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Research Article

Strong Vertex Coloring in Bipolar Fuzzy Graphs

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Abstract. Bipolar fuzzy graph (BFG) coloring techniques are used to solve many complex real world problems. The chromatic number of complement of BFG is obtained and compared with the chromatic number of the corresponding BFGs. This paper is an attempt to define coloring in a BFG based on strong edges. The strong chromatic number of complete BFG and BF tree are obtained.

Keywords. Bipolar fuzzy graphs, Bipolar Fuzzy Sets (BFS), coloring, strong edges and strong chromatic number

Mathematics Subject Classification (2020). 05C72

1. Introduction

In 1965, L.A. Zadeh [12] initiated the concept of BFS as a generalization of fuzzy sets. BFS are an extension of fuzzy sets whose membership degree range is $[-1,1]$. In a BF set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0,1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1,0)$ of an element indicates that the element somewhat satisfies the implicit counter-property. Although BFS and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with BF information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. Akram [1] introduced the concept of BFGs and defined different operations on it. Samanta *et al.* [10] introduced the concept coloring of FGs. Mathew and Sunitha [4], and Tom and Sunitha [11] introduced the concept of complement BFGs in 2014 and discussed about some connectivity concepts in BFGs. BFG coloring techniques are used to solve many complex real world problems. S.Y. Mohemed *et al.* [4,5] introduced structures of edge regular bipolar fuzzy graphs and bipolar fuzzy graphs based on eccentricity nodes. In this paper the chromatic number of complement of BFG is obtained and compared with the chromatic number of the corresponding BFGs. This paper is an attempt to define coloring in a BFG based on strong edges. The strong chromatic number of complete BFG and BF tree is obtained.

2. Preliminaries

In this section, some basic definitions and preliminary ideas are given which is useful for proving theorems.

Definition 2.1 ([1]). By a BFG, we mean a pair $G=(A,B)$ where $A=(\mu_A^P,\mu_A^N)$ is a BF set in V and $B=(\mu_B^P,\mu_B^N)$ is a BF relation on E such that

$$\mu_B^P(xy)\le\min(\mu_A^P(x),\mu_A^P(y)) \text{ and } \mu_B^N(xy)\ge\max(\mu_A^N(x),\mu_A^N(y))$$

for all $(x,y)\in E$.

Definition 2.2 ([1]). We call A the BF nodes set of V, B the BF edge set of E, respectively. Note that B is symmetric BF relation on A. We use the notation xy for an element of E. Thus,

$$G=(A,B) \text{ is a BFG of } G^*=(V,E) \text{ if } \mu_B^P(xy)\le\min(\mu_A^P(x),\mu_A^P(y))$$

and

$$\mu_B^N(xy)\ge\max(\mu_A^N(x),\mu_A^N(y)) \text{ for all } xy\in E.$$

$H=(\alpha,\beta)$ (where $\alpha=(\mu_A^P,\mu_A^N)$ is a BF subset of a set A and $\beta=(\mu_B^P,\mu_B^N)$ is a BF relation on B) is called a partial BF subgraph of G if $\alpha\le A$ and $\beta\le B$. We call $H=(\alpha,\beta)$ a spanning BF subgraph of $G=(A,B)$ if $\alpha=A$.

Definition 2.3 ([1]). A path p of length n is a sequence of distinct nodes $((\mu_A^p(u_0),\mu_A^N(u_0))(\mu_A^p(u_1),\mu_A^N(u_1))(\mu_A^p(u_2),\mu_A^N(u_2))\dots(\mu_A^p(u_n),\mu_A^N(u_n)))$ such that $(\mu_B^P(u_{i-1}u_i),\mu_B^N(u_{i-1}u_i))>0$ and degree of membership of a weakest edge is defined as its strength. If $(\mu_A^p(u_0),\mu_A^N(u_0))=(\mu_A^p(u_n),\mu_A^N(u_n))$ and $n\ge3$, then p is called a cycle and it is a BF cycle if there is more than one weak edge.

