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Research Article

Regional Analysis of Consecutive Days Maximum Rainfall Using L-Moments: A Case Study for Brahmaputra Valley Region of Assam, India

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Abstract. In this study, Regional Rainfall Frequency analysis has been carried out based on L-moment in the Brahmaputra Valley. For this purpose, the consecutive 1 to 6 days maximum rainfall for 11 sites (data varying from 14 years to 23 years) have been considered for our study. The heterogeneity measures for 1 to 6 days of the study region are calculated. The five probability distributions namely, generalized extreme value (GEV), generalized logistic (GLO), generalized Pareto (GPA), generalized normal (GNO), and the Pearson Type III (PE3) have been considered. The Z-statistics criteria and L-moment ratio diagram for goodness of fit tests have been used for identifying the best fitting distributions for 1 to 6 days of the study area. The PE3 distribution is identified as the best fitting distribution for the consecutive 1-day and 6-days maximum rainfall while GLO distribution for the consecutive 2-days and 4-days, GNO distributions for the consecutive 3-days and 5-days maximum rainfall of the study region. The regional relationships for each season of the study regions are developed.

Keywords. Probability, Distribution, L-moment, Heterogeneity: Z-statistics, L-moment ratio diagram

Mathematics Subject Classification (2020). 60E05, 62-04, 62-11

1. Introduction

Rainfall is a strange occurrence that occurs in both time and space. Rainfall distribution is extremely unequal, varying substantially not only from place to location but also from year to year. Rainfall is a critical and dominating factor in the formulation and implementation of any agricultural programme in any given region. Furthermore, the most crucial factor for the country's low rice output is a lack of sufficient and proper water supply [15]. Because of shifting patterns of precipitation and a lack of proper land and water resource usage, the irrigated agriculture and food industry are under stress [12]. Crop planning in a region is influenced by irrigation methods, drainage, irrigational design, soil properties, geography, and socioeconomic conditions. In rainfed environments, however, rainfall magnitude and distribution in space and time are the most critical determinants [17]. Again, due to multiple moisture pressures, the early termination of rainfall has an influence on productivity, particularly when Kharif crops are at crucial points of grain formation [5].

In order to maintain a consistent level of agricultural production, it is essential to handle agricultural on a scientific (technical) basis. In rain-fed locations, this means examining the pattern of dry and rainy periods in the area in order to design a crop plan. A wet and dry spell forecast by scientists can help farmers improve their fields and crop quality, thus improving their financial situation. Rainfall frequency analysis can be done in a variety of ways, including daily, weekly, month-by-month, seasonally yearly, and with the highest rainfall occurring over a period of consecutive days. All of these factors have a role in predicting rainfall patterns and assisting farmers in their agricultural planting. It also aids in the construction of small and large dams, bridges, drainage channels, and other hydrology-related engineering projects.

The accuracy of the estimate is strongly related to the length of the data; hence, long-term data sets were necessary for at-site rainfall data analysis. A longer record appears to be the most advantageous feature of regional frequency analysis. Many studies have been carried out in order to analyze the frequency of regional floods and rainfall. There was a substantial advantage to the L-moment, devised by Hosking and Wallis [11], in that it combined a *probability-weighted moment* (PWM) with the linear function of the data set. The inverse form of the various distributions used to describe the parameters was explained by Greenwood *et al*. [7]. Regional data helps to reduce sampling uncertainty by giving a better distribution choice for data. According to Bobée *et al*. [3], researchers Fowler and Kilsby [6] found that excessive precipitation in the United Kingdom has altered the character of global climate change, based on regional rainfall studies from 1961 to 2000. In developing rainfall estimation algorithms in Iran, Malekinezhad and Garizi *et al*. [13] utilized the L-moment for daily rainfall frequency analysis.

Using frequency analysis, it is possible to predict how often a given event will occur. Data on the magnitude and frequency of extreme events is being gathered through the use of probability distributions (Noto and La Loggia [14]). Deka *et al*. [4] used L-moment and LQ-moment to study maximum rainfall every day in the Northeast for 42 years. In Pakistan, Ahmad *et al*. [1] used several moments to monitor the yearly daily maximum rainfall, and they came to the conclusion that the goodness of fit estimation was used to pick the fit probability distribution. By comparing the L-moment approach with commonly used nine distributions, Alam *et al*. [2] found that the best-suited probability distribution for monthly maximum rainfall in Bangladesh used the L-moment method.

The probability distribution that is selected is critical since a poor choice could result in substantial bias and inconsistency in new model flood predictions, particularly for longer return period predictions. This can lead to either an underestimation or an overestimation of our abilities, which can have serious consequences for how we live our lives. In their research work, Rahman *et al*. [16] placed emphasis on the importance of determining the best-fitting probability distribution function for floods in Australia. For the purpose of consecutive day rainfall analysis, we are attempting to determine the best-fitting probability distribution in order to obtain an accurate return period for the season.

2. Study Region and Data Collection

Eleven locations in Assam's Brahmaputra Valley were sampled for this research, which gathered rainfall data. Among them are Beki, Dibrugarh, Golaghat, Guwahati, Jorhat, Kokrajahar, North Lakhimpur, Sibsagar, and Kampur from the Indian Meteorological Department, Guwahati, for about 15 years of rainfall data. The Brahmaputra valley region is where the most tea gardens are located, so this is where we conduct our research. Our economy relies heavily on this. Also, there are the tiniest tea gardens in this area. Rainfall is the most important factor affecting tea production. A thorough examination of rainfall frequency is required in order to predict how much rain will fall in the future. Small tea plantations can not handle the dry season's water demands like larger tea estates can.

3. Methods

3.1 Different Probability Distributions Utilizing in Our Investigation

The probability distributions employed by Hosking and Wallis [11] under the L-moment were different. In this study, we made use of these strategies. These five distributions are represented by their probability density functions (PDFs) as well as their quantile functions.

