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Research Article

A New Approach of Perfect Domination in Product of Interval-Valued Fuzzy Incidence Graphs With Application

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Abstract. Fuzzy graphs, also known as fuzzy incidence graphs, are a useful and well-organized tool for encapsulating and resolving a variety of real-world situations involving ambiguous data and information. In this investigation article, we introduced the chance of *interval-valued fuzzy incidence graphs* (IVFIGs) alongside their specific properties. The operations of *Cartesian product* (CP), *Tensor product* (TP) in IVFIGs are additionally examined. The technique to compute the *degree* (DG) of IVFIGs acquired by CP and TP is examined. Some significant hypotheses to figure the DG of the vertices of IVFIGs gained by CP and TP are explained. An innovative idea of perfect domination in CP of two IVFIGs and TP of two IVFIGs utilizing incidence pair are presented and gotten the connection between them. Eventually, genuine utilization of *perfect domination number* (PDN) to discover which countries (country) have the best education policies among various countries is inspected.

Keywords. Interval-valued Fuzzy Incidence Graph, Cartesian product of two IVFIGs, Tensor product of two IVFIGs, Perfect dominating set

Mathematics Subject Classification (2020). 05C12, 05C72

1. Introduction

Zadeh [41] introduced fuzzy set theory and related fuzzy logic for dealing with and addressing numerous issues in which variables, parameters, and relations are only imprecisely known, necessitating the use of approximate reasoning systems. This is true of practically all nontrivial and, in particular, human-centered phenomena, processes, and systems that exist in reality, and it is difficult to characterise them adequately using standard mathematics based on binary logic.

Fuzzy set theory has been developed in a variety of directions, piqueing the interest of mathematicians and computer scientists working in a variety of domains. As an extension of fuzzy sets, Zadeh presented IVFSs, in which the values of the membership degrees are intervals of numbers rather than the numbers themselves. Traditional fuzzy sets do not adequately describe uncertainty, however IVFSs do. In applications like fuzzy control, it's critical to use IVFSs. Defuzzification is one of the most computationally costly aspects of fuzzy control. We summarise Gorzalczany's work on IVFSs [9, 10] and Roy *et al*. [28] work on fuzzy relations because IVFSs are frequently applied.

Mordeson and Chang-Shyh [17] discussed fuzzy graph operations. The idea of IFSs was first initiated by Atanassov [1]. The notion of IFGs was introduced by Parvathi *et al*. [22]. Parvathi *et al*. [23] investigated operations on IFGs. In IFGs, Gani [8] established the concepts of degree, order, and size. Samanta and Pal [34, 35] have also expressed various fuzzy graphs. Rashmanlou and Pal [26] recommended irregular IVFGs.

The notion of products on IFGs was initiated by Sahoo and Pal [29]. IVFGs have been researched further by Rashmanlou and Pal [21]. Intuitionistic fuzzy competition graphs have also been expressed by Sahoo and Pal [30]. Dinesh [7] explored fuzzy incidence graphs (FIGS). Fuzzy strong graphs have also been expressed by Kalaiarasi *et al*. [13]. The idea of multiple IFGs was given by Sahoo and Pal [31, 32]. Concepts in FIGs were proposed by Mathew and Mordeson [16]. A fuzzy graph with applicability was proposed by Sahoo *et al*. [33].

Domination was first introduced by O. Ore [25] and C. Berge [2], and further studied by Somasundaram and Somasundaram [39]. The product of the new graph was produced by Nazeer *et al*. [19]. Clique coverings have also been found in IVFGs by Patra *et al*. [24]. Domination in graphs has been examined further by Haynes and Hedetniemi [11]. By utilising effective edges, Somasundaram and Somasundaram have established dominance in fuzzy graphs [39]. Xavior *et al*. [40] recommended domination in fuzzy graphs. In IVFGs, Debnath [5] has also displayed dominance. Revathi and Harinarayaman [27] proposed an equitable domination number for fuzzy graphs. Sunitha and Manjusha have also stated that they have significant dominance [15].

In a fuzzy graph, Nagoorgani and Chandrasekaran [18] have also demonstrated dominance. For fuzzy graphs, Sarala and Kavitha have also expressed (1, 2)-domination [36]. Domination parameters for fuzzy graphs have also been given by Dharmalingam and Nithya [6]. In fuzzy graphs, Manjusha *et al*. [14] have discussed paired domination. The dominating set has been discussed by Bozhenyuk *et al*. [4]. Nazeer *et al*. [20] have established dominance in FIGs. Selvam and Ponnappan [37] have discussed dominance in fuzzy graphs. The inverse dominating set of IVFGs was recommended by Shain and Shubatah [38]. Tushar *et al*. [3] proposed a new path graph definition. In fuzzy graphs, Ismayil and Begum [12] have also represented accurate split domination.

Section 2 presents some preliminary findings that are necessary to comprehend the remainder of the article. In Section 3 conveys a meaning DG of a vertex in CP of two IVFIGs. In Section 4 we examine the DG of a vertex in TP of two IVFIGs. In Section 5 perfect domination in CP and TP of two IVFIGs is given. In Section 6, a genuine utilization of PDN in the issue of education policies among various countries is clarified. In Section 7, a comparative analysis is provided.

2. Preliminaries

Definition 2.1. A fuzzy subset μ_{FS} on a set M_{FS} is a map $\mu_{FS}: M_{FS} \to [0,1]$. A map $\gamma_{FS}: M_{FS} \times M_{FS} \to [0,1]$ is known as a fuzzy relation on μ_{FS} if $\gamma_{FS}(w_{11}, w_{22}) \le \min\{\mu_{FS}(w_{11}), \mu_{FS}(w_{22})\}$ for each $w_{11}, w_{22} \in M_{FS}$. A fuzzy graph is a pair $G_{FS} = (\mu_{FS}, \gamma_{FS})$, where μ_{FS} is a fuzzy subset on a set V_{FS} and γ_{FS} is a fuzzy relation on μ_{FS}.

Definition 2.2 ([5]). An IVFS A_{IV} on a set V_{IV} defined by $A_{IV} = \{(w_{11}, [\mu^-_{A_{IV}}(w_{11}), \mu^+_{A_{IV}}(w_{11})]), w_{11} \in V_{IV}\}$, where $\mu^-_{A_{IV}}$ and $\mu^+_{A_{IV}}$ are fuzzy subsets of V_{IV} such that $\mu^-_{A_{IV}}(w_{11}) \le \mu^+_{A_{IV}}(w_{11})$ for all $w_{11} \in V_{IV}$. If $G^*_{IV} = (V_{IV}, E_{IV})$ is a crisp graph, then by an interval-valued fuzzy relation B_{IV} on V_{IV} we mean an IVFS on E_{IV} such that $\mu^-_{B_{IV}}(w_{11}w_{22}) \le \min\{\mu^-_{A_{IV}}(w_{11}), \mu^-_{A_{IV}}(w_{22})\}$ and $\mu^+_{B_{IV}}(w_{11}w_{22}) \le \max\{\mu^+_{A_{IV}}(w_{11}), \mu^+_{A_{IV}}(w_{22})\}$ for all $w_{11}w_{22} \in E_{IV}$ and we write $B_{IV} = \{(w_{11}w_{22}, [\mu^-_{B_{IV}}(w_{11}w_{22}), \mu^+_{B_{IV}}(w_{11}w_{22})]), w_{11}w_{22} \in E_{IV}\}$.

Definition 2.3 ([5]). An IVFG of a graph $G^*_{IV} = (V_{IV}, E_{IV})$ is a pair $G_{IV} = (A_{IV}, B_{IV})$, where $A_{IV} = [\mu^-_{A_{IV}}, \mu^+_{A_{IV}}]$ is an IVFS on V_{IV} and $B_{IV} = [\mu^-_{B_{IV}}, \mu^+_{B_{IV}}]$ is an interval-valued fuzzy relation on V_{IV}.

Example 2.4.

