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Research Article

M-N Homomorphism of an M-N Fuzzy Soft Subgroups and Its Level M-N Subgroups

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Abstract. We examined the notion of M-N homomorphism of fuzzy soft subgroups in this study, then defined the M-N level subsets of a fuzzy soft subgroup and discussed some of its basic aspects.

Keywords. Fuzzy group, M-N fuzzy group, M-N fuzzy soft subgroups, M-N level subset, M-N homomorphism of fuzzy soft subgroups

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1. Introduction

In the actual world, there are many different forms of uncertainties, yet few traditional mathematical techniques are suited for modelling these uncertainties. Undefined data is at the heart of many complex problems in economics, social science, engineering, medicine, and other domains. These difficulties that one encounters in life cannot be solved with traditional mathematical methods. A mathematical model of an object is devised in classical mathematics, but the concept of the exact solution of this model is not yet specified. The exact solution cannot be found because the standard mathematical model is too complex. There are a number of well-known ideas that can be used to describe uncertainty. For example, Rosenfeld [11] presented the notion of fuzzy subgroup in 1971, and Zadeh [12] inspired the theory of fuzzy sets. In addition, Molodtsov [8] introduced the concept of soft sets in 1999. Maji *et al*. [6] presented the concept of fuzzy soft sets in 2009, and Jacobson [3] introduced the concept of M-group M-subgroup.

Aktaş and Çağman [1] provided an introduction to a novel concept of soft sets and soft groups based on the inclusion relation and set intersection. Das [2] investigated fuzzy groups and level subgroups in 1981. In 2012, Massa'deh [7] wrote about the M-N homomorphism and M-N anti homomorphism of an over M-N fuzzy subgroups.

The M-homomorphism and M-anti homomorphism of an M-fuzzy subgroup and its level M-subgroups were introduced by Muthuraj *et al*. in 2010 [10]. In [4, 5], discussed the concept of M-N fuzzy soft subgroups in previous work.

We have studied the concept of M-N homomorphism of fuzzy soft subgroups based on the concept of fuzzy soft groups in this work. The basic definition, notations on M-N fuzzy soft subgroups, and needed results on fuzzy soft subgroups were presented in Section 2. The M-N homomorphism of a fuzzy soft set and the M-N level subsets of a fuzzy soft subgroup are defined in Section 3. We have also gone over the M-N homomorphism of fuzzy soft group notion and some of its basic features.

2. Preliminaries

Some basic definitions and findings are provided in this section. We have laid down the previous topics that will be used in this article for your convenience.

Definition 2.1 ([12])**.** Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \to [0,1]$.

Example 2.2. Let $X = \{1,2,3,4,5\}$ be a set. Then $A = \{(1,0.2),(2,0.4),(3,0.6),(4,0.7),(5,0.8)\}$ is a fuzzy subset of X.

Definition 2.3 ([7])**.** Let M, N be left and right operator sets of group G respectively, if $(mx)n = m(xn)$ for all $x \in G$, $m \in M$, $n \in N$. Then G is said be an M-N group.

Definition 2.4 ([5])**.** Let G be a group and (f,A) be an M-N fuzzy soft set over G. Then (f,A) is said to be a M-N fuzzy soft group over G iff for each $a \in A$ and $x, y \in G$,

(i) $f_a\{m(xy)n\} \ge \min\{f_a(x), f_a(y)\}$,

(ii) $f_a\{(mx^{-1})n\} \ge f_a(x)$ hold for each $a \in A$, $m \in M$, $n \in N$, f_a is a M-N fuzzy soft subgroup of a group G.

Definition 2.5 ([5])**.** Let G be an M-N group and (f,A) be a fuzzy soft subgroup of G if

(i) $f_a(mx) \ge f_a(x)$,

(ii) $f_a(xn) \ge f_a(x)$ hold for any $x \in G$, $m \in M$, $n \in N$ and $a \in A$, then (f,A) is said be an M-N fuzzy soft subgroup of G.