3.1.1 Generalized Logistic Distribution (GLO)

A three-parameter distribution consisting of location(ξ), scale (α), and shape (k) is GLO. The *PDF* and quantile functions for this are given by

$$f(x)=\alpha^{-1}\{1-k\frac{(x-\xi)}{\alpha}\}^{\frac{1}{k}-1}\left[1+\left\{1-k\frac{(x-\xi)}{\alpha}^{\frac{1}{k}}\right\}\right]^{-2}. \tag{3.1}$$

The quantile function $Q(F)$ is in the form

$$Q(F)=\xi+\frac{\alpha}{k}\left[1-\left\{\frac{(1-F)}{F}\right\}^{k}\right]. \tag{3.2}$$

3.1.2 Generalized extreme value distribution (GEV)

A three-parameter distribution for maximum is *GEV* which gives shape, scale and location parameters. The probability distribution function and the quantile function of the *GEV* are given by

$$f(x)=\frac{1}{\alpha}\left[1-k\frac{(x-\xi)}{\alpha}^{\frac{1}{k}-1}\right]\exp\left[-\left\{1-k\frac{(x-\xi)}{\alpha}\right\}^{\frac{1}{k}}\right], \tag{3.3}$$

$$Q(F) = \xi + \frac{\alpha}{k}\left[1-(-\ln F)^k\right]. \tag{3.4}$$

3.1.3 Generalized Pareto Distribution (GPA)

In precipitation analysis, the GPA distribution is applied. This is also three parameters distribution. The parameters can be estimated by using L-moments methods. The *pdf* and *cdf* are as follows:

$$f(x) = \alpha^{-1}\exp[-(1-k)y], \tag{3.5}$$

where

$$f(x) = \begin{cases} -k^{-1}\log\{1-k(x-\xi)/\alpha, & k \neq 0, \\ (x-\xi)/\alpha, & k = 0, \end{cases}$$

$$0 \le x \le \alpha/k \quad \text{if } k > 0, \quad 0 = x < \propto \ \text{if } k = 0,$$

$$F(x) = 1-\exp(-y),,$$

$$x(F) = f(x) = \begin{cases} \xi + \frac{\alpha\{1-(1-F)^k\}}{k}, & k \neq 0, \\ \xi - \alpha\log(1-F), & k = 0. \end{cases} \tag{3.6}$$

3.1.4 Generalized Normal (Lognormal) (*GNO*)

A three-parameter distribution is *GNO* which gives shape, scale and location parameters. The probability distribution function and quantile function of the *GNO* are given by

$$f(x) = \frac{\exp\left[-\log\left\{1-k\frac{(x-\xi)}{\alpha}\right\}-\frac{1}{2}\left[-\frac{1}{k}\log\left\{1-k\frac{(x-\xi)}{\alpha}\right\}\right]\right]^2}{\alpha(2\pi)^{\frac{1}{2}}}. \tag{3.7}$$

The quantile function $Q(F)$ is in the form

$$Q(F) = \xi + \frac{\alpha}{k}\{1-\exp(-k\phi^{-1}(F))\}. \tag{3.8}$$

3.1.5 Pearson Type III (PE3)

A three-parameter (ξ, α and β represent location, scale and shape) distribution is PE3 which gives shape, scale and location parameters. The probability distribution function and quantile function of the PE3 are given by

$$f(x) = (\beta^\alpha\Gamma\alpha)^{-1}(x-\xi)^{\alpha-1}\exp\left\{-\frac{(x-\xi)}{\beta}\right\}, \tag{3.9}$$

$$Q(x) = \mu + \sigma\left[\frac{2}{\Gamma}\left\{1+\frac{\Gamma}{6}\phi^{-1}(F)-\frac{\Gamma^2}{36}\right\}^3-\frac{2}{\Gamma}\right], \tag{3.10}$$

where $\mu = \xi + \frac{2\sigma}{\Gamma}$, $\alpha = \frac{4}{\Gamma^2}$ and $\beta = \frac{1}{2}\sigma\ \mathrm{mod}(\Gamma)$.

By putting $F = 1-\frac{1}{T}$ in the above four distribution functions, we have the Quantile Function for return period T years and is known as statistical distribution function.

3.2 Moments of Distribution

Probability Weighted Moments and L-Moments

Probability Weighted Moments (*PWMs*) was given by Greenwood *et al.* [7] first time as

$$M_{p,r,s} = E\left[X^p\{F(x)\}^r\{1-F(x)\}^s\right] = \int_0^1 [x(F)]^p F^r(1-F)^s dF\,, \tag{3.11}$$

where X is a real valued random variable with *distribution function* F and p, r, s are real. A special case for the two moments $M_{1,0.s}$ and $M_{1,r,0}$ are generally used and defined as

$$\alpha_r = M_{1,0.r} = \int_0^1 x(F)(1-F)^s dF\,, \tag{3.12}$$

$$\beta_r = M_{1,r,0} = \int_0^1 x(F)F^r dF, \quad r = 0,1,2,\dots, \tag{3.13}$$

where $p,r,s \in \mathbb{R}$, $M_{p,0,0}$ is the conventional moment of order p about the origin and $x(F)$ is the inverse *CDF* of x calculated at the probability F.