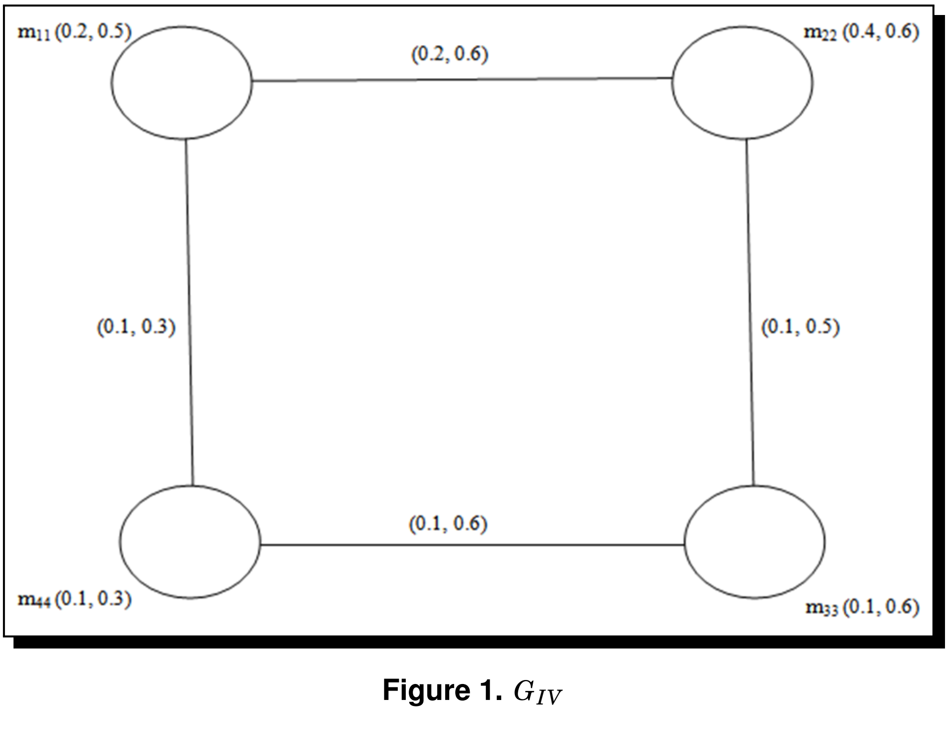

Figure 1. G_{IV}

Figure 1 indicates a IVFG $G_{IV} = (V_{IV}, E_{IV}, \mu_{A_{IV}}, \mu_{B_{IV}})$ with

$\mu_{A_{IV}}(m_{11}) = (0.2, 0.5)$, $\mu_{A_{IV}}(m_{22}) = (0.4, 0.6)$,
$\mu_{A_{IV}}(m_{33}) = (0.1, 0.6)$, $\mu_{A_{IV}}(m_{11}) = (0.2, 0.5)$,
$\mu_{B_{IV}}(m_{11}m_{22}) = (0.2, 0.6)$, $\mu_{B_{IV}}(m_{22}m_{33}) = (0.1, 0.5)$,
$\mu_{B_{IV}}(m_{33}m_{44}) = (0.1, 0.6)$, $\mu_{B_{IV}}(m_{11}m_{44}) = (0.1, 0.3)$.

Definition 2.5. Let $G_{IV} = (V_{IV}, E_{IV}, \mu_{IV}, \gamma_{IV})$ be an IVFG and $w_{11} \in V_{IV}$, then its DG is represented by $d_{G_{IV}}(w_{11}) = (d_{1G_{IV}}(w_{11}), d_{2G_{IV}}(w_{11}))$ and defined by $d_{1G_{IV}}(w_{11}) = \sum_{w_{11} \neq w_{22}} \gamma_{1IV}(w_{11}, w_{22}) = \sum_{(w_{11}, w_{22}) \in E_{IV}} \gamma_{1IV}(w_{11}, w_{22})$ and $d_{2G_{IV}}(w_{11}) = \sum_{w_{11} \neq w_{22}} \gamma_{2IV}(w_{11}, w_{22}) = \sum_{(w_{11}, w_{22}) \in E_{IV}} \gamma_{2IV}(w_{11}, w_{22})$.

Definition 2.6 ([7])**.** Assume $G_I = (V_I, E_I)$ is a graph. Then, $G_I = (V_I, E_I, I_I)$ is named as an incidence graph, where $I_I \subseteq V_I \times E_I$.

Definition 2.7 ([7])**.** Assume $G_{FS} = (V_{FS}, E_{FS})$ is a graph, μ_{FS} is a fuzzy subset of V_{FS}, and γ_{FS} is a fuzzy subset of $V_{FS} \times V_{FS}$. Let ψ_{FS} be a fuzzy subset of $V_{FS} \times E_{FS}$. If $\psi_{FS}(w_{11}, w_{11}w_{22}) \leq \min\{\mu_{FS}(w_{11}), \gamma_{FS}(w_{11}w_{22})\}$ for every $w_{11} \in V_{FS}, w_{11}w_{22} \in E_{FS}$, then ψ_{FS} is a fuzzy incidence of G_{FS}.

Definition 2.8 ([7])**.** Assume G_I is a graph and (μ_I, γ_I) is a fuzzy sub graph of G_I. If ψ_I is a fuzzy incidence of G_I, then $G_I = (\mu_I, \gamma_I, \psi_I)$ is named as FIG of G_I.

Example 2.9.

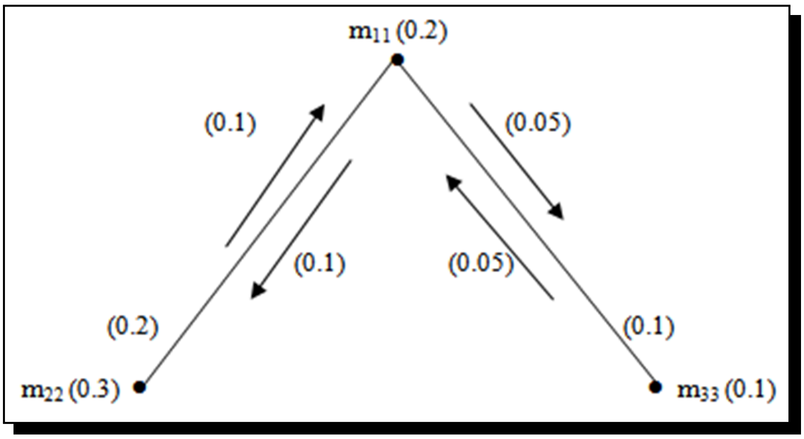

Figure 2. G_I

A FIG with $\mu_I(m_{11}) = 0.2$, $\mu_I(m_{22}) = 0.3$, $\mu_I(m_{33}) = 0.1$, $\gamma_I(m_{11}m_{22}) = 0.2$, $\gamma_I(m_{11}m_{33}) = 0.1$ and $\psi_I(m_{11}, m_{11}m_{22}) = 0.1$, $\psi_I(m_{22}, m_{11}m_{22}) = 0.1$, $\psi_I(m_{11}, m_{11}m_{33}) = 0.05$, $\psi_I(m_{33}, m_{11}m_{33}) = 0.05$ is shown in Figure 2.

3. DG of A Vertex in CP of Two IVFIGs

Nomenclature

G_{IV}	:	Interval-Valued Fuzzy Graph
G_{IVI}	:	Interval-Valued Fuzzy Incidence Graph
V_{IV}, V_{IVI}	:	Vertices
E_{IV}, E_{IVI}	:	Edges
I_{IVI}	:	Incidence Pair
IVFS	:	Interval-Valued Fuzzy Set
DG	:	Degree
IFS	:	Intuitionistic Fuzzy Set
IFG	:	Intuitionistic Fuzzy Graph
FIG	:	Fuzzy Incidence Graph
IVFIG	:	Interval-Valued Fuzzy Incidence Graph
MS	:	Membership
NMS	:	Non Membership
CP	:	Cartesian Product
TP	:	Tensor Product
PDN	:	Perfect Domination Number
PDS	:	Perfect Dominating Set