Definition 2.4 ([4]). A path P of length n is a sequence of distinct nodes $\mu_A^P(u_0),\mu_A^P(u_1)\dots\mu_A^P(u_n)$ and the degree of membership of a weakest edge is defined as its positive strength.

A path P of length n is a sequence of distinct nodes $\mu_A^N(u_0),\mu_A^N(u_1)\dots\mu_A^N(u_n)$ and the degree of membership of a greatest edge is defined as its negative strength.

Definition 2.5 ([4]). The strength of connectedness between two nodes $\mu_A^P(u)$ and $\mu_A^P(v)$ is defined as the maximum of the strengths of all paths between u and v and is denoted by $CONN_G^P(u,v)$.

The strength of connectedness between two nodes $\mu_A^N(u)$ and $\mu_A^N(v)$ is defined as the minimum of the strength of all paths between u and v and is denoted by $CONN_G^N(u,v)$.

Definition 2.6 ([4]). A u-v path p is called a strongest u-v path if its strength equals $(CONN_G^P(u,v), CONN_G^N(u,v))$.

Definition 2.7 ([4]). An edge (u,v) in G is called α-strong if ${\mu_B}^P(u,v) > CONN_{G\text{-}(u,v)}(u,v)$ and ${\mu_B}^N(u,v) < CONN_{G\text{-}(u,v)}(u,v)$.

Definition 2.8 ([4]). An edge (u,v) in G is called β-strong if ${\mu_B}^P(u,v) = CONN_{G\text{-}(u,v)}(u,v)$ and ${\mu_B}^N(u,v) = CONN_{G\text{-}(u,v)}(u,v)$.

Definition 2.9 ([4]). An edge (u,v) in G is called δ-edge if ${\mu_B}^P(u,v) < CONN_{G\text{-}(u,v)}(u,v)$ and ${\mu_B}^N(u,v) > CONN_{G\text{-}(u,v)}(u,v)$.

Definition 2.10 ([4]). A δ-edge (u,v) in G is called a δ^* edge if ${\mu_B}^P(u,v) > {\mu_B}^P(x,y)$ where (x,y) is a weakest edge of G and ${\mu_B}^N(u,v) < {\mu_B}^N(x,y)$ where (u,v) is weakest edge of G.

Definition 2.11 ([7]). An edge $((\mu_A^P(u),\mu_A^N(u)),(\mu_A^P(v),\mu_A^N(v)))$ of G is called m-strong if $\mu_B^P(u,v) = \min(\mu_A^P(u),\mu_A^P(v))$ and $\mu_B^N(u,v) = \max(\mu_A^N(u),\mu_A^N(v))$. Suppose $G:(A,B)$ be a BFG. The complement of G is denoted as $\bar{G}:(\bar{A},\bar{B})$ where $\bar{A} = A$ and

$$\bar{\mu}_B^P(xy) = \min(\mu_A^P(x),\mu_A^P(y)) - \mu_B^P(xy) \text{ and } \bar{\mu}_B^N(xy) = \max(\mu_A^N(x),\mu_A^N(y)) - \mu_B^N(xy).$$

3. Coloring of Bipolar Fuzzy Graphs

Definition 3.1 ([10]). Let $G:(A,B)$ be a BFG, where $A = (\mu_A^P,\mu_A^N)$ and $B = (\mu_B^P,\mu_B^N)$ be two BFS on a non-empty finite set V and $E \subseteq V \times V$, respectively. The positive and negative level set of BF set A is defined as

$$L_A^P = \{\alpha^P; \mu_A^P(u) = \alpha^P \text{ for some } u \in V\} \text{ and } L_A^N = \alpha^N; \mu_A^N(u) = \alpha^N \text{ for some } u \in V\}$$

and the positive and negative level set of B is defined as

$$L_B^P = \{\alpha^P; \mu_B^P(u,v) = \alpha^P \text{ for some } (u,v) \in V \times V\},$$
$$L_B^N = \{\alpha^N; \mu_B^N(u,v) = \alpha^N \text{ for some } (u,v) \in V \times V\}.$$