Example 2.6 ([5])**.** Let f_a be a fuzzy soft subgroup of an M-N group G. $a \in A$ is the parameters of the set, then f_a is defined by

$$f_a(x) = \begin{cases} 0.1 & \text{if } x \in G, \\ 0.9 & \text{if } x \notin G, \end{cases}$$

where $x = \{1,2,3,4,5,6\}$, $f_a(1) = 0.1$, $f_a(2) = 0.03$, $f_a(3) = 0.06$, $f_a(4) = 0.6$, $M = \{1,2,3\}$ and $N = \{1,3,5\}$, here $N \sqsubset A$ and $M \sqsubset A$ where A is a natural numbers.

Definition 2.7 ([7])**.** A mapping f from a group G into a group G^L is said be a homomorphism if for all $x,y \in G$, $f(xy) = f(x)f(y)$.

Example 2.8. $f : G \to G^L$ defined by $f(x) = e$ for all $x \in G$, e is the identity element in G^L is a trivial homomorphism.

Definition 2.9 ([10])**.** Let $f : G \to G^L$ be a group homomorphism we say that f is an isomorphism satisfied f is one-one and onto.

Definition 2.10 ([10])**.** Let $f : G \to G^L$ be a group homomorphism we say that f is monomorphism if f is one-one. We say that epimorphism if f is onto.

Definition 2.11 ([10])**.** A group homomorphism $f : G \to G^L$ is isomorphism, then its called on automorphism.

3. M-N Fuzzy Soft Subgroups of an M-N Group G under M-N Homomorphism

We will define the M-N homomorphism of a fuzzy soft subgroup and the M-N level subsets of a fuzzy soft subgroup in this section. We have also gone over the M-N homomorphism of fuzzy soft group notion and some of its basic features.

Definition 3.1. Let G and G^L be any two M-N groups. If (f,A) is an fuzzy soft subgroup of an M-N group G, then the function $f_a : G \to G^L$ is said be an M-N homomorphism of fuzzy soft subgroup if

(i) $f_a(xy) = f_a(x)f_a(y)$ for all $x,y \in G$, $a \in A$,

(ii) $f_a(mx) = mf_a(x)$, for all $x \in G$, $a \in A$ and $m \in M$,

(iii) $f_a(yn) = mf_a(y)$, for all $y \in G$, $a \in A$ and $n \in N$.

Note 3.2. If λ is a constant and $kerf_a$ is an M-N fuzzy soft subgroup, then

(i) $f_a(\lambda)f_a(mx) = \lambda(mx) = \lambda(x)$, for all $x \in G$, $a \in A$ and $m \in M$,

(ii) $f_a(\lambda)f_a(xn) = \lambda(xn) = \lambda(x)$, for all $x \in G$, $a \in A$ and $n \in N$.

Theorem 3.1. *Let f_a be an M-N homomorphism of fuzzy soft subgroup from an M-N group G onto an M-N group G^L. If λ is an M-N fuzzy subgroup of G and λ is an f_a-soft invariant, then $f_a(\lambda)$ is an M-N fuzzy soft subgroup of G^L.*

Proof. We know that λ is a constant and $kerf_a$ is an M-N fuzzy soft subgroup. Now

$$\begin{aligned} f_a(\lambda)(f_a(x)f_a(y)) &= f_a(\lambda)(f_a(xy)), && \text{(for all } x,y \in G, a \in A\text{)} \\ &= \lambda(xy), && \text{(since by note)} \\ &\geq \min\{\lambda(x), \lambda(y)\} \end{aligned}$$

$$\geq \min\{f_a(\lambda)(f_a(x), f_a(\lambda)f_a(y))\}.$$

Therefore

$$f_a(\lambda)(f_a(x)f_a(y)) \geq \min\{f_a(\lambda)(f_a(x), f_a(\lambda)f_a(y))\}$$

Clearly $f_a(\lambda)$ is an fuzzy soft subgroup of G^L.
To prove that $f_a(\lambda)$ is an M-N fuzzy soft subgroup of G^L.
Let $f_a(\lambda) \in G^L$, then

(i)
$$\begin{aligned} f_a(\lambda)(mf_a(x)) &= f_a(\lambda)(f_a(mx)) \\ &= \lambda(mx) \\ &\geq \lambda(x), \qquad \text{(by the definition } A(mx) \geq A(x)) \\ &= f_a(\lambda)f_a(x). \end{aligned}$$