Definition 3.1. An IVFIG is of the form $G_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K, \mu_L, \mu_M)$ where $\mu_K = (\mu_K^-, \mu_K^+)$, $\mu_L = (\mu_L^-, \mu_L^+)$, $\mu_M = (\mu_M^-, \mu_M^+)$ and $V_{IVI} = \{w_0, w_1, \dots w_n\}$ such that $\mu_K^- : V_{IVI} \to [0,1]$ and $\mu_K^+ : V_{IVI} \to [0,1]$ represent the DG of MS and NMS of the vertex $w_{ii} \in V_{IVI}$ respectively, and $\mu_K^-(w_{11}) \le \mu_K^+(w_{11})$, $0 \le \mu_K^- + \mu_K^+ \le 1$ for each $w_{ii} \in V_{IVI}$ $(i = 1,2,\dots,n)$, $\mu_L^- : V_{IVI} \times V_{IVI} \to [0,1]$ and $\mu_L^+ : V_{IVI} \times V_{IVI} \to [0,1]$ $\mu_L^-(w_{11}, w_{22})$ and $\mu_L^+(w_{11}, w_{22})$ show the DG of MS and NMS of the edge (w_{11}, w_{22}) respectively, such that$\mu_L^-(w_{11}, w_{22}) \le \min\{\mu_K^-(w_{11}), \mu_K^-(w_{22})\}$ and $\mu_L^+(w_{11}, w_{22}) \le \max\{\mu_K^+(w_{11}), \mu_K^+(w_{22})\}$, $0 \le \mu_L^-(w_{11}, w_{22}) + \mu_L^+(w_{11}, w_{22}) \le 1$ for every (w_{11}, w_{22}), $\mu_M^- : V_{IVI} \times E_{IVI} \to [0,1]$ and $\mu_M^+ : V_{IVI} \times E_{IVI} \to [0,1]$, $\mu_M^-(w_{11}, w_{11}w_{22})$ and $\mu_M^+(w_{11}, w_{11}w_{22})$ show the DG of MS and NMS of the incidence pair respectively, such that $\mu_M^-(w_{11}, w_{11}w_{22}) \le \min\{\mu_K^-(w_{11}), \mu_L^-(w_{11}, w_{22})\}$ and $\mu_M^+(w_{11}, w_{11}w_{22}) \le \max\{\mu_K^+(w_{11}), \mu_L^+(w_{11}, w_{22})\}$, $0 \le \mu_M^-(w_{11}, w_{11}w_{22}) + \mu_M^+(w_{11}, w_{11}w_{22}) \le 1$ for every $(w_{11}, w_{11}w_{22})$.

Definition 3.2. Let $G_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K, \mu_L, \mu_M)$ is an IVFIG and $w_{11} \in V_{IVI}$, then its DG is represented by $d_{G_{IVI}}(w_{11}) = (d_{1G_{IVI}}(w_{11}), d_{2G_{IVI}}(w_{11}))$ and defined by $d_{1G_{IVI}}(w_{11}) = \sum_{w_{11} \ne w_{22}} (w_{11}, w_{11}w_{22}) \in I_{IVI}$ and $d_{2G_{IVI}}(w_{11}) = \sum_{w_{11} \ne w_{22}} (w_{11}, w_{11}w_{22}) \in I_{IVI}$.

Definition 3.3. The CP of two IVFIGs $G_{IVI}^1 = (V_{IVI}^1, E_{IVI}^1, I_{IVI}^1, \mu_K^1, \mu_L^1, \mu_M^1)$ and $G_{IVI}^2 = (V_{IVI}^2, E_{IVI}^2, I_{IVI}^2, \mu_K^2, \mu_L^2, \mu_M^2)$ is defined as an IVFIG

$$G_{IVI} = G_{IVI}^1 \times G_{IVI}^2 = (V_{IVI}, E_{IVI}, I_{IVI}, \mu_K^1 \times \mu_K^2, \mu_L^1 \times \mu_L^2, \mu_M^1 \times \mu_M^2)$$

where

$$V_{IVI} = V^1_{IVI} \times V^2_{IVI},$$

and

$$E_{IVI} = \{((m_1,n_1),(m_2,n_2))/m_1 = m_2,(n_1,n_2) \in E^2_{IVI} \text{ or } n_1 = n_2,(m_1,m_2) \in E^1_{IVI}\}$$

$$I_{IVI} = \{(m_1,n_1),(m_1,n_1)(m_1,n_2)/m_1 = m_2,(n_1,n_1n_2) \in I^2_{IVI},(n_2,n_1n_2) \in I^2_{IVI} \text{ or } n_1 = n_2(m_1,m_1m_2) \in I^1_{IVI},\ (m_2,m_1m_2) \in I^1_{IVI}\}$$

with

$$(\mu^{-1}_K \times \mu^{-2}_K)(m_1,n_1) = \min\{\mu^{-1}_K(m_1),\mu^{-2}_K(n_1)\} \quad \forall\ (m_1,n_1) \in V^1_{IVI} \times V^2_{IVI},$$

$$(\mu^{+1}_K \times \mu^{+2}_K)(m_1,n_1) = \max\{\mu^{+1}_K(m_1),\mu^{+2}_K(n_1)\} \quad \forall\ (m_1,n_1) \in V^1_{IVI} \times V^2_{IVI},$$

$$(\mu^{-1}_L \times \mu^{-2}_L)((m_1,n_1)(m_2,n_2)) = \begin{cases} \min\{\mu^{-1}_K(m_1),\mu^{-2}_L(n_1,n_2)\} & \text{if } m_1 = m_2,(n_1,n_2) \in E^2_{IVI}, \\ \min\{\mu^{-1}_L(m_1,m_2),\mu^{-2}_K(n_1)\} & \text{if } n_1 = n_2,(m_1,m_2) \in E^1_{IVI}, \end{cases}$$

$$(\mu^{+1}_L \times \mu^{+2}_L)((m_1,n_1)(m_2,n_2)) = \begin{cases} \max\{\mu^{+1}_K(m_1),\mu^{+2}_L(n_1,n_2)\} & \text{if } m_1 = m_2,(n_1,n_2) \in E^2_{IVI}, \\ \max\{\mu^{+1}_L(m_1,m_2),\mu^{+2}_K(n_1)\} & \text{if } n_1 = n_2,(m_1,m_2) \in E^1_{IVI}, \end{cases}$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_1,n_1),(m_1,n_1)(m_1,n_2)] = \min\{\mu^{-1}_K(m_1),\mu^{-2}_M(n_1,n_1n_2)\} \text{ if } m_1 = m_2,(n_1,n_1n_2) \in I^2_{IVI}$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_1,n_2),(m_1,n_1)(m_1,n_2)] = \min\{\mu^{-1}_K(m_1),\mu^{-2}_M(n_2,n_1n_2)\} \text{ if } m_1 = m_2,(n_2,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_1,n_1),(m_1,n_1)(m_2,n_1)] = \min\{\mu^{-1}_M(m_1,m_1m_2),\mu^{-2}_K(n_1)\} \text{ if } n_1 = n_2,(m_1,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_2,n_1),(m_1,n_1)(m_2,n_1)] = \min\{\mu^{-1}_M(m_2,m_1m_2),\mu^{-2}_K(n_1)\} \text{ if } n_1 = n_2,(m_2,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_1,n_2),(m_1,n_2)(m_2,n_2)] = \min\{\mu^{-1}_M(m_1,m_1m_2),\mu^{-2}_K(n_2)\} \text{ if } n_1 = n_2,(m_1,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_2,n_2),(m_1,n_2)(m_2,n_2)] = \min\{\mu^{-1}_M(m_2,m_1m_2),\mu^{-2}_K(n_2)\} \text{ if } n_1 = n_2,(m_2,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_2,n_1),(m_2,n_1)(m_2,n_2)] = \min\{\mu^{-2}_M(n_1,n_1n_2),\mu^{-1}_K(m_2)\} \text{ if } m_1 = m_2,(n_1,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{-1}_M \times \mu^{-2}_M)[(m_2,n_2),(m_2,n_1)(m_2,n_2)] = \min\{\mu^{-2}_M(n_2,n_1n_2),\mu^{-1}_K(m_2)\} \text{ if } m_1 = m_2,(n_2,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_1,n_1),(m_1,n_1)(m_1,n_2)] = \max\{\mu^{+1}_K(m_1),\mu^{+2}_M(n_1,n_1n_2)\} \text{ if } m_1 = m_2,(n_1,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_1,n_2),(m_1,n_1)(m_1,n_2)] = \max\{\mu^{+1}_K(m_1),\mu^{+2}_M(n_2,n_1n_2)\} \text{ if } m_1 = m_2,(n_2,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_1,n_1),(m_1,n_1)(m_2,n_1)] = \max\{\mu^{+1}_M(m_1,m_1m_2),\mu^{+2}_K(n_1)\} \text{ if } n_1 = n_2,(m_1,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_2,n_1),(m_1,n_1)(m_2,n_1)] = \max\{\mu^{+1}_M(m_2,m_1m_2),\mu^{+2}_K(n_1)\} \text{ if } n_1 = n_2,(m_2,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_1,n_2),(m_1,n_2)(m_2,n_2)] = \max\{\mu^{+1}_M(m_1,m_1m_2),\mu^{+2}_K(n_2)\} \text{ if } n_1 = n_2,(m_1,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_2,n_2),(m_1,n_2)(m_2,n_2)] = \max\{\mu^{+1}_M(m_2,m_1m_2),\mu^{+2}_K(n_2)\} \text{ if } n_1 = n_2,(m_2,m_1m_2) \in I^1_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_2,n_1),(m_2,n_1)(m_2,n_2)] = \max\{\mu^{+2}_M(n_1,n_1n_2),\mu^{+1}_K(m_2)\} \text{ if } m_1 = m_2,(n_1,n_1n_2) \in I^2_{IVI},$$

$$(\mu^{+1}_M \times \mu^{+2}_M)[(m_2,n_2),(m_2,n_1)(m_2,n_2)] = \max\{\mu^{+2}_M(n_2,n_1n_2),\mu^{+1}_K(m_2)\} \text{ if } m_1 = m_2,(n_2,n_1n_2) \in I^2_{IVI}.$$

Example 3.4.

