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Research Article

Finite Element Analysis of an Unsteady MHD Normal Convection Flow of a Casson Fluid Past a Vertical Oscillating Plate in Porous Medium with Effect of Heat Source and Eckert Number

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Abstract. An investigation of unsteady MHD free convection flow over a changeable permeable plate with Casson fluid in the presence of heat absorption and Eckert number effects have been carried out. The governing equations of the flow are solved by Galerikin method. Velocity, temperature and concentration profiles are analyzed through the graphs with effects of Casson, permeability, phase angle, Eckert number and chemical reaction parameter, which provide excellent correlation with the previous results.

Keywords. MHD, Casson fluid, Heat absorption/generation and Eckert number

Mathematics Subject Classification (2020). 76D05, 80A19, 80M10

1. Introduction

In recent years researchers are more engrossment to study the properties of viscosity. Casson fluid is a shear thinning fluid, which is one kind of non-Newtonian, examples are jelly, honey and human blood. The non-Newtonian fluids have several applications in engineering field and industries for production of paints, chemicals and syrups. Casson fluid model with yield stress which widely used modeling of blood. The fluid mechanics of blood flow play an important role not only in the understanding of flow characteristics of blood, but also in the diagnosis and treatment of several arteries diseases. Now-a-days Researchers are concentrating study of MHD due to its application in many areas like petroleum industries, nuclear power projects and lubrication industries. Alfvén [2], Ferraro and Plumpton [13], Cramer and Pai [10] and on study Cowling [9] are contributed their study on MHD. Many scientists, engineers and researchers worked on non-Newtonian fluids properties which are difficult to analyze compared to Newtonian fluids. Some researchers study about the non-Newtonian fluids such as Walter's B (Ullah *et al*. [31]), viscoplastic (Férec [12]), Jeffrey (Nallapu and Radhakrishnamacharya [25]), Maxwell (Khan and Ali [22]), Oldroyd (Abbasbandy *et al*. [1]), and Brinkman (Zakaria [32]).

First study was started by Casson [4] on Casson fluid after many researchers have studied on Casson fluid, Khalid *et al*. [21] studied an unsteady MHD natural convection flow of Casson fluid past over an fluctuating vertical plate surrounded in a porous medium. Due to many applications of Casson fluid researchers interested to study in different aspects [15, 18, 20, 24, 29].

The application of heat transfer is very important in engineering to invent and improve the quality of the product. Heat source is system from which heat is generated, here natural convection is worked by Heat generation, it is involved wide range in physical phenomena. The study of heat generation in flowing fluids as crucial in many physical problems related with chemical reaction. It has many applications in industries as well as real life for examples boilers, radiators, Heat pumps and electrical heaters. A passive/active heat exchanger is a sink, these are generally three models free and forced convection and liquid cooled and, it is performed as thermal resistance. These are used in coolers, power amplifiers and processors etc. Chamka *et al*. [7, 8] studied the effect of heat absorption on unsteady MHD convection of heat and mass transfer through a movable semi-infinite plate. Das [11] discussed influence of inclined Hartman number and radiation on blood flow in an asymmetric channel with heat absorption and many researchers focused on related work [28] .

Heat transfer can performed in three categories of convection free, forced and mixed. The free convection of heat transfer passive mode is used in cooling of liquid metal system. Due to many applications of natural convection researcher concentrating on it. Chamka *et al*. [7] studied natural convection of micro polar fluid in a vertical channel and this type of research was extended by many authors [3, 8, 23, 26].

The Eckert number is a measure of heat transport and dissipation. About the Eckert number first investigated by Georpp in 1969 [14] and describes an Eckert number of $Ec \approx 1$ a reversal in heat transmission from a moving wall. Eckert number play a vital role in the cooling of commutators, these are used to transform electric power to the rotor in electric motors. The commutators due to friction becomes warmer faster the rotor spins at the slip-ring-contacts. To avoid high surface temperature commutators has to be cooled. Experimental investigations of Eckert number on heat transfer from flowing wall of rotating cylinder was studied by Gschwendtner [17], and Chamkha and Issa [6], and Govardhan *et al*. [16] were extended their research on this number.

Chemical reaction plays an important role of analysis of transport processes and chemical industries. The effect of chemical reaction and magnetic field on convective flow with mass and heat transfer plays a vital role in many areas such as cooling of nuclear reactors and petroleum industries. Patil and Kulkarni [27] studied influence of chemical reaction on natural convection flow of polar fluid through porous medium. Chamka *et al*. [5] investigated the influence of the chemical reaction and Heat source/sink on MHD flow from rotating vertical plate. Ibrahim *et al*. [19] studied effect of the chemical reaction and heat suction on the unsteady MHD natural convection on permeable flowing plate. Effect of chemical reaction and mass diffusion on steady electrically conducting fluid was studied by Salem and El-Aziz [30]. In the present paper we discussed effect of chemical reaction; Eckert number and Heat source/absorption on MHD free convectional flow of Casson fluid on a vertical plate through porous medium these results agreement with earlier results.