The fundamental set of the BFG $G = (A,B)$ is defined as

$$L^P = L_A^P \cup L_B^P \text{ and } L^N = L_A^N \cup L_B^N.$$

We define for

$$\alpha^P \in L^P,\ \alpha^N \in L^N,\ G_\alpha^P = (V_\alpha^P, E_\alpha^P)$$

where $V_\alpha^P = \{v \in V; \mu_A^P \geq \alpha^P\}$, $E_\alpha^P = \{e \in E; \mu_B^P \geq \alpha^P\}$ and

$$G_\alpha^N = (V_\alpha^N, E_\alpha^N),$$

where $V_\alpha^N = \{v \in V; \mu_A^N \leq \alpha^N\}$ and $E_\alpha^N = \{e \in E; \mu_B^N \leq \alpha^N\}$.

Definition 3.2. The chromatic number of G is defined as $(\chi_G^P,\chi_G^N) = \{\max\chi(G_\alpha^P), \min\chi(G_\alpha^N)\}$ where χ_α^P and χ_α^N are the chromatic number of G_α^P and G_α^N respectively and $\propto^P$ and $\propto^N$ are the positive and negative different membership value of vertices of G. Moreover, the chromatic number of BFG $G:(A,B)$ is fuzzy number $\chi(G) = \{(\chi_\alpha^P,\propto^P),(\chi_\alpha^N,\propto^N)\}$ where χ_α^P and χ_α^N are the chromatic number of G_α^P and G_α^N which $\propto^P \in L^P \cup \{0\}$ and $\propto^N \in L^N \cup \{0\}$.

Two vertices u and v, for any strong edge in G, are called adjacent if

$$\mu_B^P(u,v) = \min(\mu_A^P(u), \mu_A^P(v)) \text{ and } \mu_B^N(u,v) = \max(\mu_A^N(u), \mu_A^N(v)).$$

A family $\Gamma = (\Gamma^P, \Gamma^N) = (\{\gamma_1^P, \gamma_2^P, \dots \gamma_r^P\}, \{\gamma_1^N, \gamma_2^N \dots \gamma_s^N\})$ of BFS on set V is called a (r,s) BF coloring of $G = (A,B)$ if

(i) (a) $(\cup\Gamma^P, \cap\Gamma^N) = (\mu_i^P, \mu_i^N) = A$, (b) $\gamma_i^P \cap \gamma_j^P = 0$, $\gamma_i^N \cup \gamma_j^N = 0$.

(ii) For every strong edge (x,y) of G,

$$(\min\{\gamma_i^P(x), \gamma_i^P(y)\}, \max\{\gamma_j^N(x), \gamma_j^N(y)\}) = (0,0).$$

The minimum numbers r and s which there exists a (r,s)-BF coloring is called the BF chromatic number of G and denoted by $\chi^{BF}(G) = (\chi^P(G), \chi^N(G))$.

Theorem 3.1. *For a BFG $G:(A,B)$, $\chi(G) = \chi^{BF}(G)$.*

Proof. Let $G:(A,B)$ be a BFG on n vertices, $\{v_1, v_2, \dots v_n\}$.
Let $\chi^{BF}(G) = (\chi^P(G), \chi^N(G)) = (K, K')$.
Then $\Gamma = (\{\gamma_1^P, \gamma_2^P, \dots \gamma_K^P\}, \{\gamma_1^N, \gamma_2^N \dots \gamma_{K'}^N\})$ is a (K,K') BF coloring.
If C_j^P and C_j^N are the colors assigned to vertices in $\gamma_{j^P}^*$ and $\gamma_{j^N}^*$ where

$$\gamma_{j^P}^* = \{v \epsilon V, \gamma_j(v) > 0\}, \quad \gamma_{j^N}^* = \{v \epsilon V, \gamma_j(v) < 0\}$$

