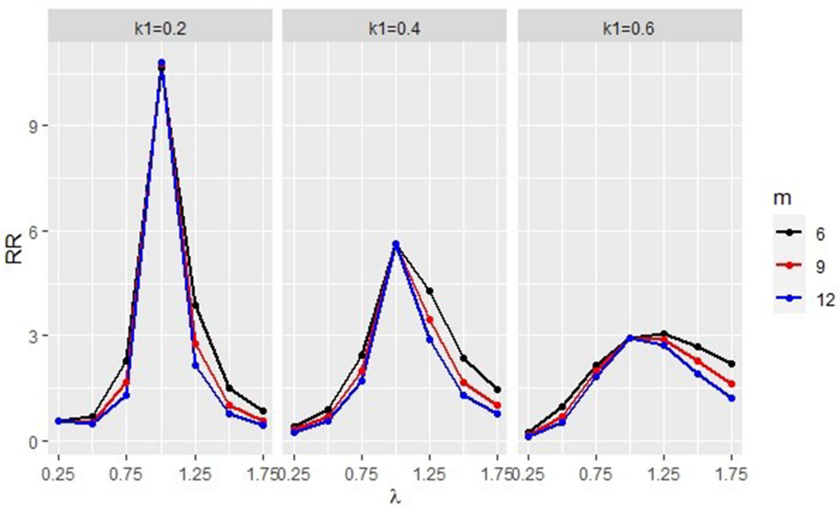

Figure 6. For $\alpha = 0.01$

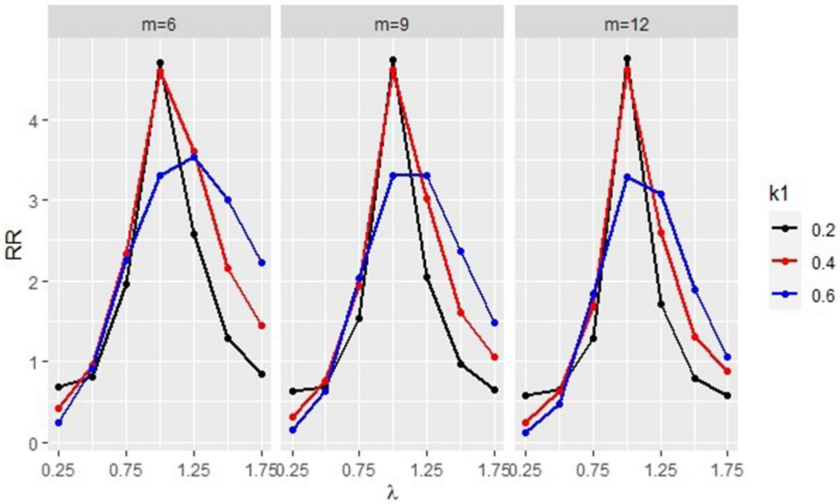

Figure 7. For $\alpha = 0.05$

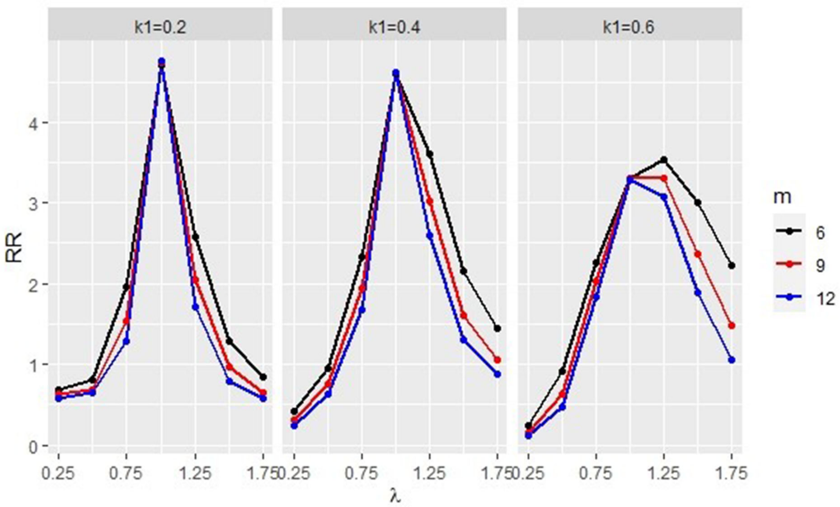

Figure 8. For $\alpha = 0.05$

6. Conclusion

From Tables 1-3 and from Figures 1-8 following conclusions are drawn: Relative risks of both proposed estimators are high in and around $\lambda = 1$, i.e., true value of θ is closer to θ_0, under SELF as well as LLF and $RR_{LLF}(\widetilde{I}_1(\theta))$ is higher than $RR_{SELF}(\widetilde{I}_1(\theta))$ and $RR_{SELF}(\widetilde{I}_2(\theta))$ is higher than $RR_{SELF}(\widetilde{I}_1(\theta))$.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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