2. Formulation and Problem Solution

We consider a kind of non-Newtonian fluid i.e. Casson fluid is travel through an infinite plate which is embedded in a porous media. Consider that y-axis is normal to the plate, which is towards the x-axis direction. Here we assumed that uniform magnetic field is applied on an electrically conducting fluid normal to the plate. Due to infinitely small magnetic Reynolds number of the flow, the induced magnetic field is neglected. When $t = 0$ fluid along with the plate are kept at rest with uniform temperature. Plate oscillating starts at $y = 0$ in view of

$$\bar{\boldsymbol{u}} = UH(\bar{t})\cos(\bar{\omega}\bar{t})i \quad \text{(or)} \quad \bar{\boldsymbol{u}} = U\sin(\bar{\omega}\bar{t})i, \quad \text{for} \quad t > 0. \tag{2.1}$$

Here U amplitude plate oscillations (t) unit step function, the oscillation frequency of plate ω, the temperature is increased to T_w and maintained to be fixed.

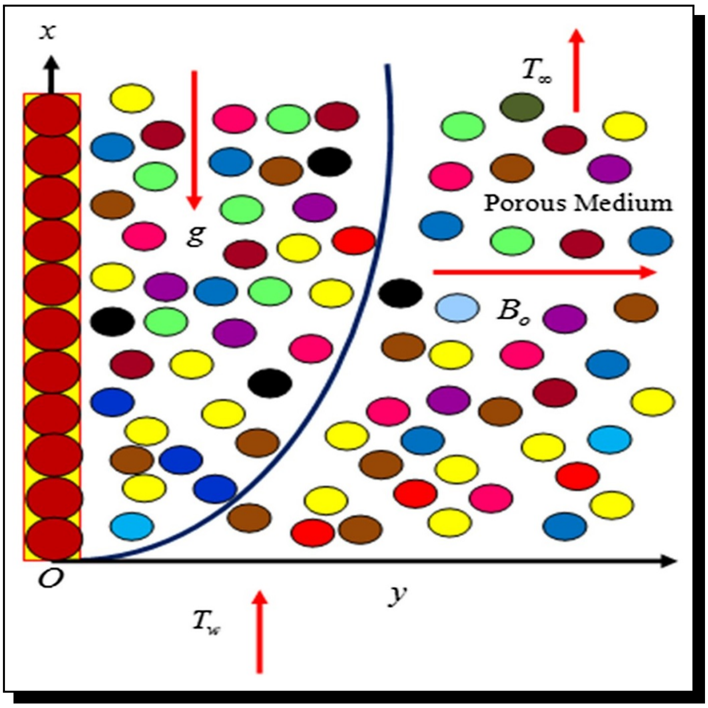

Figure 1

Cauchy tensor rheological state equation of Casson fluid as follows (see [17]):

$$\tau = \tau_0 + \mu\gamma',$$

$$\tau_{i,j} = \begin{cases} 2\left(\mu_B + \dfrac{P_Y}{\sqrt{2\pi}}\right)e_{i,j}, & \pi > \pi_c, \\ 2\left(\mu_B + \dfrac{P_Y}{\sqrt{2\pi}}\right)e_{i,j}, & \pi < \pi_c. \end{cases}$$

Here μ_B = dynamic viscosity plastic of Newtonian fluid, P_y = fluid stress yield, $\pi = e_{i,j}$ component rate deformation, π : the product of component rate deformation with itself, π_e = value critical. Under the following assumptions incompressible and unsteady flow, one-directional flow, rigid plate, non-Newtonian, unconfined convection, vertical plate oscillating, unidirectional streams, viscous dissipation terms in energy equation are neglected, then the governing equations and flow configuration of the problem are as follows (see Figure 1).