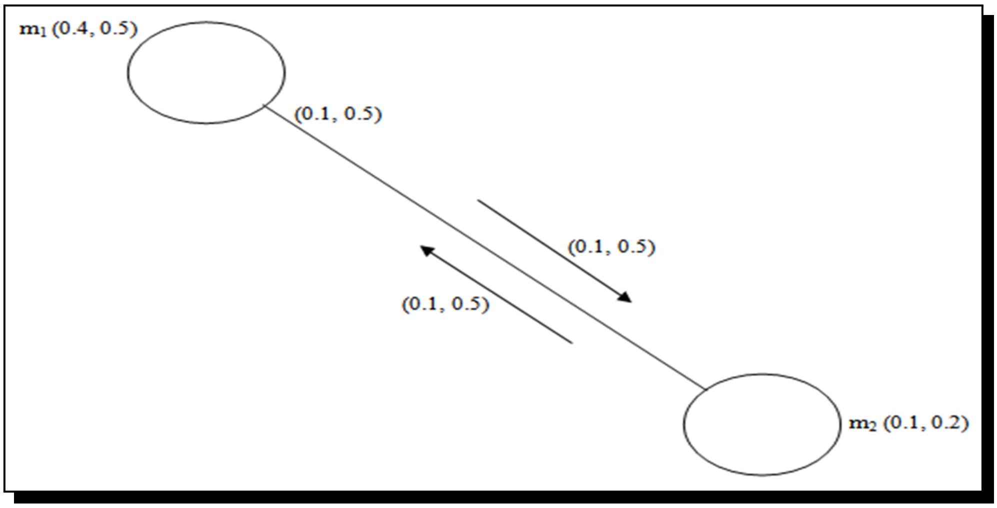

Figure 3. G^1_{IVI}

Figure 3 indicates a IVFIG $G^1_{IVI} = (V^1_{IVI}, E^1_{IVI}, I^1_{IVI}, \mu^1_K, \mu^1_L, \mu^1_M)$ with

$\mu^1_K(m_1) = (0.4, 0.5)$, $\mu^1_K(m_2) = (0.1, 0.2)$, $\mu^1_L(m_1m_2) = (0.1, 0.5)$, $\mu^1_M(m_1, m_1m_2) = (0.1, 0.5)$, $\mu^1_M(m_2, m_1m_2) = (0.1, 0.5)$.

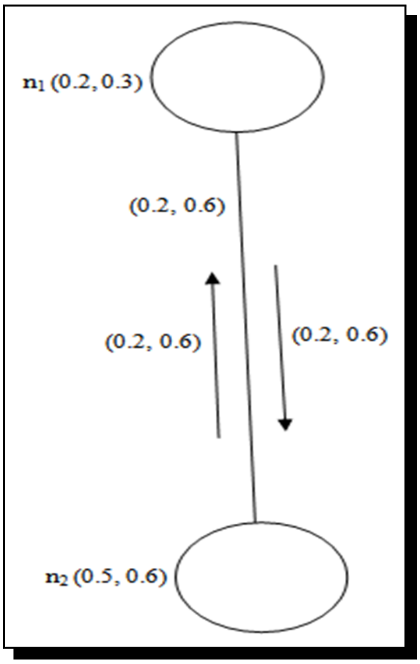

Figure 4. G^2_{IVI}

Figure 4 indicates a IVFIG $G^2_{IVI} = (V^2_{IVI}, E^2_{IVI}, I^2_{IVI}, \mu^2_K, \mu^2_L, \mu^2_M)$

$\mu^1_K(n_1) = (0.2, 0.3)$, $\mu^1_K(n_2) = (0.5, 0.6)$, $\mu^1_L(n_1n_2) = (0.2, 0.6)$, $\mu^1_M(n_1, n_1n_2) = (0.2, 0.6)$, $\mu^1_M(n_2, n_1n_2) = (0.2, 0.6)$.

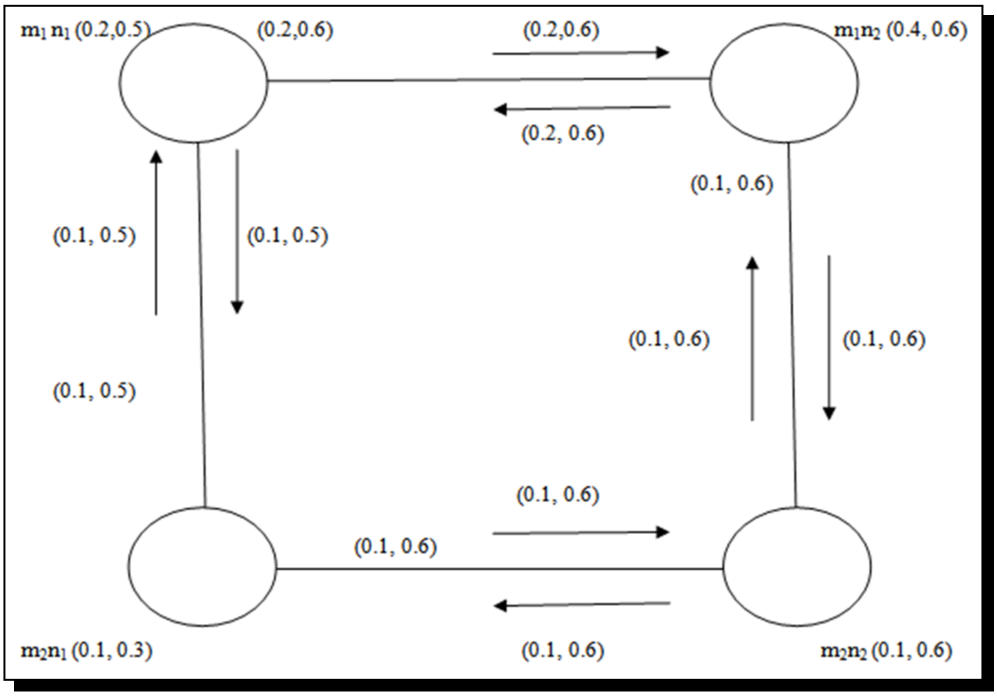

Figure 5. $G^1_{IVI} \times G^2_{IVI}$ of Figure 3 and 4

Figure 5 indicates a CP of two IVFIGs

$G^1_{IVI} \times G^2_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu^1_K \times \mu^2_K, \mu^1_L \times \mu^2_L, \mu^1_M \times \mu^2_M)$

$(\mu^1_K \times \mu^2_K)(m_1n_1) = (0.2, 0.5)$,

$(\mu^1_K \times \mu^2_K)(m_1n_2) = (0.4, 0.6)$,

$(\mu^1_K \times \mu^2_K)(m_2n_1) = (0.1, 0.3)$,

$(\mu^1_K \times \mu^2_K)(m_2n_2) = (0.1, 0.6)$,

$(\mu^1_L \times \mu^2_L)(m_1n_1, m_1n_2) = (0.2, 0.6)$,

$(\mu^1_L \times \mu^2_L)(m_1n_1, m_2n_1) = (0.1, 0.5)$,

$(\mu^1_L \times \mu^2_L)(m_1n_2, m_2n_2) = (0.1, 0.6)$,

$(\mu^1_L \times \mu^2_L)(m_2n_1, m_2n_2) = (0.1, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_1n_1, m_1n_1m_1n_2) = (0.2, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_1n_2, m_1n_1m_1n_2) = (0.2, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_1n_2, m_1n_2m_2n_2) = (0.1, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_2n_2, m_1n_2m_2n_2) = (0.1, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_2n_1, m_2n_1m_2n_2) = (0.1, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_2n_2, m_2n_1m_2n_2) = (0.1, 0.6)$,

$(\mu^1_M \times \mu^2_M)(m_1n_1, m_1n_1m_2n_1) = (0.1, 0.5)$,

$(\mu^1_M \times \mu^2_M)(m_2n_1, m_1n_1m_2n_1) = (0.1, 0.5)$.

