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Research Article

$\widetilde{g}$-Open Sets in Fuzzy Topological Spaces

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Abstract. New category of fuzzy generalized closed sets, specifically fuzzy $\widetilde{g}$-closed sets is to launch and argue of fuzzy topological spaces. Further, compare with related to various types of fuzzy generalized closed sets are investigated. Moreover, the properties of fuzzy $\widetilde{g}$-closed sets are given of this paper.

Keywords. Fuzzy open sets, Fuzzy g-open sets, Fuzzy $\widetilde{g}$-open sets

Mathematics Subject Classification (2020). 54A05, 54A10, 54C08, 54C10

1. Introduction

Zadeh [12] was introduced and discussed the novel model of a fuzzy subsets. The consequent research behavior in this area and the linked areas have originate relevance in various branches of science and engineering. Chang [4] by the idea of generalization of fuzzy topological spaces. Another researchers similar to Azad [1], Shahna [8], Wong [10] and any more authors donate to the growth of fuzzy topological spaces and so on. In this paper, new category of fuzzy generalized closed sets, we specifically fuzzy $\widetilde{g}$-closed sets is to launch and argue of fuzzy topological spaces. Further, compare with related to various types of fuzzy generalized closed sets are investigated. Also, the properties of fuzzy $\widetilde{g}$-closed sets are given.

2. Preliminaries

In this paper (X,F_τ) (briefly, X) will denote fuzzy topological spaces or space (X,F_τ). We remember the following basic definitions which are apply in this paper: A fuzzy subset A of a fuzzy topological space (X,F_τ) is called a fuzzy semi-open (Azad [1]), α-open (Shahna [8]), and regular open (Thakur and Singh [9]), the complement of open sets are called closed in (X,F_τ). The operators namely, fuzzy semi-closure (Yalvaç [11]), fuzzy α-closure (Prasad *et al.* [7]), fuzzy semi-preclosure (Yalvaç [11]) in (X,F_τ). Further, some fuzzy generalized closed sets are indicated (resp. shortly denotes fg-closed (Balasubramanian and Sundaram [2], fsg-closed, fgs-closed (Maki [6])), $f\alpha g$-closed (Saraf *et al.*[1]), $fgsp$-closed (El-Shafei and Zakari [5]), the complement of closed sets are called open in (X,F_τ).

3. $f\widetilde{g}$-Open Sets

Definition 3.1. A fuzzy subset A of a space (X,F_τ) is said to be a fuzzy $\widetilde{g}$-open set (shortly denotes $f\widetilde{g}$-open set) if A^c is fuzzy $\widetilde{g}$-closed (Balasubramanian and Sundaram [3]). The family of all fuzzy $\widetilde{g}$-open sets in X is denoted by $F\widetilde{G}O(X)$.

Proposition 3.2. *In a space (X,F_τ), entire a fuzzy open set is $f\widetilde{g}$-open.*

Proof. A is a fuzzy open set $\Rightarrow A^c$ is a fuzzy closed set in (X,F_τ). Since entire a fuzzy closed set is $f\widetilde{g}$-closed in (X,F_τ). As a result A^c is a $f\widetilde{g}$-closed set. Thus A is $f\widetilde{g}$-open in (X,F_τ). □

Remark 3.3. The converse of Proposition 3.2 is not true as seen from the follows.

Example 3.4. Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = 1$, $n(v) = 0$. In the space (X,F_τ), then α defined by $\alpha(u) = 0.5$, $\alpha(v) = 0$ is a $f\widetilde{g}$-open set but not fuzzy open.

Proposition 3.5. *In a space (X,F_τ), entire a $f\widetilde{g}$-open set is fgsp-open.*

Proof. A is a $f\widetilde{g}$-open set $\Rightarrow A^c$ is a $f\widetilde{g}$-closed set in (X,F_τ). Since entire a $f\widetilde{g}$-closed set is $fgsp$-closed in (X,F_τ). As a result A^c is $fgsp$-closed. Thus A is a $fgsp$-open set in (X,F_τ). □

Remark 3.6. The converse of Proposition 3.5 is not true as seen from the follows.

Example 3.7. Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$, where n is a fuzzy set in X defined by $n(u) = n(v) = 0.4$. In the space (X,F_τ), then α defined by $\alpha(u) = \alpha(v) = 0.5$ is a $fgsp$-open set but not $f\widetilde{g}$-open.

Proposition 3.8. *In a space (X,F_τ), entire a $f\widetilde{g}$-open set is fg-open.*

Proof. A is $f\widetilde{g}$-open set $\Rightarrow$ A^c is $f\widetilde{g}$-closed set in (X,F_τ). Since entire a $f\widetilde{g}$-closed set is fg-closed. As a result A^c is fg-closed. Thus A is fg-open set in (X,F_τ). □

Remark 3.9. The converse of Proposition 3.8 is not true as seen from the follows.