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}} = \mu_B\left(1+\frac{1}{y}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \sigma B_0^2\bar{u} + \rho g\beta(\bar{T}-\bar{T}_\infty) + \rho g\beta(\bar{C}-\bar{C}_\infty) - \frac{\mu\varphi}{K_1}\bar{u}, \tag{2.2}$$

$$\rho c_p\frac{\partial \bar{T}}{\partial \bar{t}} = k\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \rho c_p Q(\bar{T}-\bar{T}_\infty) + \vartheta\left(\frac{\partial \bar{u}}{\partial y}\right)^2, \tag{2.3}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - K(\bar{C}-\bar{C}_\infty). \tag{2.4}$$

Initial boundary conditions as

$$\begin{aligned} &\bar{u} = 0,\ \bar{T} = \bar{T}_\infty,\ \bar{C} = \bar{C}_\infty \text{ for } y > 0, \text{ if } t < 0, \\ &\bar{u} = UH(\bar{t})\cos(\bar{\omega}\bar{t}) \text{ or } \bar{u} = U\sin(\bar{\omega}\bar{t}),\ \bar{T} = \bar{T}_w,\ \bar{C} = \bar{C}_\infty \text{ for } y = 0 \text{ if } t \ge 0, \\ &\bar{u} \to 0,\ \bar{T} \to \bar{T}_\infty,\ \bar{C} \to \bar{C}_\infty \text{ as } y \to \infty. \end{aligned} \tag{2.5}$$

Consider the non dimensional variables as

$$\begin{aligned} &u = \frac{\bar{u}}{U},\ y = \frac{v\bar{y}}{U},\ Sc = \frac{v}{D},\ p_r = \frac{\mu c_p}{k},\ t = \frac{\bar{t}U^2}{v},\ M^2 = \frac{\sigma\beta_0^2 v}{\rho U^2},\ T = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ \omega = \frac{\bar{\omega}U^2}{v}, \\ &Gm = \frac{g\beta^*\vartheta(\bar{C}_w-\bar{C}_\infty)}{u_0^3},\ Gr = \frac{g\beta\vartheta(\bar{T}_w-\bar{T}_\infty)}{u_0^3},\ K_c = \frac{\vartheta K}{u_0^2},\ C = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}-\bar{C}_\infty},\ K = \frac{k_1 U^2}{\phi\vartheta^2}, \\ &Ec = \frac{U^2}{\rho c_p(\bar{T}_w-\bar{T}_\infty)},\ \gamma = \frac{\mu_B\sqrt{2\pi_c}}{P_y},\ \tau = \frac{\tau'}{\rho u^2}. \end{aligned} \tag{2.6}$$

Then eqs. (2.2)-(2.4) transforms as

$$\frac{\partial u}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - M^2 u - \frac{1}{K}u + Gr\,T + Gc\,C, \tag{2.7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{p_r}\frac{\partial^2 T}{\partial \bar{y}^2} + QT + Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{2.8}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_c C. \tag{2.9}$$

Boundary conditions are

$$u=0,\ T=0,\ C=0,\ \text{ for } y>0;\ \text{if } t<0$$
$$u=H(t)\cos(\omega t) \text{ or } u=\sin(\omega t),\ t=1,\ C=1,\ \text{at } y=0 \text{ if } t\geq 0,$$
$$u\to 0,\ T\to 0,\ C\to 0,\ \text{as } y\to\infty. \tag{2.10}$$

Method of solution

In Cartesian form plate is towards x-axis and y-axis is perpendicular to the plate. Velocity components u, v are along with x and y directions. The equations (2.7)-(2.9) are solved by Galerkin finite element technique.

Consider $r=\frac{k}{h^2}$ where k and h are mesh sizes in time and special y-direction respectively

$$\int_{y_j}^{y_k} N^{(e)^T}\left[\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial \tau}-\left(M^2+\frac{1}{K}\right)u+R\right]dy, \tag{2.11}$$

where $R=Gr\,T+Gc\,C$

$$u^{(e)}=N_j(y)u_j(t)+N_k(y)u_k(y),$$
$$N_j=\frac{y_k-y}{y_k-y_j},\ N_k\frac{y-y_j}{y_k-y_j},\ l^{(e)}=y_k-y_j=h,$$
$$\left[N^{(e)^T}\left(1+\frac{1}{\gamma}\right)\frac{\partial u^{(e)}}{\partial y}\right]_{y_j}^{y_k}$$
$$-\left\{\int_{y_j}^{y_k}\left(1+\frac{1}{\gamma}\right)\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)^T}\left(\frac{\partial u^{(e)}}{\partial t}+\left(M^2+\frac{1}{K}\right)u^{(e)}-R\right)\right\}dy=0. \tag{2.12}$$