Definition 3.5. Let $G_{IVI} = G^1_{IVI} \times G^2_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu^1_K \times \mu^2_K, \mu^1_L \times \mu^2_L, \mu^1_M \times \mu^2_M)$ be the CP of two IVFIGs $G^1_{IVI} = (V^1_{IVI}, E^1_{IVI}, I^1_{IVI}, \mu^1_K, \mu^1_L, \mu^1_M)$ and $G^2_{IVI} = (V^2_{IVI}, E^2_{IVI}, I^2_{IVI}, \mu^2_K, \mu^2_L, \mu^2_M)$. Then the DG of $V_{IVI} = (m_1, n_1)$ is represented by

$$d_{G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) = (d_{1G^1_{IVI} \times G^2_{IVI}}(m_1, n_1), d_{2G^1_{IVI} \times G^2_{IVI}}(m_1, n_1))$$

and defined by

$$d_{1G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) = \sum_{m_1 = m_2, (n_1, n_1 n_2) \in I^2} \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_1, n_1 n_2)\} + \sum_{n_1 = n_2, (m_1, m_1 m_2) \in I^1} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_K^{-2}(n_1)\},$$

$$d_{2G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) = \sum_{m_1 = m_2, (n_1, n_1 n_2) \in I^2} \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_1, n_1 n_2)\} + \sum_{n_1 = n_2, (m_1, m_1 m_2) \in I^1} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_1)\}.$$

Theorem 3.6. *Let $G^1_{IVI} = (V^1_{IVI}, E^1_{IVI}, I^1_{IVI}, \mu^1_K, \mu^1_L, \mu^1_M)$ and $G^2_{IVI} = (V^2_{IVI}, E^2_{IVI}, I^2_{IVI}, \mu^2_K, \mu^2_L, \mu^2_M)$ be two IVFIGs. If $\mu_K^{-1} \le \mu_K^{+1}$, $\mu_K^{-1} \ge \mu_M^{-2}$, $\mu_K^{+1} \le \mu_M^{+2}$ and $\mu_K^{-2} \le \mu_K^{+2}$, $\mu_K^{-2} \ge \mu_M^{-1}$, $\mu_K^{+2} \le \mu_M^{+1}$ then $d_{G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) = (d_{G^1_{IVI}}(m_1) + d_{G^2_{IVI}}(n_1))$.*

Proof. In CP by the definition of the DG of a vertex, we have

$$\begin{aligned}
d_{1G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) &= \sum_{m_1 = m_2, (n_1, n_1 n_2) \in I^2} \min\{\mu_K^{-1}(m_1), \mu_M^{-2}(n_1, n_1 n_2)\} \\
&\quad + \sum_{n_1 = n_2, (m_1, m_1 m_2) \in I^1} \min\{\mu_M^{-1}(m_1, m_1 m_2), \mu_M^{-2}(n_1)\} \\
&= \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{-2}(n_1, n_1 n_2) + \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{-1}(m_1, m_1 m_2) \\
&\qquad \text{since } \mu_K^{-1} \le \mu_K^{+1},\ \mu_K^{-1} \ge \mu_M^{-2},\ \mu_K^{-2} \ge \mu_M^{-1} \\
&= \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{-1}(m_1, m_1 m_2) + \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{-2}(n_1, n_1 n_2) \\
&= d_{1G^1_{IVI}}(m_1) + d_{1G^2_{IVI}}(n_1)
\end{aligned}$$

$$\begin{aligned}
d_{2G^1_{IVI} \times G^2_{IVI}}(m_1, n_1) &= \sum_{m_1 = m_2, (n_1, n_1 n_2) \in I^2} \max\{\mu_K^{+1}(m_1), \mu_M^{+2}(n_1, n_1 n_2)\} \\
&\quad + \sum_{n_1 = n_2, (m_1, m_1 m_2) \in I^1} \max\{\mu_M^{+1}(m_1, m_1 m_2), \mu_K^{+2}(n_1)\} \\
&= \sum_{(n_1, n_1 n_2) \in I^2} \mu_M^{+2}(n_1, n_1 n_2) + \sum_{(m_1, m_1 m_2) \in I^1} \mu_M^{+1}(m_1, m_1 m_2) \\
&\qquad \text{since } \mu_K^{-2} \le \mu_K^{+2},\ \mu_K^{+1} \le \mu_M^{+2},\ \mu_K^{+2} \le \mu_M^{+1}
\end{aligned}$$

$$= \sum_{(m_1,m_1m_2)\in I^1} \mu_M^{+1}(m_1,m_1m_2) + \sum_{(n_1,n_1n_2)\in I^2} \mu_M^{+2}(n_1,n_1n_2)$$

$$= d_{2G^1_{IVI}}(m_1) + d_{2G^2_{IVI}}(n_1).$$

Hence $d_{G^1_{IVI}\times G^2_{IVI}}(m_1,n_1) = (d_{G^1_{IVI}}(m_1) + d_{G^2_{IVI}}(n_1))$. □

Example 3.7. Let G^1_{IVI} and G^2_{IVI} be two IVFIGs as shown in Figures 3 and 4, and their CP is provided in Figure 5 with

$$\mu_K^{-1} \le \mu_K^{+1},\ \mu_K^{-1} \ge \mu_M^{-2},\ \mu_K^{+1} \le \mu_M^{+2} \text{ and } \mu_K^{-2} \le \mu_K^{+2},\ \mu_K^{-2} \ge \mu_M^{-1},\ \mu_K^{+2} \le \mu_M^{+1}.$$

Then, by Theorem 3.6, we have

$$d_{1G^1_{IVI}\times G^2_{IVI}}(m_1,n_1) = d_{1G^1_{IVI}}(m_1) + d_{1G^2_{IVI}}(n_1) = 0.1 + 0.2 = 0.3,$$

$$d_{2G^1_{IVI}\times G^2_{IVI}}(m_1,n_1) = d_{2G^1_{IVI}}(m_1) + d_{2G^2_{IVI}}(n_1) = 0.5 + 0.6 = 1.1.$$

So $d_{G^1_{IVI}\times G^2_{IVI}}(m_1,n_1) = (0.3, 1.1)$.

4. DG of A Vertex in TP of two IVFIGs

Definition 4.1. The TP of two IVFIGs $G^1_{IVI} = (V^1_{IVI}, E^1_{IVI}, I^1_{IVI}, \mu^1_K, \mu^1_L, \mu^1_M)$ and $G^2_{IVI} = (V^2_{IVI}, E^2_{IVI}, I^2_{IVI}, \mu^2_K, \mu^2_L, \mu^2_M)$ is defined as an IVFIG,

$$G_{IVI} = G^1_{IVI} \diamondsuit G^2_{IVI} = (V_{IVI}, E_{IVI}, I_{IVI}, \mu^1_K \diamondsuit \mu^2_K, \mu^1_L \diamondsuit \mu^2_L, \mu^1_M \diamondsuit \mu^2_M)$$

where

$$V_{IVI} = V^1_{IVI} \times V^2_{IVI},$$

$$E_{IVI} = \{((m_1,n_1),(m_2,n_2))/(m_1,m_2) \in E^1_{IVI}, (n_1,n_2) \in E^2_{IVI}\}$$

and

$$I_{IVI} = \{(m_1,n_1),(m_1,n_1)(m_1,n_2)/(m_1,m_1m_2) \in I^1_{IVI}, (m_2,m_1m_2) \in I^1_{IVI}, (n_1,n_1n_2) \in I^2_{IVI}, (n_2,n_1n_2) \in I^2_{IVI}\}$$

with

$$(\mu_K^{-1} \diamondsuit \mu_K^{-2})(m_1,n_1) = \min\{\mu_K^{-1}(m_1), \mu_K^{-2}(n_1)\} \ \ \forall\ (m_1,n_1) \in V^1_{IVI} \diamondsuit V^2_{IVI},$$

$$(\mu_K^{+1} \diamondsuit \mu_K^{+2})(m_1,n_1) = \max\{\mu_K^{+1}(m_1), \mu_K^{+2}(n_1)\} \ \ \forall\ (m_1,n_1) \in V^1_{IVI} \diamondsuit V^2_{IVI},$$

$$(\mu_L^{-1} \diamondsuit \mu_L^{-2})((m_1,n_1)(m_2,n_2)) = \min\{\mu_L^{-1}(m_1,m_2), \mu_L^{-2}(n_1,n_2)\} \ \ \forall\ (m_1,m_2) \in E^1_{IVI}, (n_1,n_2) \in E^2_{IVI},$$