then $(\{\gamma_1^P, \gamma_2^P, \dots \gamma_K^P\}, \{\gamma_1^N, \gamma_2^N \dots \gamma_{K'}^N\})$ is a family of BFS where

$$\gamma_j^P(v_i) = \{v_j, \mu_A^P(v_j) \cup \{v_i, \mu_A^P(v_i); \mu_B^P(v_i, v_j) = 0, i \neq j\}\}$$

and

$$\gamma_j^N(v_i) = \{v_j, \mu_A^N(v_j) \cup \{v_i, \mu_A^N(v_i); \mu_B^N(v_i, v_j) = 0, i \neq j\}\}$$

Also, $\cup_{j=1}^K \gamma_j^{P^*} = \cup_{j=1}^{K'} \gamma_j^{N^*} = V$ and $\gamma_i^{P^*} \cap \gamma_j^{P^*} = \gamma_i^{N^*} \cap \gamma_j^{N^*} = \emptyset$, $i^P \neq j^P$, $i^N \neq j^N$.
Hence $\gamma_i^{P^*}$ and $\gamma_j^{N^*}$ are independent sets of vertices.
Then

$$\chi(G) = (\chi(G_t), \chi(G_{t'})) = (K, K')$$

where

$$t = \min\{\alpha^P, \alpha^P \in L^P\} = \min\{\chi_\alpha^P, \alpha^P \in L^P\}$$

and

$$t' = \max\{\alpha^N, \alpha^N \in L^N\} = \max\{\chi_\alpha^N, \alpha^N \in L^N\}.$$

Therefore $\chi(G) = \chi^{BF}(G)$. □

Theorem 3.2. *Consider the following BFG G and the crisp graphs $G_{0.9}$, $G_{0.7}$, $G_{0.4}$, $G_{0.3}$, G_0, $G_{-0.7}$, $G_{-0.5}$, $G_{-0.3}$, $G_{-0.1}$.*

Given in Figures 1 and 2 corresponding to the values in the positive and negative level set of

$$L^P = \{0.9, 0.7, 0.5, 0.3, 0.2, 0.1\} \cup \{0\} \quad \text{and} \quad L^N = \{-0.7, -0.5, -0.3, -0.1\}.$$