The element equations given by

$$\int_{y_j}^{y_k}\left(1+\frac{1}{\gamma}\right)\begin{bmatrix}N_j'N_j' & N_j'N_k'\\ N_k'N_j' & N_k'N_k'\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}dy+\left(M^2+\frac{1}{K}\right)\begin{bmatrix}N_jN_j & N_jN_k\\ N_kN_j & N_kN_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}dy$$
$$+\begin{bmatrix}N_jN_j & N_jN_k\\ N_kN_j & N_kN_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}dy-R\begin{bmatrix}N_j\\ N_k\end{bmatrix}dy=0, \tag{2.13}$$

$$\frac{A}{l^{(e)}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{l^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}+\mathbf{B}\frac{l^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=R\frac{l^{(e)}}{2}\begin{bmatrix}1\\ 1\end{bmatrix}, \tag{2.14}$$

where $A=\left(1+\frac{1}{\gamma}\right)$, $B=\left(M^2+\frac{1}{K}\right)$ and dot denotes the differentiation with respect to t, assembling the element equation for two consecutive elements ($y_{i-1}\leq y\leq y_i$) and ($y_i\leq y\leq y_{i+1}$) the following is obtained

$$\frac{A}{l^{(e)}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{l^{(e)}B}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}+\frac{l^{(e)}}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_1\\ \dot{u}_{i+1}\end{bmatrix}=R\frac{l^{(e)}}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix},$$
$$\frac{A}{b^2}(-u_{i-1}+2u_i+u_{i+1})+\frac{B}{6}(u_{i-1}+4u_i+u_{i+1})+\frac{1}{6}(\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1})=R^*. \tag{2.15}$$

By applying the trapezoidal rule, along with Crank-Nicolson method to eq. (2.15) the following system are obtained

$$A_1u_{i-1}^{j+1}+A_2u_i^{j+1}+A_3u_{i+1}^{j+1}=A_4u_{i-1}^{j}+A_5u_i^{j}+A_6u_{i+1}^{j}+R^*, \tag{2.16}$$
$$R^*=12(Gr\,Tk+Gc\,Ck).$$

Similarly, for eq. (2.8)-(2.9) we obtained

$$B_1 u_{i-1}^{j+1} + B_2 u_i^{j+1} + B_3 u_{i+1}^{j+1} = B_4 u_{i-1}^{j} + B_5 u_i^{j} + B_6 u_{i+1}^{j} + Q^*, \tag{2.17}$$

$$Q^* = 12kEc\left(\frac{u(i+1)-u(i)}{h}\right)^2,$$

$$D_1 u_{i-1}^{j+1} + D_2 u_i^{j+1} + D_3 u_{i+1}^{j+1} = D_4 u_{i-1}^{j} + D_5 u_i^{j} + D_6 u_{i+1}^{j}. \tag{2.18}$$

The system of equations are obtained of the form $A_i X_i = B_i$, where $i = 1,2,3,\dots$.

To obtain velocity, temperature and concentration Thomas algorithm is applied on the system equations. For the convergence, stability of Galerkin FEM and to execute the numerical solution MATLAB is used.

$$A_1 = 2 - 6rA + Bk;\quad A_2 = 8 + 12rA + 4Bk;\quad A_3 = 2 - 6rA + Bk;$$

$$A_4 = 2 + 6rA - Bk;\quad A_5 = 8 - 12rA - 4Bk;\quad A_6 = 2 + 6rA - Bk;$$

$$B_1 = 2Pr - 6r - PrQk;\quad B_2 = 8Pr + 12r - 4PrQk;\quad B_3 = 2Pr - 6r - PrQk;$$

$$B_4 = 2r + 6r + PrQk;\quad B_5 = 8Pr + 12r - 4PrQk;\quad B_6 = 2Pr + 6r + PrQk;$$

$$D_1 = 2Sc + ScK_c rk - 6r;\quad D_2 = 8Sc + 12r + 4ScK_c rk;\quad D_3 = 2Sc + ScK_c rk - 6r;$$

$$D_4 = 2Sc - ScK_c rk + 6r;\quad D_5 = 8Sc - 12r - 4ScK_c rk;\quad D_6 = 2Sc - ScK_c rk + 6r.$$

3. Results and Discussion

The non-dimensional coupled nonlinear differential equations are solved through the boundary conditions by using FEM. To identify the physical characters of velocity, temperature and concentration profiles with effect of permeability parameter (K), Eckert number (Ec), Grashof number (Gr), Casson parameter (γ), Magnetic parameter (M), phase angle (ωt) and Prandtl number (Pr).