$$(\mu_L^{+1} \diamondsuit \mu_L^{+2})((m_1,n_1)(m_2,n_2)) = \max\{\mu_L^{+1}(m_1,m_2), \mu_L^{+2}(n_1,n_2)\} \ \ \forall\ (m_1,m_2) \in E^1_{IVI}, (n_1,n_2) \in E^2_{IVI},$$

$$(\mu_M^{-1} \diamondsuit \mu_M^{-2})[(m_1,n_1),(m_1,n_1)(m_2,n_2)] = \min\{\mu_M^{-1}(m_1,m_1m_2), \mu_M^{-2}(n_1,n_1n_2)\} \quad \forall\ (m_1,m_1m_2) \in I^1_{IVI}, (n_1,n_1n_2) \in I^2_{IVI},$$

$$(\mu_M^{-1} \diamondsuit \mu_M^{-2})[(m_2,n_2),(m_1,n_1)(m_2,n_2)] = \min\{\mu_M^{-1}(m_2,m_1m_2), \mu_M^{-2}(n_2,n_1n_2)\} \quad \forall\ (m_2,m_1m_2) \in I^1_{IVI}, (n_2,n_1n_2) \in I^2_{IVI},$$

$$(\mu_M^{-1}\Diamond\mu_M^{-2})[(m_1,n_2),(m_1,n_2)(m_2,n_1)] = \min\{\mu_M^{-1}(m_1,m_1m_2),\mu_M^{-2}(n_2,n_1n_2)\}$$
$$\forall\ (m_1,m_1m_2)\in I^1_{IVI},(n_2,n_1n_2)\in I^2_{IVI},$$
$$(\mu_M^{-1}\Diamond\mu_M^{-2})[(m_2,n_1),(m_1,n_2)(m_2,n_1)] = \min\{\mu_M^{-1}(m_2,m_1m_2),\mu_M^{-2}(n_1,n_1n_2)\}$$
$$\forall\ (m_2,m_1m_2)\in I^1_{IVI},(n_1,n_1n_2)\in I^2_{IVI},$$
$$(\mu_M^{+1}\Diamond\mu_M^{+2})[(m_1,n_1),(m_1,n_1)(m_2,n_2)] = \max\{\mu_K^{+1}(m_1,m_1m_2),\mu_M^{+2}(n_1,n_1n_2)\}$$
$$\forall\ (m_1,m_1m_2)\in I^1_{IVI},(n_1,n_1n_2)\in I^2_{IVI},$$
$$(\mu_M^{+1}\Diamond\mu_M^{+2})[(m_2,n_2),(m_1,n_1)(m_2,n_2)] = \max\{\mu_K^{+1}(m_2,m_1m_2),\mu_M^{+2}(n_2,n_1n_2)\}$$
$$\forall\ (m_2,m_1m_2)\in I^1_{IVI},(n_2,n_1n_2)\in I^2_{IVI},$$
$$(\mu_M^{+1}\Diamond\mu_M^{+2})[(m_1,n_2),(m_1,n_2)(m_2,n_1)] = \max\{\mu_M^{+1}(m_1,m_1m_2),\mu_M^{+2}(n_2,n_1n_2)\}$$
$$\forall\ (m_1,m_1m_2)\in I^1_{IVI},(n_2,n_1n_2)\in I^2_{IVI},$$
$$(\mu_M^{+1}\Diamond\mu_M^{+2})[(m_2,n_1),(m_1,n_2)(m_2,n_1)] = \max\{\mu_M^{+1}(m_2,m_1m_2),\mu_M^{+2}(n_1,n_1n_2)\}$$
$$\forall\ (m_2,m_1m_2)\in I^1_{IVI},(n_1,n_1n_2)\in I^2_{IVI}.$$

Example 4.2.

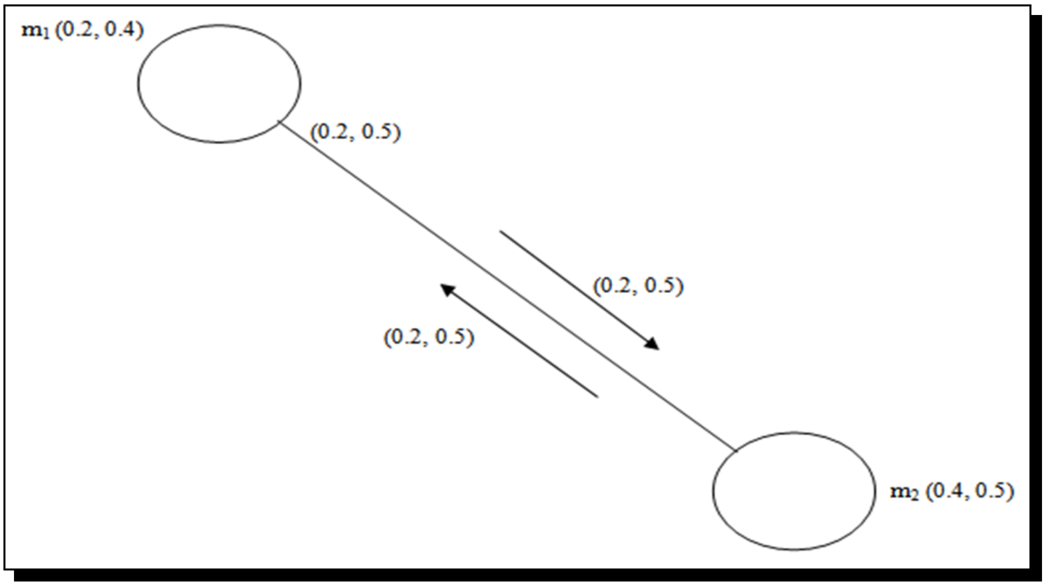

Figure 6. G^1_{IVI}

Figure 6 indicates a IVFIG $G^1_{IVI} = (V^1_{IVI}, E^1_{IVI}, I^1_{IVI}, \mu^1_K, \mu^1_L, \mu^1_M)$

$\mu^1_K(m_1) = (0.2, 0.4)$, $\mu^1_K(m_2) = (0.4, 0.5)$,
$\mu^1_L(m_1m_2) = (0.2, 0.5)$, $\mu^1_M(m_1, m_1m_2) = (0.2, 0.5)$,
$\mu^1_M(m_2, m_1m_2) = (0.2, 0.5)$.

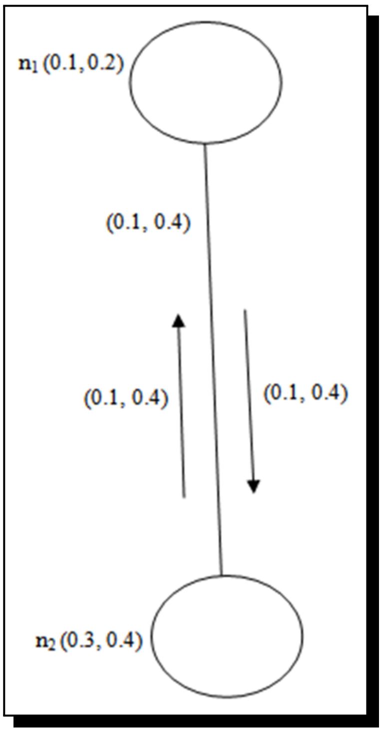

Figure 7. G^2_{IVI}

Figure 7 indicates a IVFIG $G^2_{IVI} = (V^2_{IVI}, E^2_{IVI}, I^2_{IVI}, \mu^2_K, \mu^2_L, \mu^2_M)$

$\mu^1_K(n_1) = (0.1, 0.2)$, $\mu^1_K(n_2) = (0.3, 0.4)$, $\mu^1_L(n_1n_2) = (0.1, 0.4)$,
$\mu^1_M(n_1, n_1n_2) = (0.1, 0.4)$, $\mu^1_M(n_2, n_1n_2) = (0.1, 0.4)$.