[1] R. K. Saraf, M. Caldas and S. Mishra, Results via $fg\alpha$-closed sets and $f\alpha g$-closed sets, *preprint*.

Example 3.10. Let $X = \{u,v\}$ and $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = n(v) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(u) = \alpha(v) = 0.6$ is fg-open set but not $f\widetilde{g}$-open set.

Proposition 3.11. *In a space (X, F_τ), entire a $f\widetilde{g}$-open set is $f\alpha g$-open.*

Proof. A is a $f\widetilde{g}$-open set $\Rightarrow A^c$ is $f\widetilde{g}$-closed set. Since entire a $f\widetilde{g}$-closed set is $f\alpha g$-closed. As a result A^c is a $f\alpha g$-closed. Thus A is $f\alpha g$-open set in (X, F_τ). □

Remark 3.12. The converse part of Proposition 3.11 is not true as seen from the follows.

Example 3.13. Let $X = \{u,v\}$ and $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = 1$, $n(v) = 0$. In the space (X, F_τ), then α defined by $\alpha(u) = 1$, $\alpha(v) = 0.5$ is $f\alpha g$-open set but not $f\widetilde{g}$-open set.

Proposition 3.14. *In a space (X, F_τ), entire a $f\widetilde{g}$-open set is fgs-open.*

Proof. A is $f\widetilde{g}$-open set $\Rightarrow A^c$ is $f\widetilde{g}$-closed set in (X, F_τ). Since entire a $f\widetilde{g}$-closed set is fgs-closed. Therefore A^c is fgs-closed. Hence A is fgs-open set in (X, F_τ). □

Remark 3.15. The converse part of Proposition 3.14 is not true as seen from the follows.

Example 3.16. Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = n(v) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(u) = \alpha(v) = 0.4$ is fgs-open set but not $f\widetilde{g}$-open set.

Proposition 3.17. *In a space (X, F_τ), entire a $f\widetilde{g}$-open set is fuzzy αgs-open.*

Proof. A is $f\widetilde{g}$-open set $\Rightarrow A^c$ is $f\widetilde{g}$-closed set in (X, F_τ). Since entire $f\widetilde{g}$-closed set is fuzzy αgs-closed. As a result A^c is fuzzy αgs-closed. Thus A is $f\alpha gs$-open set in (X, F_τ). □

Remark 3.18. The converse part of Proposition 3.17 is not true as seen from the follows.

Example 3.19. Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = 1$, $n(v) = 0$. In a space (X, F_τ), then α defined by $\alpha(u) = 1$, $\alpha(v) = 0.5$ is fuzzy αgs-open set but not $f\widetilde{g}$-open set.

Remark 3.20. The following Example shows that the family of $f\widetilde{g}$-open sets are independent of the family of fuzzy α-open sets and fuzzy semi-open sets in (X, F_τ).

Example 3.21.
(i) Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = n(v) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(u) = \alpha(v) = 0.6$ is $f\widetilde{g}$-open but it is neither fuzzy α-open nor fuzzy semi-open.
(ii) Let $X = \{u,v\}$ with $F_\tau = \{0_X, n, 1_X\}$ where n is a fuzzy set in X defined by $n(u) = 1$, $n(v) = 0$. In the space (X, F_τ), then α defined by $\alpha(u) = 0.5, \alpha(v) = 1$ is fuzzy α-open as well as fuzzy semi-open but not $f\widetilde{g}$-open set.

Remark 3.22. The above discussions are shown in the following implications, where $A \to B$ (resp. A implies B, $A \nleftrightarrow B$) (resp. A and B are independent of each other).

$$\begin{array}{ccccc}
f\alpha gs\text{-open} & & f\text{-open} & & fg\text{-open} \\
& \nwarrow & \downarrow & \nearrow & \\
\text{fuzzy } \alpha\text{-open} & \nleftrightarrow & f\widetilde{g}\text{-open} & \nleftrightarrow & \text{fuzzy semi-open} \\
& \swarrow & \downarrow & \searrow & \\
f\alpha g\text{-open} & & fgsp\text{-open} & & fgs\text{-open}
\end{array}$$

Remark 3.23. In a space (X,F_τ), if A and B are two $f\widetilde{g}$-open sets then $A \vee B$ is not $f\widetilde{g}$-open as seen from the following example.

Example 3.24. In Example 5.2, then m and n defined by $m(u) = 0.7$, $n(v) = 0$ and $n(u) = 0$, $n(v) = 0.3$ are $f\widetilde{g}$-open set but $m \vee n = (0.7, 0.3)$ is not $f\widetilde{g}$-open.