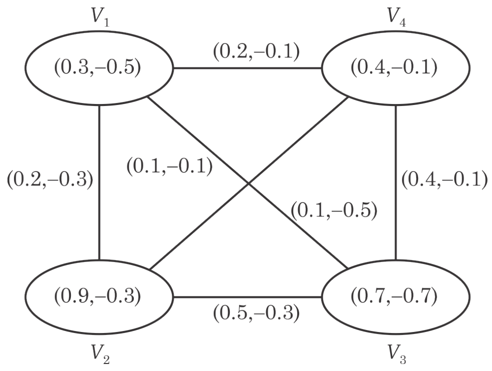

Figure 1

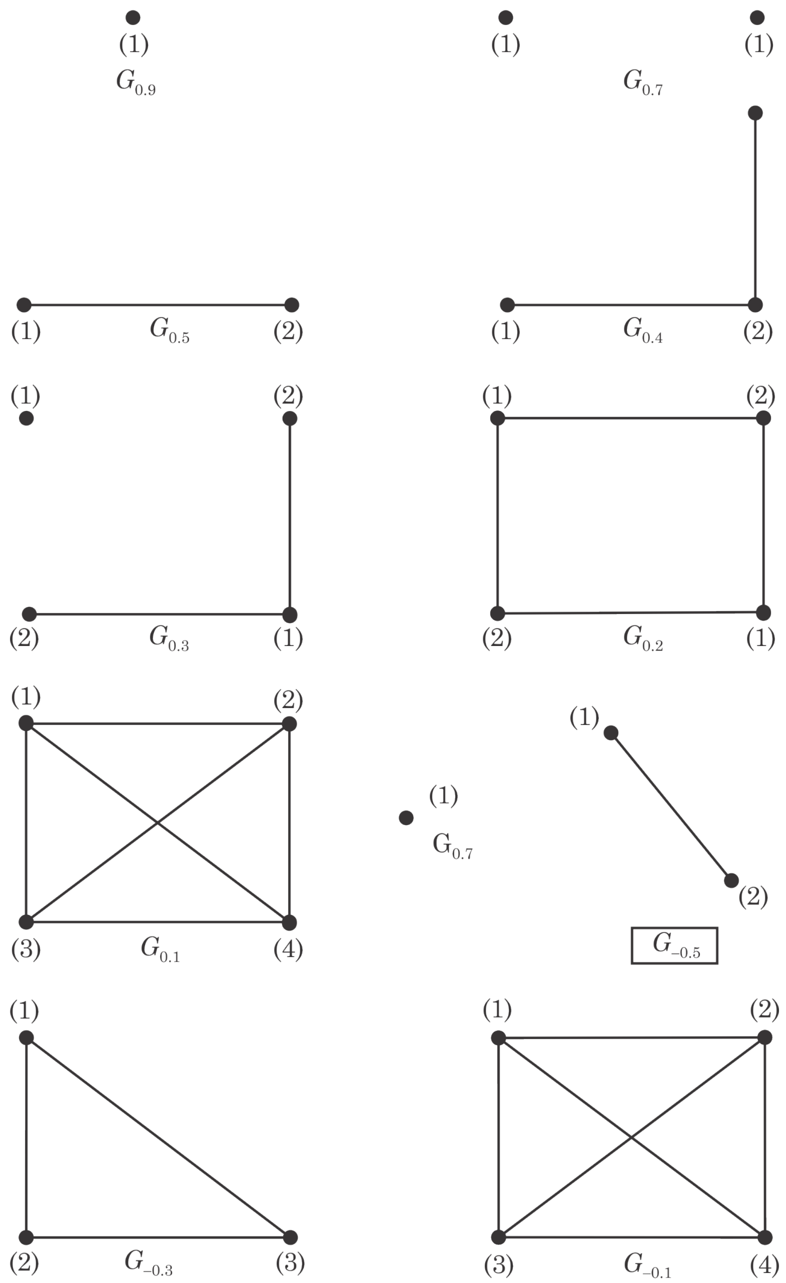

Figure 2

The BF coloring is $\Gamma = (\{\gamma_1^P, \gamma_2^P, \gamma_3^P, \gamma_4^P\}, \{\gamma_1^N, \gamma_2^N, \gamma_3^N, \gamma_4^N\})$

$$\gamma_1^P(x_i) = \begin{cases} 0.3 & i=1 \\ 0 & \text{otherwise,} \end{cases} \quad \gamma_2^P(x_i) = \begin{cases} 0.9 & i=2 \\ 0 & \text{otherwise,} \end{cases}$$

$$\gamma_3^P(x_i) = \begin{cases} 0.7 & i=3 \\ 0 & \text{otherwise,} \end{cases} \quad \gamma_4^P(x_i) = \begin{cases} 0.4 & i=4 \\ 0 & \text{otherwise,} \end{cases}$$

$$\gamma_1^N(x_i) = \begin{cases} -0.5 & i=1 \\ 0 & \text{otherwise,} \end{cases} \quad \gamma_2^N(x_i) = \begin{cases} -0.3 & i=2 \\ 0 & \text{otherwise,} \end{cases}$$

$$\gamma_3^N(x_i) = \begin{cases} -0.7 & i=3 \\ 0 & \text{otherwise,} \end{cases} \quad \gamma_4^N(x_i) = \begin{cases} -0.1 & i=4 \\ 0 & \text{otherwise.} \end{cases}$$