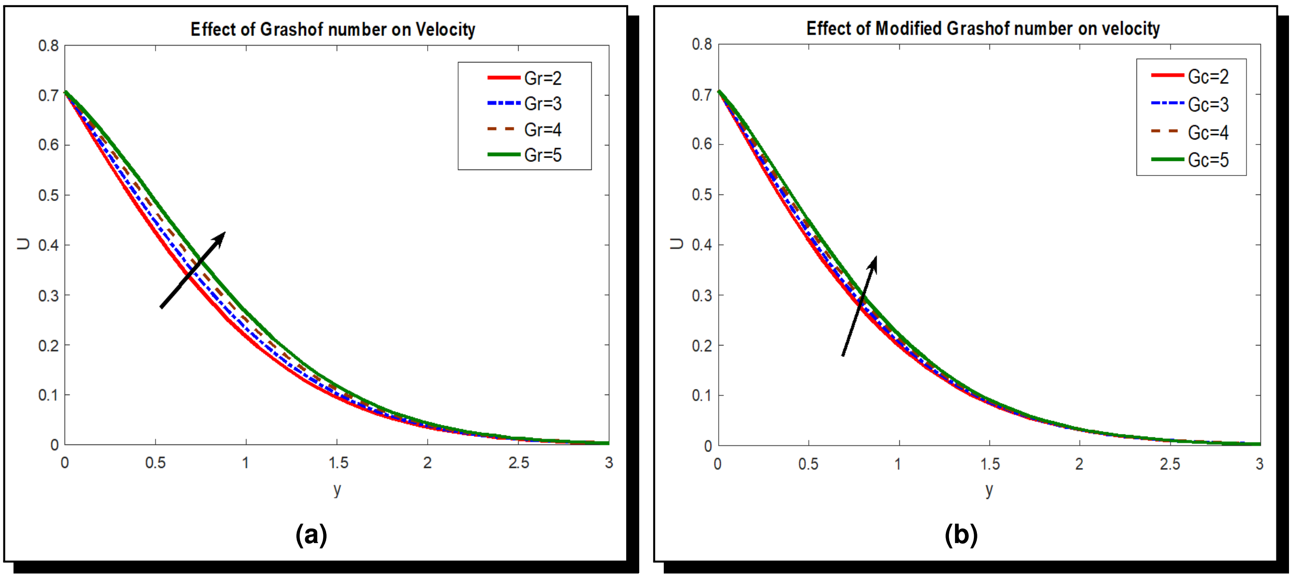

Figure 2

Figure 2(a,b) explains velocity profiles for the different values Grashof number (Gr) and modified (Gc). Gr and Gc are the ratio to buoyant force and viscous force. As Gr increases viscous forces acting on fluid are decreases due to this fluid velocity is raised.

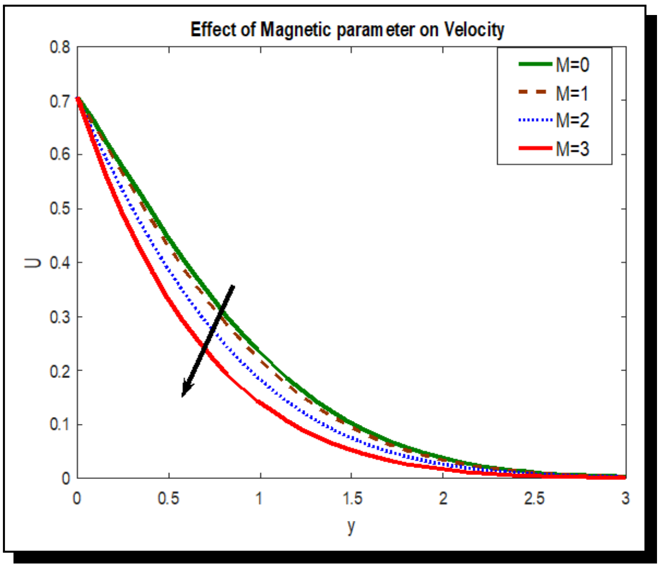

Figure 3

Figure 3 shows the variation in velocity for different values of magnetic field parameter (M), velocity of the flow depreciate with increasing of M values, because of an electrically conducting fluid produces drag force similar to Lorentz force this moderate the fluid velocity.

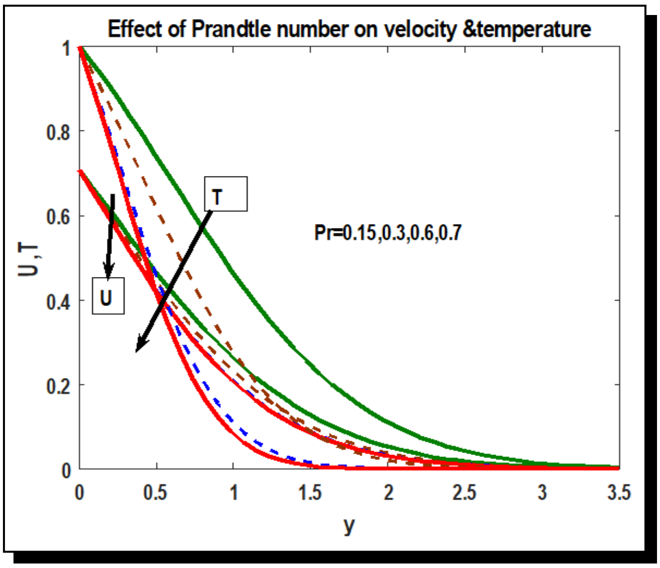

Figure 4

Figure 4 explains the velocity and temperature profiles for different values of Prandtl number (Pr). Pr is the ratio of momentum diffusion to thermal diffusion as momentum diffusion dominates the thermal diffusion value of the Pr increases which leads to the increase in the collusion of the fluid particle due to this slowdown the velocity of the fluid. As *Pr* increases thermal conductivity decrease because of reduction in thermal boundary layer which leads to depreciate the temperature of the fluid.