4. Properties of $f\widetilde{g}$-Open Sets

Theorem 4.1. *Let A be a fuzzy subset of a space (X,F_τ), the following statements are equivalent.*

(i) *a fuzzy subset A is $f\widetilde{g}$-open.*

(ii) $G \le \text{int}(A)$ *whenever* $1-G$ *is* fsg*-open and* $G \le A$.

Proof. (i)⇒(ii): Assuming that A is $f\widetilde{g}$-open in (X,F_τ). Let $1-G$ be fsg-open such that $G \le A$. Then $1-A \le 1-G$ where $1-A$ is $f\widetilde{g}$-closed. Hence $cl(1-A) \le 1-G$ and $G \le 1-cl(1-A) = int(A)$.

(ii) ⇒(i): Suppose that A is $f\widetilde{g}$-open under the given conditions, we prove $1-A$ is $f\widetilde{g}$-closed in (X,F_τ). Let M be any fsg-open set such that $1-A \le M$. Then $1-M \le A$. Taking $B = 1-M$, we have $B \le A$ where $1-B$ is $f\widetilde{g}$-open. By assumption $B \le int(A)$ which implies $1-M \le int(A)$ and hence $1-int(A) \le M$ thus $cl(1-A) \le U$ which proves that $1-A$ is $f\widetilde{g}$-closed and A is $f\widetilde{g}$-open. □

Theorem 4.2. *In a space (X,F_τ), if A is a $f\widetilde{g}$-open subset such that* $\text{int}(A) \le B \le A$ *then B is $f\widetilde{g}$-open.*

Proof. Assuming that $int(A) \le B \le A \Rightarrow 1-A \le 1-B \le 1-int(A) = cl(1-A)$ where $1-A$ is $f\widetilde{g}$-closed in (X,τ). $1-B$ is $f\widetilde{g}$-closed and hence B is $f\widetilde{g}$-open in (X,F_τ). □

Theorem 4.3. *In a space (X,F_τ), if A is a fsg-open and $f\widetilde{g}$-closed then A is fuzzy closed.*

Proof. Since A is fsg-open and $f\widetilde{g}$-closed, $cl(A) \le A$ and hence A is fuzzy closed in (X,F_τ). □

5. Some Related $f\widetilde{g}$-Subsets

Proposition 5.1. *In a space (X,F_τ), fuzzy closed forward of $f\widetilde{g}$-closed forward of fsg-closed.*

Proof. In a space, if A is fuzzy closed then A is $f\widetilde{g}$-closed. Let A be $f\widetilde{g}$-closed in (X,F_τ) and M be any fuzzy semi-open set such that $A \le M$. Then M is fsg-open in (X,F_τ). Since A is $f\widetilde{g}$-closed, $cl(A) \le M$ and $scl(A) \le cl(A) \le M$ which means A is fsg-closed in (X,F_τ). Thus $f\widetilde{g}$-closed forward of sg-closed. Hence fuzzy closed forward of $f\widetilde{g}$-closed forward of fsg-closed. □

We consider the following some discussion.

Example 5.2. Let $X = \{u,v\}$ with $F_\tau = \{0_X, m, n, m \vee n, 1_X\}$ where m, n are fuzzy sets in X defined by $m(u) = 0.6$, $m(v) = 0$ and $n(u) = 0$, $n(v) = 0.3$. Then (X, F_τ) is a space then the following classes of fuzzy subsets.

(i) $FSC(X) = \{(\frac{u}{a}, \frac{v}{b}) | a \in [0.6,1],\ b \in [0.3,0.7]$ and $a \in [0,0.4],\ b \in [0.3,1]$ and $a = 0,\ b = 0$ and $a = 1,\ b = 1\}$.

(ii) $FSO(X) = \{(\frac{u}{a}, \frac{v}{b}) | a \in [0,0.4],\ b \in [0.3,0.7]$ and $a \in [0.6,1],\ b \in [0,0.7]$ and $a = 0,\ b = 0$ and $a = 1,\ b = 1\}$.

(iii) $FSGC(X) = \{(\frac{u}{a}, \frac{v}{b}) | a = 0,\ b \in [0,1]$ and $a \in [0,0.4],\ b \in [0,1]$ and $a \in [0.4,1],\ b \in [0.3,1]\}$.

(iv) $FSGO(X) = \{(\frac{u}{a}, \frac{v}{b}) | a = 1,\ b \in [0,1]$ and $a \in [0.6,1],\ b \in [0,1]$ and $a \in [0,0.6],\ b \in [0,0.7]\}$.

(v) $F\tilde{G}C(X) = \{(\frac{u}{a}, \frac{v}{b}) | a = 0,\ b = 0$ and $a \in [0,1],\ b = 1$ and $a = 0.4,\ b = 0.7$ and $a = 1,\ b = 0.7\}$.

Remark 5.3. Reverse implications of Proposition 5.1 is not true as seen from follows.