Vertices	(γ_1^P, γ_1^N)	(γ_2^P, γ_2^N)	(γ_3^P, γ_3^N)	(γ_4^P, γ_4^N)	$\max(\gamma_i^P, \gamma_i^N)$
V_1	$(0.3, -0.5)$	$(0,0)$	$(0,0)$	$(0,0)$	(0.3,-0.5)
V_2	$(0,0)$	$(0.9, -0.3)$	$(0,0)$	$(0,0)$	$(0.9, -0.3)$
V_3	$(0,0)$	$(0,0)$	$(0.7, -0.7)$	$(0,0)$	$(0.7, -0.7)$
V_4	$(0,0)$	$(0,0)$	$(0,0)$	$(0.4, -0.1)$	$(0.4, -0.1)$

Hence

$$\chi^{BF}(G) = (\chi^P(G), \chi^N(G)) = (4,4)$$

and the crisp graphs coloring yields $\chi(G) = (\max \chi(G_\alpha^P), \min \chi(G_\alpha^N)) = (4,4)$.

Example 3.1. Consider Figure 3 where the graph $G:(V,E)$,

$$\Gamma = (\{\gamma_1^P, \gamma_2^P, \gamma_3^P, \gamma_4^P, \gamma_5^P\}, \{\gamma_1^N, \gamma_2^N, \gamma_3^N, \gamma_4^N, \gamma_5^N\}).$$

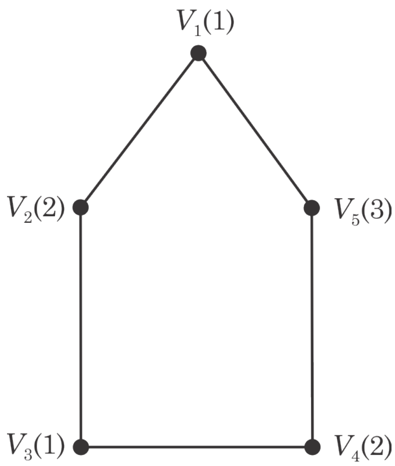

Figure 3

$\gamma_1 = \{(V_1^P, V_1^N), (V_3^P, V_3^N)\}$, $\gamma_2 = \{(V_2^P, V_2^N), (V_4^P, V_4^N)\}$, $\gamma_3 = \{(V_3^P, V_3^N), (V_5^P, V_5^N)\}$,
$\gamma_4 = \{(V_1^P, V_1^N), (V_4^P, V_4^N)\}$ and $\gamma_5 = \{(V_2^P, V_2^N), (V_5^P, V_5^N)\}$.

Out of the subfamilies of Γ satisfying the condition

$$\gamma_i^P \cap \gamma_j^P = \emptyset \text{ and } \gamma_i^N \cup \gamma_j^N = \emptyset,\ i \neq j \text{ is } \Gamma' = \{(\gamma_1^P, \gamma_1^N), (\gamma_2^P, V_2^N)\}.$$

Hence $T = (u_{5,}^P u_5^N)$. Assigning colors C_1 and C_2 to γ_1 and γ_2, respectively, $k = 2$ and $k' = 2$. Put $G = (T)$. Since $|T| = 1$, assign a different color C_3 to $\gamma = (u_5^P, u_5^N)$ and $\chi(G) = 1$.
Hence new $k' = 3$, i.e., the chromatic number $\chi^{BF}(G) = 3$.

4. Strong Chromatic Number of Bipolar Fuzzy Graphs

Coloring of graphs play a vital role in network problems. In any network, modeled as a BFG, the role of δ-edge is negligible, as the flow is minimum along δ-edge and there is an alternate strong path (maximum flow) between the corresponding nodes. Hence strong edges are more significant in networks. This motivated us in defining a new coloring of BFG based on strong edges.

Definition 4.1. The coloring $C : V(G) \to N$ (where N is the set of all positive integers) such that $[C^P(u), C^N(u)] \neq [C^P(v), C^N(v)]$ if $[\mu_B^P(u,v), \mu_B^N(u,v)]$ is a strong edge in G is called strong coloring.

A BFG G is k-strong colorable if there exists a strong coloring of G from a set of k colors. The minimum number k for which G is k-strong colorable is called strong chromatic number of G denoted by $\chi_S(G)$.