In 1993, Hosking and Wallis [10] gave the general form L-moments in connection with PWM as

$$\lambda_{r+1} = (-1)^r \sum_{k=0}^{r} p^*_{r,k}\alpha_k = \sum_{k=0}^{r} p^*_{r,k}\beta_k, \quad r = 0,1,\dots, \tag{3.14}$$

where $p^*_{r,k}$ defined by Hosking and Wallis [11] as

$$p^*_{r,k} = \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!}. \tag{3.15}$$

Therefore, the first four L-moments, which are the linear combination of PWMs, are

$$\lambda_1 = \beta_0, \tag{3.16}$$

$$\lambda_2 = 2\beta_1 - \beta_0, \tag{3.17}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0, \tag{3.18}$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0. \tag{3.19}$$

Also, we have

$$\lambda_r = \int_0^1 xF(x)P^*_{r-1}(F)dF, \quad r = 1,2,\dots, \tag{3.20}$$

where $P^*_r(F) = \sum_{k=0}^{r} P^*_{r,k}F^k$ is the rth shifted Legendre polynomial.

The L-moments ratio (LMRs) is given by Hosking and Wallis [11] as:

$$\text{coefficient of L-variation}: \quad \tau = \frac{\lambda_2}{\lambda_1} \tag{3.21}$$

$$\text{coefficient of L-skewness}: \quad t_3 = \frac{\lambda_3}{\lambda_2} \tag{3.22}$$

$$\text{coefficient of L-Kurtosis}: \quad t_4 = \frac{\lambda_4}{\lambda_2} \tag{3.23}$$

t, t_3 and t_4 are denoted in short by LC_v, LC_s and LC_k.

The sample estimation of L-moments can be defined as:

$$\hat{\lambda}_{r+1} = \sum_{k=0}^{r} p_{r,k}^{*} b_k \tag{3.24}$$

with

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} x_j \tag{3.25}$$

where x_j, for $j = 1,2,\dots n$ is the ordered sample and n is the sample size. The sample estimates of β_r and λ_r are unbiased.

Again, another equivalent statistical tool was given by Hosking and Wallis [11] for the L-moments ratio which can be used to judge the consistency with the data sample. These are

$$\hat{\lambda} = \frac{\hat{\lambda}_2}{\hat{\lambda}_1}, \quad \hat{\lambda}_3 = \frac{\hat{\lambda}_3}{\hat{\lambda}_2}, \quad \hat{\lambda}_4 = \frac{\hat{\lambda}_4}{\hat{\lambda}_2}. \tag{3.26}$$

3.3 Regional Rainfall Frequency Analysis

At our study site, we use the steps outlined by Hosking and Wallis [11] for regional rainfall frequency analysis. Step-by-step instructions are provided in the subsections below.

3.3.1 Screening of Data by using the Discordancy Measure

Hosking and Wallis [10] developed two tools for Discordancy measure (D_i). It is a measure of dissimilarity. The difference between a site's L-moment ratios and the average L-moment ratios of similar sites is used to calculate the similar sites. This data can also be used to find out incorrect data sets. The discordance D_i assessment calculates the distance between the given site and the group's middle.

Hosking and Wallis ([10], [11]) established the formula for discordance measure by taking the sites NS in a group as follows:

Let $u_i = [\hat{\tau}^{(i)} \quad \hat{\tau}_3^{(i)} \quad \hat{\tau}_4^{(i)}]$ be a vector containing the LMRs, $\hat{\tau}^{(i)}, \hat{\tau}_3^{(i)}, \hat{\tau}_4^{(i)}$ for the site (i), then the group average for NS sites is given equation

$$\overline{u} = \frac{1}{NS} \sum_{i=1}^{NS} u_i. \tag{3.27}$$

The sample Covariance matrix is given by the equation

$$S = \sum_{i=1}^{NS} (u_i - \overline{u})(u_i - \overline{u})^T. \tag{3.28}$$

The discordancy measure is given for the site i by the equation

$$D_i = \frac{1}{3} NS (u_i - \overline{u})^T S^{-1} (u_i - \overline{u}). \tag{3.29}$$

If D_i is high, a site i is declared to be unusual. Hosking and Wallis [11] calculated a critical value for the discordancy statistic D_i against the number of sites in an area. If D_i is greater than the critical value, the site i is declared to be discordant.

Following are the tabulated form for discordancy measures for seasonal rainfall of 11 gauging site.

We use FORTRAN-77 source code for these functions on request from *IBM Watson* (https://www.ibm.com/watson/) or from the STATLIB software repository (http://lib.stat.emu.edu/general/LMOMENTS) which is free package.

Table 1. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive one day maximum rainfall of Brahmaputra Valley

Name of site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	79.28	0.1961	0.2151	0.1815	0.47
Dibrugarh	15	89.89	0.1771	0.1966	0.0378	0.90
Goalpara	15	74.97	0.2476	0.2859	0.2146	0.37
Golaghat	15	74.56	0.2193	0.1604	0.0679	0.17
Guwahati	15	77.61	0.2652	0.0198	0.0273	2.07
Jorhat	23	87.21	0.1645	0.1090	0.0842	0.64
Kokrajhar	15	108.76	0.2697	0.4682	0.3782	2.04
North Lakhimpur	15	81.21	0.1933	0.0329	−0.0080	0.40
Sibsagar	15	74.47	0.1691	0.1421	−0.0184	1.18
Tezpur	15	67.33	0.1761	−0.0298	0.0925	2.13
Kampur	14	72.56	0.2596	0.2424	0.1424	0.62

Table 2. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive two day maximum rainfall of Brahmaputra Valley

Name of site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	129.85	0.1743	−0.0741	−0.0482	0.98
Dibrugarh	15	123.37	0.1692	0.0415	0.0435	0.28
Goalpara	15	138.92	0.2917	0.3725	0.4149	1.72
Golaghat	15	101.23	0.1184	−0.0247	0.1600	2.12
Guwahati	15	95.64	0.2566	0.3702	0.2728	0.62
Jorhat	23	97.79	0.1506	0.1479	0.0470	1.08
Kokrajhar	15	168.13	0.2406	0.2530	0.1761	0.45
North Lakhimpur	15	130.37	0.2070	0.3313	0.4056	1.10
Sibsagar	15	101.82	0.1924	0.3701	0.2249	1.30
Tezpur	15	91.71	0.2152	0.3424	0.3221	0.35
Kampur	14	81.57	0.1946	−0.0415	−0.0033	1.01