Example 5.4. In Example 5.2, then

(i) α defined by $\alpha(a) = 0.2$, $\alpha(b) = 1$ is $f\tilde{g}$-closed but not fuzzy closed.

(ii) α defined by $\alpha(a) = 0$, $\alpha(b) = 0.5$ is fsg-closed but not $f\tilde{g}$-closed.

Theorem 5.5. *In a space (X, F_τ), the family of all $f\tilde{g}$-closed sets properly lies between the family of all fuzzy closed sets and the family of all fsg-closed sets.*

Proof. By Proposition 5.1, fuzzy closed forward of $f\tilde{g}$-closed forward of fsg-closed. Hence $FC(X) \le F\tilde{G}C(X) \le FSGC(X)$. By Example 5.2, fsg-closed $\Rightarrow f\tilde{g}$-closed is not implies fuzzy closed. Hence $FC(X) < F\tilde{G}C(X) < FSGC(X)$ and this proves the theorem. □

Remark 5.6. In a space (X, F_τ), the family of fsg-open sets and the family of $f\tilde{g}$-closed sets are independent of each other as seen from the following example.

Example 5.7. In Example 5.2, then

(i) α defined by $\alpha(a) = 1$, $\alpha(b) = 0.5$ is fsg-open but not $f\tilde{g}$-closed.

(ii) α defined by $\alpha(a) = 0.3$, $\alpha(b) = 1$ is $f\tilde{g}$-closed but not fsg-open.

Proposition 5.8. *In a space (X, F_τ), fuzzy open forward of $f\tilde{g}$-open forward of fsg-open.*

Proof. Considering the complements of fuzzy closed, $f\tilde{g}$-closed and fsg-closed, follows from Proposition 5.1. □

Remark 5.9. The following example show that backward of Proposition 5.8 is not true.

Example 5.10. In Example 5.2, then

(i) α defined by $\alpha(a) = 0.8$, $\alpha(b) = 0$ is $f\tilde{g}$-open but not fuzzy open.

(ii) α defined by $\alpha(a) = 1$, $\alpha(b) = 0.5$ is fsg-open but not $f\tilde{g}$-open.

Theorem 5.11. *In a space (X, F_τ), the family of all $f\tilde{g}$-open sets properly lies between the family of all fuzzy open sets and the family of all fsg-open sets.*

Proof. Follows from Proposition 5.8, and Example 5.10. □

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] K. K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, *Journal of Mathematical Analysis and Applications* **82**(1) (1981), 14 – 32, DOI: 10.1016/0022-247X(81)90222-5.

[2] G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy continuous functions, *Fuzzy Sets and Systems* **86**(1) (1997), 93 – 100, DOI: 10.1016/0165-0114(95)00371-1.

[3] K. Balasubramaniyan and R. Prabhakaran, On fuzzy $\widetilde{g}$-closed sets and it's properties, *Advances in Mathematics: Scientific Journal* **9**(4) (2020), 2167 – 2175, DOI: 10.37418/amsj.9.4.77.

[4] C. L. Chang, Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications* **24**(1) (1968), 182 – 190, DOI: 10.1016/0022-247X(68)90057-7.

[5] M. E. El-Shafei and A. Zakari, Semi-generalized continuous mappings fuzzy topological spaces, *Journal of the Egyptian Mathematical Society* **15**(1) (2007), 57 – 67.

[6] H. Maki, T. Fukutaka, M. Kojima and H. Harada, Generalized closed sets in fuzzy topological spaces I. *Meeting on Topological Spaces, Theory and Applications* (1998), 23 – 26.

[7] R. Prasad, S. S. Thakur and R. K. Saraf, Fuzzy α-irresolute mappings, *Journal of Fuzzy Mathematics* **2**(2) (1994), 335 – 339.

[8] A. S. B. Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems* **44**(2) (1991), 303 – 308, DOI: 10.1016/0165-0114(91)90013-G.

[9] S. S. Thakur and S. Singh, On fuzzy semi-preopen sets and fuzzy semi-precontinuity, *Fuzzy Sets and Systems* **98**(3) (1998), 383 – 391, DOI: 10.1016/S0165-0114(96)00363-6.

[10] C. K. Wong, Fuzzy points and local properties of fuzzy topology, *Journal of Mathematical Analysis and Applications* **46**(2) (1974), 316 – 328, DOI: 10.1016/0022-247X(74)90242-X.

[11] T. H. Yalvaç, Semi-interior and semi-closure of a fuzzy set, *Journal of Mathematical Analysis and Applications* **132**(2) (1988), 356 – 364, DOI: 10.1016/0022-247X(88)90067-4.

[12] L. A. Zadeh, Fuzzy sets, *Information and Control* **8**(3) (1965), 338 – 353, DOI: 10.1016/S0019-9958(65)90241-X.