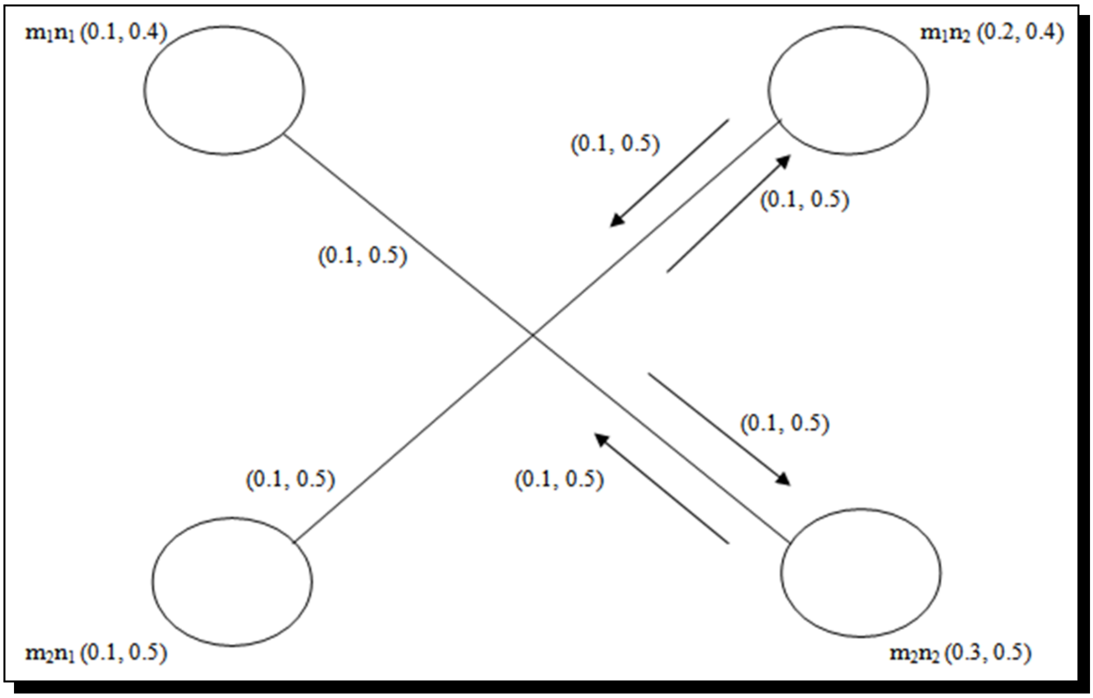

Figure 8. $G^1_{IVI} \Diamond G^2_{IVI}$ of Figure 6 and 7

Figure 8 indicates a TP of two IVFIGs $G^1_{IVI}\Diamond G^2_{IVI}=(V_{IVI},E_{IVI},I_{IVI},\mu^1_K\Diamond\mu^2_K,\mu^1_L\Diamond\mu^2_L,\mu^1_M\Diamond\mu^2_M)$

$(\mu^1_K\Diamond\mu^2_K)(m_1n_1)=(0.1,0.4)$,

$(\mu^1_K\Diamond\mu^2_K)(m_1n_2)=(0.2,0.4)$,

$(\mu^1_K\Diamond\mu^2_K)(m_2n_1)=(0.1,0.5)$,,

$(\mu^1_K\Diamond\mu^2_K)(m_2n_2)=(0.3,0.5)$,,

$(\mu^1_L\Diamond\mu^2_L)(m_1n_1,m_2n_2)=(0.1,0.5)$,

$(\mu^1_L\Diamond\mu^2_L)(m_1n_2,m_2n_1)=(0.1,0.5)$,

$(\mu^1_M\Diamond\mu^2_M)(m_1n_1,m_1n_1m_2n_2)=(0.1,0.5)$,

$(\mu^1_M\Diamond\mu^2_M)(m_2n_2,m_1n_1m_2n_2)=(0.1,0.5)$,

$(\mu^1_M\Diamond\mu^2_M)(m_1n_2,m_1n_2m_2n_1)=(0.1,0.5)$,

$(\mu^1_M\Diamond\mu^2_M)(m_2n_1,m_1n_2m_2n_1)=(0.1,0.5)$.

Definition 4.3. Let $G_{IVI}=G^1_{IVI}\Diamond G^2_{IVI}=(V_{IVI},E_{IVI},I_{IVI},\mu^1_K\Diamond\mu^2_K,\mu^1_L\Diamond\mu^2_L,\mu^1_M\Diamond\mu^2_M)$ be the TP of two IVFIGs $G^1_{IVI}=(V^1_{IVI},E^1_{IVI},I^1_{IVI},\mu^1_K,\mu^1_L,\mu^1_M)$ and $G^2_{IVI}=(V^2_{IVI},E^2_{IVI},I^2_{IVI},\mu^2_K,\mu^2_L,\mu^2_M)$. Then the DG of $V_{IVI}=(m_1,n_1)$ is represented by

$$d_{G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=(d_{1G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1),d_{2G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1))$$

and defined by

$$d_{1G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=\sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2}\min\{\mu^{-1}_M(m_1,m_1m_2),\mu^{-2}_M(n_1,n_1n_2)\},$$

$$d_{2G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=\sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2}\max\{\mu^{+1}_M(m_1,m_1m_2),\mu^{+2}_M(n_1,n_1n_2)\}.$$

Theorem 4.4. *Let $G^1_{IVI}=(V^1_{IVI},E^1_{IVI},I^1_{IVI},\mu^1_K,\mu^1_L,\mu^1_M)$ and $G^2_{IVI}=(V^2_{IVI},E^2_{IVI},I^2_{IVI},\mu^2_K,\mu^2_L,\mu^2_M)$ be two IVFIGs. If $\mu^{-1}_K\le\mu^{+1}_K,\mu^{-2}_M\ge\mu^{-1}_M,\mu^{+2}_M\le\mu^{+1}_M$, then $d_{G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=d_{G^1_{IVI}}(m_1)$ and if $\mu^{-2}_K\le\mu^{+2}_K,\mu^{-1}_M\ge\mu^{-2}_M,\mu^{+1}_M\le\mu^{+2}_M$ then $d_{G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=d_{G^2_{IVI}}(n_1)$.*

Proof. Suppose $\mu^{-1}_K\le\mu^{+1}_K,\mu^{-2}_M\ge\mu^{-1}_M,\mu^{+2}_M\le\mu^{+1}_M$, then

$$d_{1G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=\sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2}\min\{\mu^{-1}_M(m_1,m_1m_2),\mu^{-2}_M(n_1,n_1n_2)\}$$
$$=\sum\mu^{-1}_M(m_1,m_1m_2)=d_{1G^1_{IVI}}(m_1)$$
$$d_{2G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=\sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2}\max\{\mu^{+1}_M(m_1,m_1m_2),\mu^{+2}_M(n_1,n_1n_2)\}$$
$$=\sum\mu^{+1}_M(m_1,m_1m_2)=d_{2G^1_{IVI}}(m_1).$$

This implies $d_{G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=d_{G^1_{IVI}}(m_1)$. Similarly if $\mu^{-2}_K\le\mu^{+2}_K,\mu^{-1}_M\ge\mu^{-2}_M,\mu^{+1}_M\le\mu^{+2}_M$, then

$$d_{1G^1_{IVI}\Diamond G^2_{IVI}}(m_1,n_1)=\sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2}\min\{\mu^{-1}_M(m_1,m_1m_2),\mu^{-2}_M(n_1,n_1n_2)\}$$
$$=\sum\mu^{-2}_M(n_1,n_1n_2)=d_{1G^2_{IVI}}(n_1)$$

$$d_{2G^1_{IVI}\diamond G^2_{IVI}}(m_1,n_1) = \sum_{(m_1,m_1m_2)\in I^1,(n_1,n_1n_2)\in I^2} \max\{\mu^{+1}_M(m_1,m_1m_2),\mu^{+2}_M(n_1,n_1n_2)\}$$
$$= \sum \mu^{+2}_M(n_1,n_1n_2) = d_{2G^2_{IVI}}(n_1).$$

This implies $d_{G^1_{IVI}\diamond G^2_{IVI}}(m_1,n_1) = d_{G^2_{IVI}}(n_1)$. □

Example 4.5. In Figure 6 and 7 $\mu^{-1}_K \le \mu^{+1}_K$, $\mu^{-2}_M \ge \mu^{-1}_M$, $\mu^{+2}_M \le \mu^{+1}_M$ and $\mu^{-2}_K \le \mu^{+2}_K$, $\mu^{-1}_M \ge \mu^{-2}_M$, $\mu^{+1}_M \le \mu^{+2}_M$. Then, by Theorem 4.4, we have

$$d_{1G^1_{IVI}\diamond G^2_{IVI}}(m_1,n_1) = 0.1 = d_{1G^1_{IVI}}(m_1),$$
$$d_{2G^1_{IVI}\diamond G^2_{IVI}}(m_1,n_1) = 0.5 = d_{2G^2_{IVI}}(n_1).$$

Hence $d_{G^1_{IVI}\diamond G^2_{IVI}}(m_1,n_1) = (0.1,0.5)$.