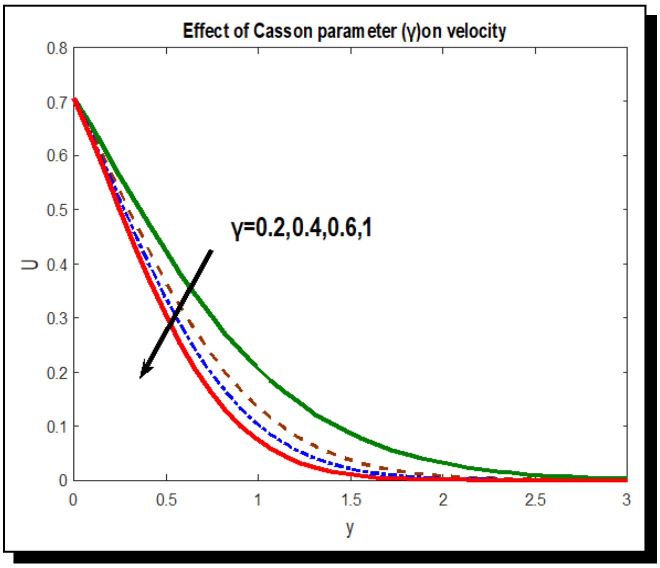

Figure 5

Figure 5 shows the effect on velocity with different values of Casson parameter. As Casson parameter increases, the boundary layer thickness will be thinner, which leads to reduce the velocity of the flow.

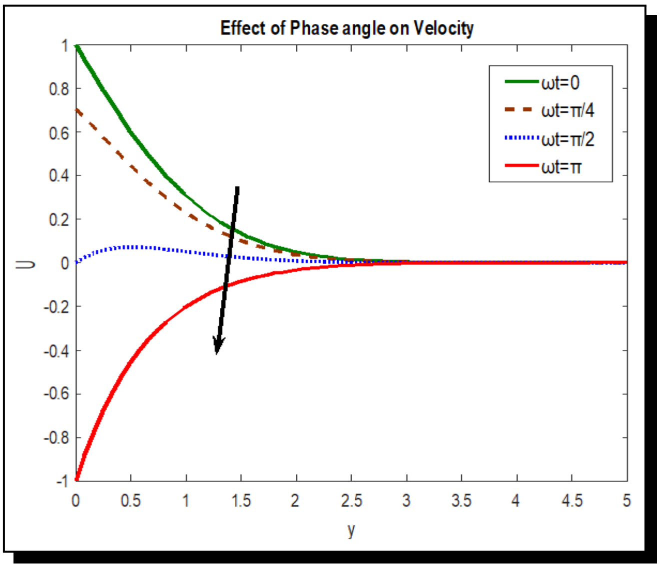

Figure 6

Figure 6 shows the influence of phase angle on velocity as phase angle varies from 0 to π velocity of the fluid slow down. The fluid velocity varying from -1 to 1, this indicates that velocity of the fluid is higher near the plate as far from the plate it is tends to 0.

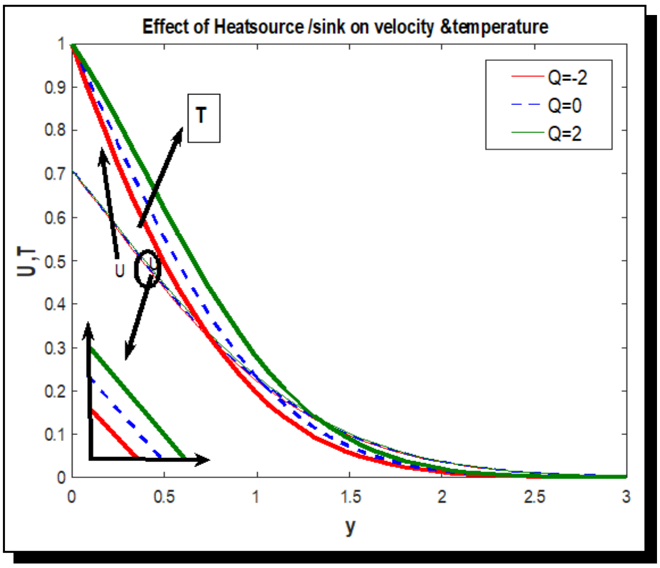

Figure 7

Figure 7 explains the effect on velocity and temperature for different values of Eckert number. The Eckert number describes the relationship between kinetic energy flow and boundary layer enthalpy difference. It represents the transformation of kinetic energy in to internal energy by doing work against the viscous fluid stresses. As Eckert number increases kinetic energy increases due to faster movement of fluid particle and this leads to enhance the temperature of the fluid.