Table 3. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive three day maximum rainfall of Brahmaputra Valley

Name of site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	203.21	0.1669	−0.0845	0.0198	1.42
Dibrugarh	15	149.69	0.1446	−0.0805	−0.0534	0.82
Goalpara	15	185.25	0.2332	0.6270	0.4504	1.77
Golaghat	15	111.93	0.1163	−0.0562	0.0376	0.81
Guwahati	15	120.85	0.2240	0.3163	0.2050	0.76
Jorhat	23	111.57	0.1586	0.1909	0.1978	0.35
Kokrajar	15	215.00	0.2326	0.3326	0.1480	1.62
North Lakhimpur	15	162.75	0.1685	0.2378	0.2240	0.33
Sibsagar	15	119.65	0.1624	0.2017	0.0848	1.46
Tezpur	15	109.39	0.1559	0.3415	0.2559	1.12
Kampur	14	103.20	0.1450	−0.0667	0.0227	0.53

Table 4. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive four day maximum rainfall of Brahmaputra Valley

Name of Site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	249.92	0.1567	0.0138	0.0177	1.13
Dibrugarh	15	185.05	0.1543	0.2654	0.1700	0.64
Goalpara	15	230.63	0.2190	0.3851	0.2188	2.14
Golaghat	15	113.43	0.1789	0.0011	0.0621	1.03
Guwahati	15	147.83	0.1710	0.2947	0.3242	0.79
Jorhat	23	140.72	0.1436	0.2126	0.2357	0.64
Kokrajhar	15	285.83	0.1921	0.2065	0.1518	0.21
North Lakhimpur	15	202.49	0.1567	0.2519	0.3457	1.40
Sibsagar	15	140.95	0.1474	0.2621	0.1059	1.72
Tezpur	15	132.39	0.1900	0.1701	0.1919	0.34
Kampur	14	109.14	0.2039	0.0824	0.0602	0.95

Table 5. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive five day maximum rainfall of Brahmaputra Valley

Name of site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	290.37	0.1202	0.1223	-0.0119	2.26
Dibrugarh	15	205.66	0.1495	0.1491	0.3255	1.59
Goalpara	15	265.73	0.2325	0.1561	0.0443	0.76
Golaghat	15	115.99	0.1697	0.1634	0.1356	0.06
Guwahati	15	180.43	0.1803	0.1312	0.1937	0.21
Jorhat	23	155.40	0.1725	0.1865	0.0827	0.34
Kokrajhar	15	360.56	0.2188	0.0340	0.0498	0.94
North Lakhimpur	15	248.95	0.1221	0.2180	0.2689	1.09
Sibsagar	15	167.22	0.2102	0.2343	0.1506	0.65
Tezpur	15	148.34	0.2131	0.2889	0.2059	1.50
Kampur	14	120.67	0.2478	0.0225	0.1195	1.61

Table 6. Name of sites, Number of records, 1st L-moment, L-CV, L-skewness, L-Kurtosis and discordancy measures for consecutive six day maximum rainfall of Brahmaputra Valley

Name of Site	Number of record	1st L-moment	L-CV	L-skewness	L-Kurtosis	Discordancy D_i
Beki	15	331.64	0.1238	0.1008	0.1404	0.84
Dibrugarh	15	221.31	0.1552	0.0911	0.2487	1.32
Goalpara	15	295.68	0.2012	0.0041	0.1544	1.24
Golaghat	15	116.35	0.2424	0.1393	0.0807	0.34
Guwahati	15	196.55	0.1813	0.0962	0.0850	0.29
Jorhat	23	174.30	0.1831	0.2016	0.0988	0.35
Kokrajhar	15	415.75	0.2079	0.1106	-0.0060	1.33
North Lakhimpur	15	268.71	0.1449	0.1366	0.1273	0.43
Sibsagar	15	162.11	0.2435	0.2454	0.0368	1.14
Tezpur	15	143.05	0.1886	0.2720	0.1857	1.71
Kampur	14	130.97	0.2963	0.1220	0.1519	2.01

Hosking and Wallis [11] gave the critical value for D_i for discordancy measure corresponding to the number of study sites considered.

Table 7. Critical values for D_i

Number of sites	5	6	7	8	9	10	**11**	12	13	14	≥ 15
Critical value	1.333	1.648	1.917	2.140	2.329	2.491	**2.623**	2.757	2.869	2.971	3.00

(The bold numbers represent the critical value for our study area)

Result and Discussion. As shown in Table 1 to Table 6, the interval ranges for D_i as $0.37 \leq D_i \leq 2.13$ for consecutive one day maximum rainfall, $0.28 \leq D_i \leq 2.12$ for consecutive two days maximum rainfall, $0.35 \leq D_i \leq 1.77$ for consecutive three days maximum rainfall, $0.21 \leq D_i \leq 2.14$ for consecutive four days maximum rainfall, $0.06 \leq D_i \leq 2.26$ for consecutive five days maximum rainfall, and $0.29 \leq D_i \leq 2.01$ for consecutive six days maximum rainfall for $N = 11$ sites is 2.623 in Table 7. So, for 1-day, 2-day, 3-day, 4-day, 5-day, and 6-day maximum rainfall, there is no site discordance in our study region.