Therefore,

$$f_a(\lambda)(mf_a(x)) = f_a(\lambda)f_a(x).$$

(ii)
$$\begin{aligned} f_a(\lambda)(f_a(x)n) &= f_a(\lambda)(f_a(xn)) \\ &= \lambda(xn) \\ &\geq \lambda(x) \qquad \text{(by the definition } A(xn) \geq A(x)) \\ &= f_a(\lambda)f_a(x). \end{aligned}$$

Therefore

$$f_a(\lambda)(f_a(x)n) = f_a(\lambda)f_a(x).$$

Hence $f_a(\lambda)$ is an M-N fuzzy soft subgroup of G^L. □

Theorem 3.2. *The M-N homomorphic pre- image of an M-N fuzzy soft subgroup of an M-N group G^L is an M-N fuzzy soft subgroup of an M-N group G.*

Proof. Let $f_a : G \to G^L$ is said be an M-N homomorphism of fuzzy soft subgroup.
Let μ be an fuzzy set on the M-N fuzzy subgroup of G^L.
Now

$$\begin{aligned} \lambda(xy) &= \mu(f_a(xy)), && \text{(for all } a \in A \text{ and } x, y \in G) \\ &= \mu(f_a(x)f_a(y)) && \text{(since } f_a \text{ is an homomorphism)} \\ &\geq \min\{\mu f_a(x), \mu f_a(y)\} && \text{(since } \mu \text{ is an fuzzy subgroup of } G^L) \\ &= \min\{\lambda(x), \lambda(y)\}. \end{aligned}$$

That is,

$$\lambda(xy) \geq \min\{\lambda(x), \lambda(y)\},$$

Let $x \in G$,

$$\begin{aligned} \lambda(X^{-L}) &= \mu(f_a(x^{-l})) \\ &= \mu(f_a(x)^{-i}) \qquad \text{(since } f_a \text{ is an homomorphism of fuzzy soft subgroup)} \end{aligned}$$

$$= \mu(f_a(x)) \qquad \text{(since } \mu \text{ is an M-N fuzzy subgroup of } G^L\text{)}$$
$$= \lambda(x)$$
$$\lambda(X^{-l}) \geq \lambda(x)$$

Clearly,

$$\begin{aligned} \lambda(mx) &= \mu(f_a(mx)) && \text{(for some } m \in M \text{ and } x \in G\text{)} \\ &= \mu(mf_a(x)) && \text{(since } f_a \text{ is an M-N homomorphism of an fuzzy soft group)} \\ &\geq \mu f_a(x) && \text{(since } \mu \text{ M-N fuzzy subgroup of } G\text{)} \\ &= \lambda(x). \end{aligned}$$

That is ,

$$\lambda(mx) \geq \lambda(x).$$

Then

$$\begin{aligned} \lambda(xn) &= \mu(f_a(xn)) && \text{(for some } n \in N \text{ and } x \in G\text{)} \\ &= \mu(nf_a(x)) && \text{(since } f_a \text{ is an M-N homomorphism of an fuzzy soft group)} \\ &\geq \mu f_a(x) && \text{(since } \mu \text{ M-N fuzzy subgroup of } G\text{)} \\ &= \lambda(x). \end{aligned}$$

That is

$$\lambda(xn) \geq \lambda(x).$$

Hence λ is an M -N fuzzy subgroup of G. □

Theorem 3.3. *If $f_a : G \to G^L$ is an M-N homomorphism of an fuzzy soft subgroup of a group G, then,*

(i) *$f_a(e) = e^L$, where e^L is the unity element of G^L,*

(ii) *$f_a(x^{-l}) = f_a(x)^{-l}$ for all $x \in G$.*

Proof. Given that $f_a : G \to G^L$ is an M-N homomorphism of an fuzzy soft subgroup of a group G,

(i) $\Rightarrow$ Suppose

$$\begin{aligned} f_a(mx)e^L &= f_a(mx) = f_a(x) && \text{(for some } m \in M,\ a \in A \text{ and } x \in G\text{)} \\ &= f_a(xe) && \text{(since } e \text{ is an identity element in } G\text{)} \\ &= f_a(x)f_a(e) && \text{(since } f_a \text{ is an homomorphism of an fuzzy soft subgroup)} \\ f_a(x)e^L &= f_a(x)f_a(e) && \text{(by left cancellation law)} \end{aligned}$$

Therefore, $f_a(e) = e^L$.