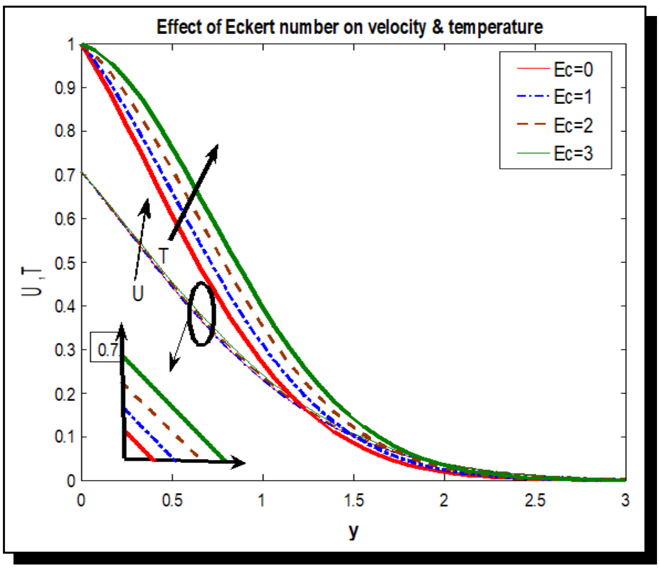

Figure 8

Figure 8 represents the influence of heat source on velocity and temperature. It is observed that enhancement in heat source increases thermal conductivity because of enrichment in the thermal boundary layer thickness due to this velocity and temperature of the fluid is increased.

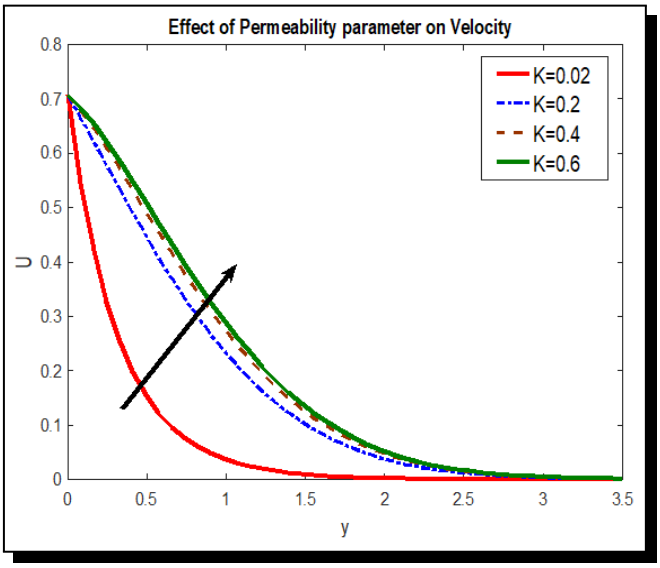

Figure 9

Figure 9 shows the influence of permeability parameter on velocity. As increase the value of permeability parameter, drag force acting on the fluid will decrease, at that time expand in size of the pores of the porous medium which leads to raise the velocity of the fluid.