3.3.2 Heterogeneity Measure Means Identification of Homogeneous Region

A heterogeneity measure is the second statistic, which is used to estimate the level of heterogeneity in a grouping of sites and whether they can be called homogeneous. Hosking and Wallis [11] suggested this test. The heterogeneity measure H_i, for a group of sites that would be expected in a homogeneous area. V_1, V_2 and $V_3$are the three levels of variability. The heterogeneity test is then formulated as

$$H_i = \frac{V_i - \mu_{V_i}}{\sigma_{V_i}}, \quad i = 1,2,3. \tag{3.30}$$

and V_1, V_2 and V_3 are given by

$$V_1 = \sum_{i=1}^{NS} N_i(\hat{\tau}^{(i)} - \hat{\tau}^R)^2 \Big/ \sum_{i=1}^{NS} N_i, \tag{3.31}$$

$$V_2 = \sum_{i=1}^{NS} N_i \sqrt{\{(\hat{\tau}^{(i)} - \hat{\tau}^R)\}^2 + (\hat{\tau}_3^{(i)} - \hat{\tau}_3^R)^2} \Big/ \sum_{i=1}^{NS} N_i, \tag{3.32}$$

$$V_3 = \sum_{i=1}^{NS} N_i \sqrt{\{(\hat{\tau}_3^{(i)} - \hat{\tau}_3^R)^2 + (\hat{\tau}_4^{(i)} - \hat{\tau}_4^R)^2\}} \Big/ \sum_{i=1}^{NS} N_i, \tag{3.33}$$

where NS represents the number of site, N_i is the record length at each site and $\tau^{(R)}$, τ_3^R and τ_4^R are the regional average L-moments, can be obtained using the following formulation

$$\hat{\tau}^R = \frac{\sum_{i=1}^{NS} N_i \hat{\tau}^{(i)}}{\sum_{i=1}^{NS} N_i}, \tag{3.34}$$

$$\hat{\tau}_3^R = \frac{\sum_{i=1}^{NS} N_i \hat{\tau}_3^{(i)}}{\sum_{i=1}^{NS} N_i}, \tag{3.35}$$

$$\hat{\tau}_4^R = \frac{\sum_{i=1}^{NS} N_i \hat{\tau}_4^{(i)}}{\sum_{i=1}^{NS} N_i}. \tag{3.36}$$

Also, μ_{V_i} and σ_{V_i} represent the standard deviation of V_i where $i = 1,2,3$.

A Kappa distribution (Hosking [8]) is useful for grouping average L-moments 1, τ^R, τ_3^R, τ_4^R in order to examine heterogeneity metrics. N_{sim} areas are simulated using this Kappa distribution. Cross-correlation and serial correlation are considered to be absent from the data, and the regions are taken to be homogeneous. The heterogeneity H_i can be calculated by using the equation (3.30). If H_i is big enough, an area is said to be heterogeneous. An area is appropriate homogenous, according to Hosking and Wallis [11] if $H_i < 1$, probably heterogeneous if $1 = H_i < 2$, and certainly heterogeneous if $H_i = 2$.

According to Hosking and Wallis [10], statistics H_2 and H_3 based on the measures V_2 and V_3 do not have enough capacity to tell the difference between homogeneous and heterogeneous areas, whereas H_1 based on V_1 does far better. As a result, the H_1 statistic, which is based on V_1, is suggested as a primary predictor of heterogeneity.

Data Analysis

Table 8. Heterogeneity Measures of our study area for consecutive 1-day, 2-day, 3-day, 4-day, 5-day and 6-day maximum rainfall

Heterogeneity Measures	Values									
	1-day		2-day		3-Day	4-day	5-day		6-day	
	11 sites	10 sites	11 sites	10 sites	11 sites	11 sites	11 sites	10 sites	11 sites	10 sites
H_1	1.14	0.50	1.03	0.28	0.91	−1.09	1.25	0.84	2.11	0.96
H_2	0.70	0.69	1.87	1.39	3.09	−0.76	−1.42	−1.87	−0.91	−1.06
H_3	0.43	0.69	1.94	1.78	2.43	−0.76	−1.13	−1.28	−1.89	−1.73

Result and discussion. Table 8 indicates that for consecutive 1-day maximum rainfall, the region is heterogeneous with 11 sites. However, after eliminating Jorhat, the research area's heterogeneity measure implies that the region is homogeneous. Similarly, for consecutive 2-day, 5-day, and 6-day maximum rainfalls, the region is found to be heterogeneous among all 11 sites. The region becomes homogeneous if the sites of Golaghat for consecutive 2-day and Kampur (Nagaon) for consecutive 5-day as well as 6-day maximum rainfall are removed. The test showed that the region's maximum rainfall for 3-day and 4-day was homogeneous.

3.3.3 Goodness of Fit Measure Means Z-statistics Criteria and L-Moment Ratio Diagram

The Z-test and the L-moments ratio diagram can be utilised for these reasons.

(a) Z-test Criteria. Hosking and Wallis [11] provided the Z-test formula, which we are utilising for each of the five distributions by using the following formula:

$$Z^{Dist} = (\tau_4^{\text{dist}} - \tau_4^R + B_4)/\sigma_4, \tag{3.37}$$

where τ_4^R is the regional average of τ_4 and B_4, σ_4 are the bias and standard deviation of τ_4, respectively, and these are defined as

$$B_4 = \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} (\tau_4^m - \tau_4^R), \tag{3.38}$$

$$\sigma_4 = \sqrt{\left[\frac{1}{(N_{\text{sim}}-1)}\left\{\sum_{m=1}^{N_{\text{sim}}} (\tau_4^m - \hat{\tau}_4^R)^2 - N_{\text{sim}} B_4^2\right\}\right]}. \tag{3.39}$$

N_{sim} represents the number of simulated data set and m is the mth simulated region caused by Kappa distribution.