Similarly, we can prove that

$$f_a(xn)e^L = f_a(xn) = f_a(x). \qquad \text{(for some } n \in N,\ a \in A \text{ and } x \in G\text{)}$$

That implies $f_a(e) = e^L$.

(ii) We know that

$$\begin{aligned} e^L &= f_a(me) && (\text{since } A(mx) \geq A(x)) \\ &= f_a(e) \\ &= f_a(xx^{-l}) \\ &= f_a(x) f_a(x^{-l}) && (\text{since } f_a \text{ is an homomorphism of} \\ &&& \text{an fuzzy soft subgroup}) \\ e^L (f_a(x))^{-l} &= f_a(x^{-l}) \\ (f_a(x))^{-l} &= f_a(x^{-l}). \end{aligned}$$

Similarly, we can prove that

$$e^L = f_a(en) \qquad (\text{since } A(xn) \geq A(x)).$$

That implies

$$(f_a(x))^{-l} = f_a(x^{-l}).$$

Hence the proof. □

Definition 3.3. Let μ be an M-N fuzzy subgroup of an M-N group G. Then M-N subgroup μ_t for $t \in [0,1]$ and $t \geq \mu(e)$, are called level M-N subgroup of μ.

Theorem 3.4. *The M-N homomorphic image of a level M-N subgroup of an M-N fuzzy subgroup μ of an M-N group G is a level M-N subgroup of an M-N fuzzy soft subgroup $f_a(\mu)$ of an M-N soft subgroup G^L, where μ is f_a-soft invariant.*

Proof. Let G and G^L be any two M-N group.
Let $f_a : G \to G^L$ be an M-N homomorphism of an fuzzy soft subgroup of a group G.
Let μ be an M-N fuzzy subgroup of G.
Clearly, $f_a(\mu)$ is an M-N fuzzy soft subgroup of G^L.
Let μ_t be a level M-N subgroup of an M-N fuzzy subgroup μ of G.
Since f_a is an M-N homomorphism fuzzy soft subgroup, $f_a(\mu)$ is an M-N soft subgroup $f_a(\mu)$ of G^L and $f_a(\mu_t) = (f_a(\mu))_t$.
Hence $(f_a(\mu))_t$ is a level M-N soft subgroup $f_a(\mu)$ of G^L. □

Theorem 3.5. *The M-N homomorphism pre-image of a level M-N soft subgroup of an M-N fuzzy subgroup μ of an M-N group G^L is a level M-N subgroup of an M-N fuzzy soft subgroup $f_a^{-l}(\mu)$ of an M-N group G.*

Proof. Let $a \in A$, $m \in M$ and $n \in N$.
Let G and G^L be any two M-N group.
Let $f_a : G \to G^L$ be an M-N homomorphism of an fuzzy soft subgroup of a group G.
Let μ be an M-N fuzzy subgroup of G^L.

Clearly, $f_a^{-l}(m\mu) = f_a^{-l}(\mu)$ and $f_a^{-l}(\mu n) = f_a^{-l}(\mu)$ is an M-N fuzzy soft subgroup of G.
Let μ_t be a level M-N subgroup of an M-N fuzzy subgroup μ of G^L.
Since f_a is an M-N homomorphism fuzzy soft subgroup, $f_a^{-l}(\mu_t)$ is an M-N soft subgroup of $f_a^{-l}(\mu)$ of G and $f_a^{-l}(\mu_t) = (f_a^{-l}(\mu))_t$ is an M-N soft subgroup of an M-N fuzzy soft subgroup $f_a^{-l}(\mu)$ of G.
That is $(f_a^{-l}(\mu))_t$ is a level M-N subgroup of an M-N fuzzy soft subgroup $f_a^{-l}(\mu)$ of G.
Hence the proof. □

4. Conclusion

The primary findings in this paper are based on the concept of fuzzy soft subgroups under M-N homomorphism [7, 10]. We also defined the M-N level subsets of the fuzzy soft subgroup and highlighted some of its basic features.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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