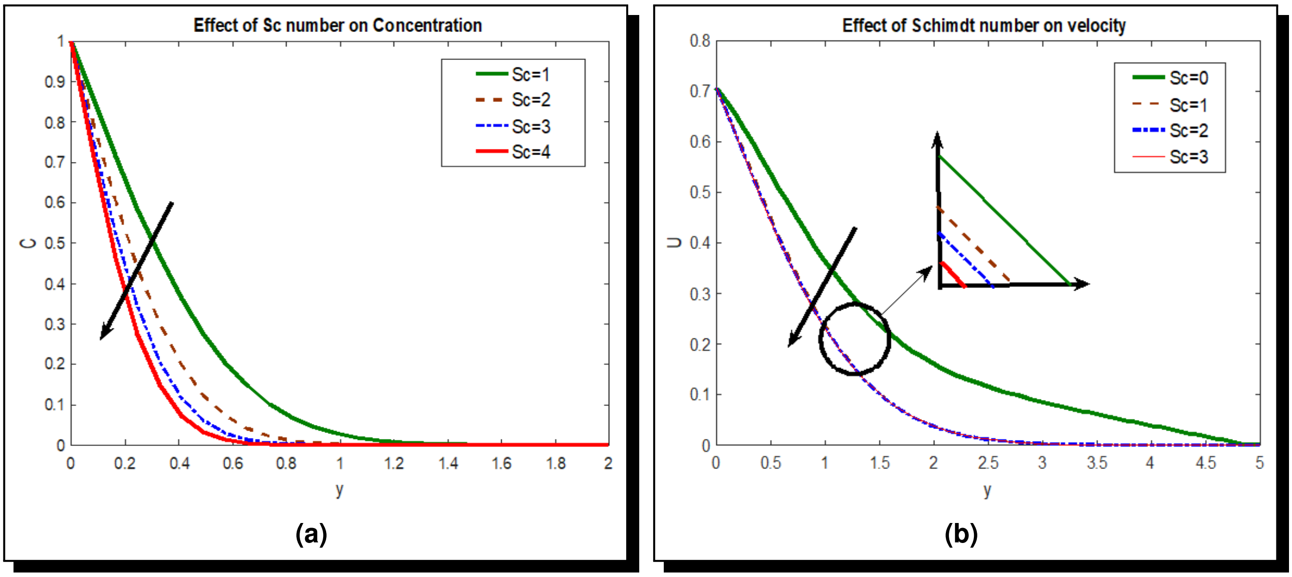

Figure 10

Figure 10(a,b) explains the influence of Schmidt number on velocity and concentration. As we know that Schmidt number is the ratio of kinematic viscosity and mass diffusivity. Due to enhancement of the Schmidt number kinematic viscosity dominates the mass diffusivity which leads to slowdown the velocity of the fluid and as mass diffusivity dominates the kinematic viscosity it provides the depreciation in concentration.

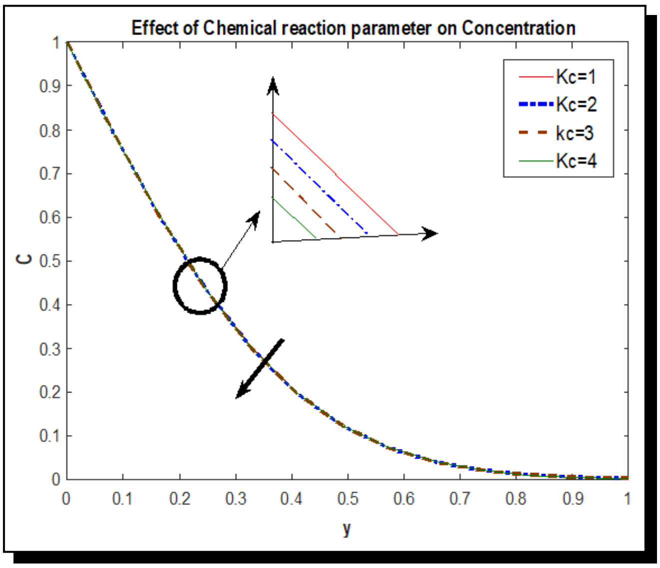

Figure 11

Figure 11 shows the influence of chemical reaction parameter on concentration, as K_C increases the concentration of the fluid also increases.

Table 1

γ	K	Gr	M	$\left(\frac{\partial U}{\partial y}\right)_{y=0}$
0.2	0.2	3	0.2	0.4171 ↑
0.3				0.4611
0.2	0.3	3	0.2	0.3315 ↓
	0.4			0.2860
0.2	0.2	4		0.3511
		5		0.2852
0.2	0.2	3	0.3	0.4195 ↑
			0.4	0.4230

Table 2

Pr	Ec	Q	$-\left(\frac{\partial T}{\partial y}\right)_{y=0}$
0.3	0.2	2	0.5450 ↑
0.6			0.7987
	0.4		0.5171 ↓
	0.5		0.5032
		3	0.4219 ↓
		4	0.2908

Table 3

K_c	Sc	$-\left(\frac{\partial C}{\partial y}\right)_{y=0}$
1	2	2.0489
	3	2.4922
2	2	2.0512
3		2.0534

4. Conclusions

The unsteady MHD free convection flow over past a vertical plate in porous medium by Casson fluid in the influence of Eckert number, Heat source/sink and chemical reaction parameter phenomenon has been discovered thoroughly. The conclusions are, the velocity increases with enhance in Permeability, Grashof, Heat source and Eckert number and depreciate with enrich in magnetic field, Prandtl number and Phase angle. Temperature profile based on variation in boundary layer as it is increase with enhancement of Heat source and Eckert number and reduces with Pr number. Concentration increases with Sc number and decreases with chemical reaction parameter K_c.

- From Table 1, it is also concluded that Shear stress is depreciates with an increase in Gr, K and increases with γ, M.
- From Table 2, as Pr increases Nu also increase and decreases with Heat source, Ec number.
- From Table 3, as Sc and K_c increases the value of Sh also increased.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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