Table 9. $|Z|$-statistics measures of the study area for consecutive one day maximum rainfall

Distributions	$\|Z\|$-statistics values					
	1-day	2-day	3-day	4-day	5-day	6-day
GLO	2.37	**0.39**	1.30	**0.46**	1.40	2.30
GEV	1.19	0.57	0.10	0.60	0.18	0.87
GNO	1.04	0.84	**0.08**	0.82	**0.04**	0.83
PE3	**0.63**	1.36	0.54	1.31	0.36	**0.51**
GPA	1.41	2.81	2.56	3.00	2.48	2.13

Result and discussion. According to Hosking and Wallis ([10], [11]), a distribution's fit is acceptable at a 90% confidence level if $|Z^{\text{dist}}| = 1.64$. The best-fitting distribution is the one with the lowest $|Z^{\text{dist}}|$ among much potential.

Table 3.9 shows that the PE3 has the lowest Z-statistic (0.63) for successive 1-day maximum rainfall, making it the best fitting distribution for our study area. The best fit probability distributions for the consecutive 2-day, 3-day, 4-day, 5-day, and 6-day maximum rainfall are GLO (lowest Z-statistics value of 0.39), GNO (lowest Z-statistics value of 0.08), GLO (lowest Z-statistics value of 0.46), GNO (lowest Z-statistics value of 0.04), and PE3 (lowest Z-statistics value of 0.51).

(b) L-moment ratio diagram. The L-moment ratio diagram is a curve that displays the theoretical correlations between L-skewness and L-Kurtosis of several candidate distributions, according to Hosking [9]. Followings are the L-moments ratio diagram for consecutive 1 to 6 days maximum rainfall of our study area.