Example 4.1. In Figure 4, G is strong coloring Bipolar fuzzy graph.

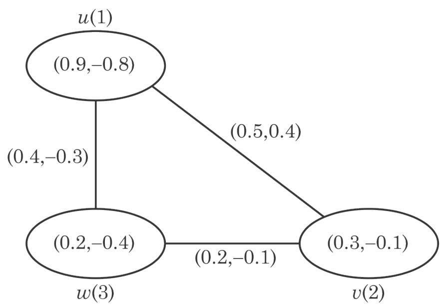

Figure 4

The number inside the brackets represents colors assigned to the vertices.
Here $\chi^{BF}(G) = 3$. In Figure 4 we find that edge (u,v) is α-strong.
Since

$$\mu_B^p(u,v) = 0.5 > CONN_{G\text{-}(u,v)}^P(u,v) = 0.2$$

and

$$\mu_B^N(u,v) = -0.4 < CONN_{G\text{-}(u,v)}^N(u,v) = -0.1.$$

Similarly, edge (u,w) is α-strong. Since,

$$\mu_B^p(u,w) = 0.4 > CONN_{G\text{-}(u,w)}^P(u,w) = 0.2$$

and

$$\mu_B^N(u,w) = -0.3 < CONN_{G\text{-}(u,w)}^N(u,w) = -0.1.$$

But

$$\mu_B^p(w,v) = 0.2 < CONN^P_{G\text{-}(w,v)}(w,v) = 0.4$$

and

$$\mu_B^N(w,v) = -0.1 > CONN^N_{G\text{-}(w,v)}(w,v) = -0.4$$

and hence (w,v) is a δ-edge.
Hence strong coloring gives colors 1 for u and 2 for both w and v. Thus $\chi_S(G) = 2$.

The results on strong coloring of complete BFG and fuzzy tree are as follows.

Proposition 4.1. *For a BFG $G:(A,B)\chi_S(G) = \chi^{BF}(G)$, if G is complete BF cycle.*

Proof. $\chi_S(G) = \chi^{BF}(G) = P$, if G is complete with P vertices since there are no δ-edges in a complete BFG and if G is a BF cycle on P vertices then G contains only strong edges.
Thus $\chi_S(G) = \chi^{BF}(G) = 2$, when P is even and $\chi_S(G) = \chi^{BF}(G) = 3$ when P is odd. □

The converse of the above proposition is not true.

Example 4.2. Consider the following BFG (Figure 5).

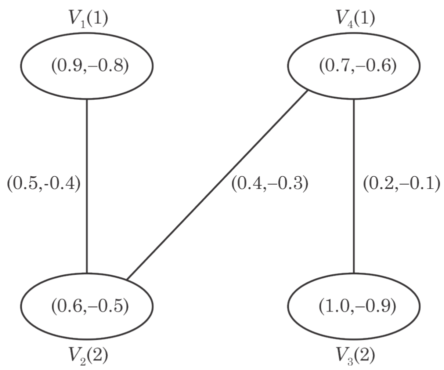

Figure 5

Here $\chi_S(G) = \chi^{BF}(G) = 2$, but the BFG in Figure 5 is neither complete nor a BF cycle.

Proposition 4.2. *If G is a BF tree, then $\chi_S(G) = \chi^{BF}(T)$ where T is the maximum spanning tree of G.*

Proof. Note that an edge $[\mu_B^P(x,y), \mu_B^N(x,y)]$ in a BF tree of G is strong if and only if $[\mu_B^P(x,y), \mu_B^N(x,y)]$ is an edge in T.
Hence strong coloring of G is exactly same as coloring of T and hence $\chi_S(G) = \chi^{BF}(T)$. □

5. Conclusion

The modeling of real life situations using BFGs is widely investigated area of current research. The relation between chromatic number of a BFG and its complement is studied. This paper is an attempt to solve network flow problems using the concept of strong coloring. We try to extend our concept to other application domains.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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