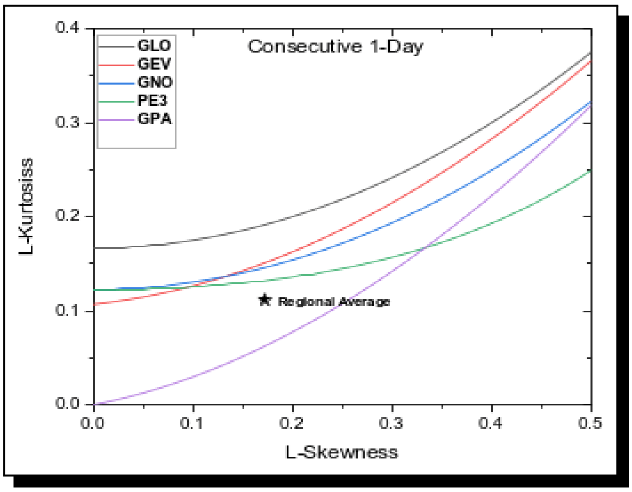

Figure 1

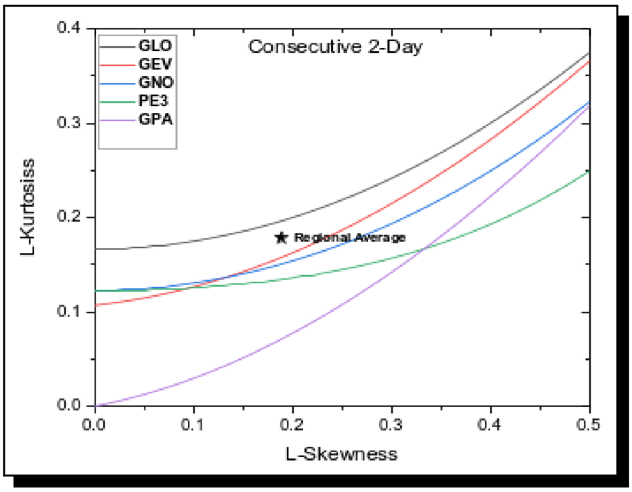

Figure 2

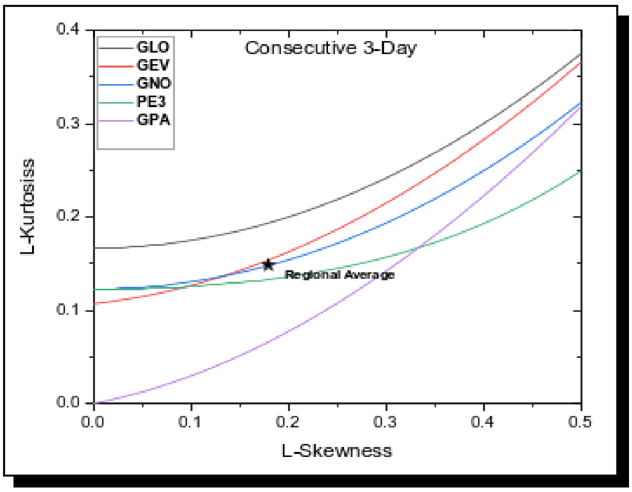

Figure 3

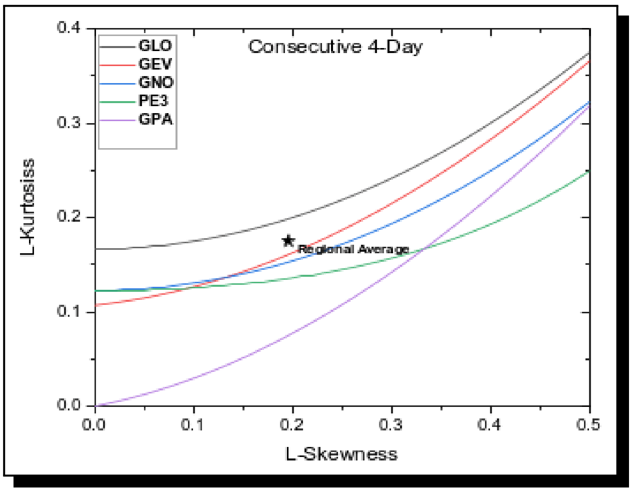

Figure 4

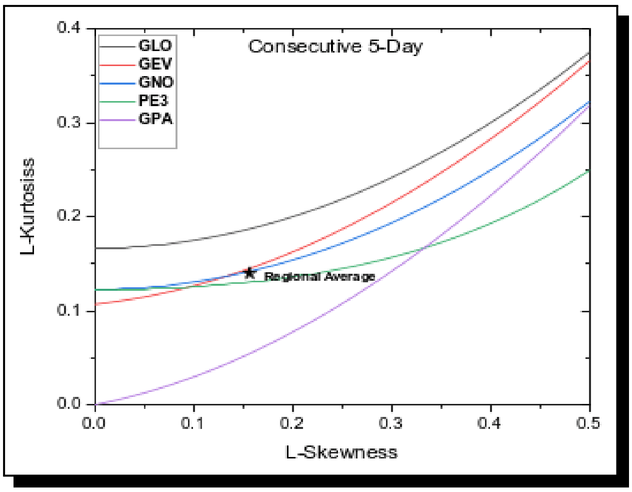

Figure 5

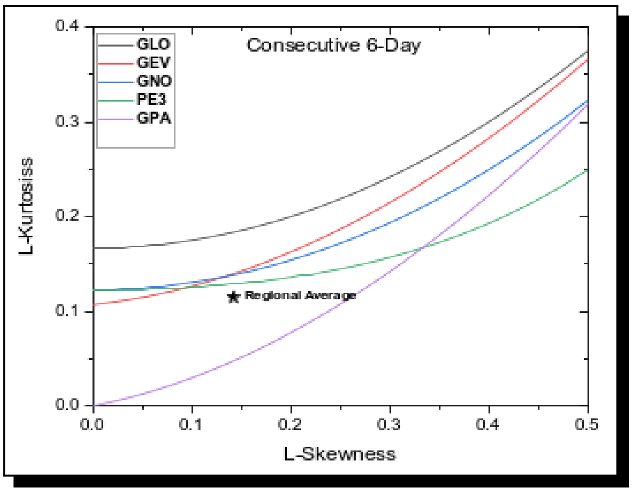

Figure 6

Table 10. Regional average L-moments

Consecutive days	Regional average		
	τ^R	τ_3^R	τ_4^R
1	0.2170	0.1729	0.1114
2	0.2063	0.2097	0.1797
3	0.1729	0.1802	0.1480
4	0.1724	0.1965	0.1749
5	0.1786	0.1693	0.1414
6	0.1870	0.1429	0.1144

Result and discussion. Figure 1 shows the L-skewness and L-Kurtosis of consecutive 1-day maximum rainfall averages. According to the graph, the region's average L-skewness and L-Kurtosis values are PE3. So, for our research area, PE3 is the strongest probability distribution for 1-day maximum rainfall. The GLO, GNO, and PE3 probability distributions are most suitable for our study site's 2-day to 6-day maximum rainfall (from Figure 2 to 6).

3.3.4 Development of Regional Parameters and Quantiles or Estimation of Selected Distribution and Return Periods

Parameter of Identified distribution. Followings are the best fitting probability distributions for regional frequency analysis in Brahmaputra valley region.

Table 11. Parameters of GEV, GNO, PE3 and GPA distribution for consecutive 1-day maximum rainfall and Parameters of GLO, GEV, GNO and PE3 distributions for consecutive 2-day maximum rainfall

Distribution	Parameters (Regional)						Distribution
	1-day			2-day			
	Location	Scale	Shape	Location	Scale	Shape	
GEV	0.819	0.312	−0.005	**0.930**	**0.192**	**−0.210**	**GLO**
GNO	0.933	0.365	−0.356	0.820	0.280	−0.061	GEV
PE3	**1.000**	**0.398**	**1.049**	0.923	0.338	−0.434	GNO
GPA	0.477	0.738	0.410	1.000	0.384	1.267	PE3

Table 12. Parameters of GLO, GEV, GNO and PE3 distributions for consecutive 3-day and 4-day maximum rainfall

Distribution	Parameters (Regional)						Distribution
	3-day			4-day			
	Location	Scale	Shape	Location	Scale	Shape	
GLO	0.950	0.164	−0.180	**0.945**	**0.162**	**−0.197**	**GLO**
GEV	0.854	0.246	−0.016	0.852	0.239	−0.041	GEV
GNO	**0.944**	**0.289**	**−0.372**	0.940	0.285	−0.406	GNO
PE3	1.000	0.318	1.092	1.000	0.319	1.189	PE3

Table 13. Parameters of GLO, GEV, GNO and PE3 distributions for consecutive 5-day maximum rainfall and parameters of GEV, GNO and PE3 distributions for consecutive 6-day maximum rainfall

Distribution	Parameters (Regional)						Distribution
	5-day			6-day			
	Location	Scale	Shape	Location	Scale	Shape	
GLO	0.951	0.170	−0.169	0.850	0.280	0.043	GEV
GEV	0.851	0.258	0.001	0.952	0.320	−0.294	GNO
GNO	**0.946**	**0.301**	**−0.349**	**1.000**	**0.339**	**0.870**	**PE3**
PE3	1.000	0.327	1.028				

(The bold figures indicate the regional parameters of best fitting distribution)

Result and discussion. From Table 11 to Table 13, we have observed that PE3 is the best fitting distribution for consecutive 1-day and 6-day maximum rainfall, GLO is the best distribution for consecutive 2-day and 4-day maximum rainfall. Again, for consecutive 3-day and 5-day maximum rainfall, GNO is the best fitting distribution.

Regional Parameters of different return periods for consecutive days maximum rainfall

Table 14. For Consecutive 1-day, 2-day and 3-day maximum rainfall

1-day							
Distribution	Return period						
	2	5	10	20	50	75	100
GEV	0.933	1.289	1.524	1.751	2.048	2.187	2.268
GNO	0.933	1.291	1.525	1.749	2.038	2.173	2.255
PE3	**0.932**	**1.299**	**1.534**	**1.751**	**2.020**	**2.142**	**2.216**
GPA	0.922	1.347	1.576	1.749	1.915	1.974	2.003
2-day							
GLO	**0.930**	**1.239**	**1.465**	**1.711**	**2.086**	**2.285**	**2.413**
GEV	0.924	1.260	1.497	1.733	2.054	2.210	2.309
GNO	0.923	1.266	1.503	1.735	2.043	2.191	2.282
PE3	0.921	1.278	1.515	1.738	2.019	2.148	2.227
3-day							
GLO	0.950	1.208	1.391	1.586	1.875	2.025	2.121
GEV	0.945	1.227	1.417	1.602	1.845	1.959	2.027
GNO	**0.944**	**1.230**	**1.419**	**1.601**	**1.835**	**1.946**	**2.014**
PE3	0.943	1.237	1.427	1.602	1.821	1.920	1.980

Table 15. For Consecutive 4-day, 5-day and 6-day maximum rainfall

Distribution	Return period						
	2	5	10	20	50	75	100
4-day							
GLO	**0.945**	**1.203**	**1.389**	**1.590**	**1.893**	**2.052**	**2.152**
GEV	0.940	1.222	1.416	1.607	1.863	1.986	2.062
GNO	0.940	1.226	1.419	1.607	1.854	1.971	2.044
PE3	0.938	1.234	1.428	1.609	1.836	1.940	2.003
5-day							
GLO	0.951	1.217	1.404	1.601	1.887	2.036	2.135
GEV	0.946	1.238	1.431	1.616	1.856	1.967	2.035
GNO	**0.946**	**1.241**	**1.432**	**1.614**	**1.850**	**1.959**	**2.025**
PE3	0.945	1.247	1.439	1.615	1.835	1.935	1.994
6-day							
GEV	0.952	1.257	1.451	1.631	1.856	1.958	2.020
GNO	0.952	1.258	1.450	1.628	1.854	1.958	2.019
PE3	**0.951**	**1.262**	**1.454**	**1.629**	**1.842**	**1.938**	**1.997**

Result and discussion. Tables 14 and 15 show that for consecutive 1-day and 6-day maximum rainfall, PE3 is the best-fit probability distribution for the varied return periods from 2 to 100 years, and the GLO distributions for consecutive 2-day and 4-day maximum rainfall are similar. For return periods of 2 to 100 years, GNO is the best matched probability distribution. It can have more return periods.

3.3.5 Development of Regional Relationship for Consecutive Days Maximum Rainfall

The regional consecutive days maximum rainfall frequency relationship for 11 gauging stations in the Brahmaputra Valley is shown in Table 10 for Location, Scale, and Shape.

Consecutive 1-day: PE3 has no explicit analytical form. So, for estimating quantiles for each site of the region we can multiply the regional growth curve given by Table 10 for PE3 distribution by the 1st L-moments of the site can be used.

Consecutive 2-days:

$$Q_T^j = \left[0.016 + 0.914 \times \left(\frac{1}{T-1}\right)^{-0.210}\right] \times \text{1st L-moment of site } j. \tag{3.40}$$

where Q_T^j represents the quantile estimates for the period T of the site j.

Consecutive 3-days: GNO has no explicit analytical form. So, for estimating quantiles for each site of the region we can multiply the regional growth curve given by Table 10 for GNO distribution by the 1st L-moments of the site can be used.

Consecutive 4-days:

$$Q_T^j = \left[0.123 + 0.822 \times \left(\frac{1}{T-1}\right)^{-0.197}\right] \times \text{1st L-moment of site } j. \tag{3.41}$$

Consecutive 5-days: GNO has no explicit analytical form

Consecutive 6-days: PE3 has no explicit analytical form.

4. Conclusions

In this paper, we have studied the rainfall frequency analysis for the consecutive 1 to 6 days of maximum rainfall in the Brahmaputra Valley by taking 11 gauging stations and considering five probability distribution functions: GLO, GEV, GPA, GNO, and PE3 under the L-moment given by Hosking and Wallis [11] for regional rainfall analysis. To do this, we perform our research in five steps. Steps include data analysis, validating homogeneous regions, determining best-fit probability distributions, searching for regional variables (growth factors), and finally analysing regional relationships. Discordancy measures are taken with L-moment and find that among all the 11 sites, none is discordant. Also, when we apply regional heterogeneity testing, all the sites become homogeneous. The Z-statistics method shows that PE3 is the best fitting distribution for consecutive 1-day and 6-day maximum rainfall, and for consecutive 2-day and 4-day maximum rainfall, it is GLO, while GNO is the best probability distribution for consecutive 3-day and 5-day maximum rainfall with the least critical value compared with the standard critical value. Heterogeneity measures show that after excluding the site Jorhat, the remaining 10 sites become homogeneous. The region becomes homogeneous if the sites of Golaghat for consecutive 2-day and Kampur (Nagaon) for 5-day as well as 6-day maximum rainfall are eliminated. The L-moment ratio diagram yields the same conclusion. Then we find the regional parameters for 1-day to 6-day maximum rainfall. The regional quantile estimates for different consecutive days are calculated. Finally, we use the best-fitting probability distributions to establish a regional association for consecutive days of maximum rainfall.

5. Data Report

We have collected the rainfall data for Jorhat station for 23 years, from 1 January 1996 to 31 March 2019. Again, data were obtained from IMD (Indian Meteorological Department), Guwahati, for the sites Beki, Dibrugarh, Goalpara, Golaghat, Guwahati, Kokrajahar, North Lakhimpur, Sibsagar, and Tezpur for 15 years from 01-01-2005 to 31-12-2019, and Kampur for 14 years and 4 months from 01-01-2005 to 31-12-2019.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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