5. Perfect Domination in CP and TP of two IVFIGs

Definition 5.1. A vertex w_{11} in an G_{IV} dominates to vertex w_{22} if $\mu^-_L(w_{11},w_{22}) = \min\{\mu^-_K(w_{11}),\mu^-_K(w_{22})\}$ and $\mu^+_M(w_{11},w_{22}) = \max\{\mu^+_K(w_{11}),\mu^+_K(w_{22})\}$. Then (w_{11},w_{22}) edge is called dominates edge.

Definition 5.2. A subset W_{IV} of V_{IV} is said to be a perfect dominating set (PDS) if for each vertex w_{11} not in W_{IV}, w_{11} is dominates exactly one vertex of W_{IV}.

Definition 5.3. A PDS W_{IV} of the G_{IV} is said to be a minimal PDS if each vertex w_{11} in W_{IV}, $W_{IV}-\{w_{11}\}$ is not a PDS.

Definition 5.4. A PDS with the lowest vertex cardinality is called a minimum PDS.

Definition 5.5. A vertex cardinality of a minimum PDS is called PDN of the G_{IV}. It is denoted by γ_{PIV}.

Example 5.6.

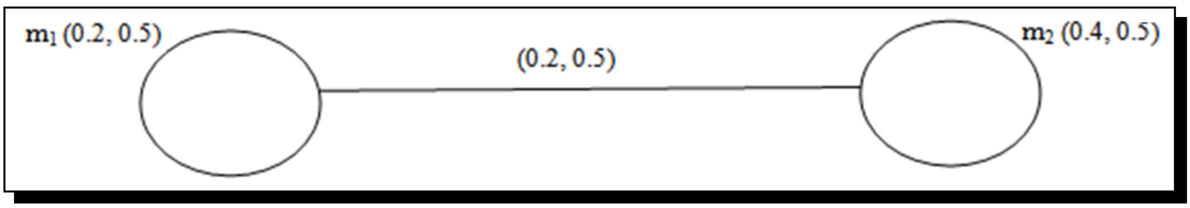

Figure 9. G_{IV}

Figure 9 indicates a $G_{IV} = (V_{IV},E_{IV},\mu_K,\mu_L)$, $\mu_K(m_1) = (0.2,0.5)$, $\mu_K(m_2) = (0.4,0.5)$, $\mu_L(m_1m_2) = (0.2,0.5)$. Figure 9, the dominates edge is $\{m_1,m_2\}$ and the PDSs are $S_{11} = \{m_1\}$, $S_{22} = \{m_2\}$. After calculating the vertex cardinality of S_{11} and S_{22}, we obtain $|S_{11}| = 0.7$, $|S_{22}| = 0.6$. The vertex cardinality of a minimum PDS is $|S_{22}| = 0.6$ and $\gamma_{PIV} = 0.6$.

Definition 5.7. A vertex w_{11} in an $G^1_{IVI} \times G^2_{IVI}$ (or $G^1_{IVI} \Diamond G^2_{IVI}$) incidentally dominates to vertex w_{22} if $\mu^-_M(w_{11}, w_{11}w_{22}) = \min\{\mu^-_K(w_{11}), \mu^-_L(w_{11}, w_{22})\}$ and $\mu^+_M(w_{11}, w_{11}w_{22}) = \max\{\mu^+_K(w_{11}), \mu^+_L(w_{11}, w_{22})\}$. Then (w_{11}, w_{22}) edge is called incidentally dominates edge.

Definition 5.8. A subset W_{IVI} of $V_{IVI}$is said to be a perfect dominating set (PDS) if for each vertex w_{11} not in W_{IVI}, $w_{11}$is incidentally dominates exactly one vertex of W_{IVI}.

Definition 5.9. A PDS W_{IVI} of the $G^1_{IVI} \times G^2_{IVI}$ (or $G^1_{IVI} \Diamond G^2_{IVI}$) is said to be a minimal PDS if each vertex w_{11} in W_{IVI}, $W_{IVI} - \{w_{11}\}$ is not a PDS.

Definition 5.10. A PDS with the lowest vertex cardinality is called a minimum PDS.

Definition 5.11. A vertex cardinality of a minimum PDS is called PDN of the $G^1_{IVI} \times G^2_{IVI}$(or$G^1_{IVI} \Diamond G^2_{IVI}$). It is denoted by$\gamma_{PIVI}$.

Example 5.12. In Figure 5, the incidentally dominates edge are $\{m_1n_1, m_1n_2\}$, $\{m_1n_2, m_2n_2\}$, $\{m_2n_2, m_2n_1\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$, $S_{55} = \{m_1n_1, m_2n_2\}$, $S_{66} = \{m_1n_2, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, S_{22}, \dots S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|S_{66}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

In Figure 8, the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is $|S_{22}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

Theorem 5.13. *If $G^1_{IVI} \times G^2_{IVI}$ be a CP of two IVFIGs without isolated vertices and W_{IVI} is the minimal PDS in $G^1_{IVI} \times G^2_{IVI}$, then $V_{IVI} - W_{IVI}$ is a PDS.*

Proof. Assume W_{IVI} is any minimal PDS of $G^1_{IVI} \times G^2_{IVI}$ and vertex $w_{11} \in W_{IVI}$ is not incidentally dominated by any vertex in $V_{IVI} - W_{IVI}$. Since $G^1_{IVI} \times G^2_{IVI}$ has no isolated vertex, w_{11} must incidentally be dominated by at least one vertex in $W_{IVI} - \{w_{11}\}$, then $W_{IVI} - \{w_{11}\}$ is a PDS, which is a contradiction with the minimality of W_{IVI}. Therefore any vertex in W_{IVI} incidentally dominated by at least one vertex in $V_{IVI} - W_{IVI}$ and so $V_{IVI} - W_{IVI}$ is a PDS. □

Example 5.14. Let $G^1_{IVI} \times G^2_{IVI}$be a CP of two IVFIGs shown in Figure 5 with the incidentally dominates edges are $\{m_1n_1, m_1n_2\}$, $\{m_1n_2, m_2n_2\}$, $\{m_2n_2, m_2n_1\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$, $S_{55} = \{m_1n_1, m_2n_2\}$, $S_{66} = \{m_1n_2, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, S_{22}, \dots S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is S_{66}, then $V_{IVI} - S_{66}$ is also a PDS.

Remark 5.15. The above theorem is also true for TP of two IVFIGs

Example 5.16. Let $G^1_{IVI}\diamondsuit G^2_{IVI}$be a TP of two IVFIGs shown in Figure 8 with the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is S_{22}, then $V_{IVI} - S_{22}$ is also a PDS.

Theorem 5.17. *For a $G^1_{IVI}\diamondsuit G^2_{IVI}$ without isolated vertices, then $\gamma_{PIVI} \le \frac{p}{2}$.*

Proof. If W_{IVI} is a minimal PDS of $G^1_{IVI}\diamondsuit G^2_{IVI}$, then $V_{IVI} - W_{IVI}$ is a PDS. Therefore $p_{IVI} = |V_{IVI}| = |W_{IVI}| + |V_{IVI} - W_{IVI}|$. Thus, at least one of the sets W_{IVI} or $V_{IVI} - W_{IVI}$has the cardinality equal $\frac{p_{IVI}}{2}$ or less. □

Example 5.18. (i) Let $G^1_{IVI}\diamondsuit G^2_{IVI}$ be a TP of two IVFIGs with

$t(\mu^1_K\diamondsuit\mu^2_K)(m_1n_1) = (0.1, 0.4)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_1n_2) = (0.2, 0.4)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_2n_1) = (0.1, 0.5)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_2n_2) = (0.3, 0.5)$,
$(\mu^1_L\diamondsuit\mu^2_L)(m_1n_1, m_2n_2) = (0.1, 0.5)$,
$(\mu^1_L\diamondsuit\mu^2_L)(m_1n_2, m_2n_1) = (0.1, 0.5)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_1n_1, m_1n_1m_2n_2) = (0.1, 0.5)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_2n_2, m_1n_1m_2n_2) = (0.1, 0.5)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_1n_2, m_1n_2m_2n_1) = (0.1, 0.5)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_2n_1, m_1n_2m_2n_1) = (0.1, 0.5)$,

the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is S_{22} with $\gamma_{PIVI} = 1.2$ and vertex cardinality (p_{IVI}) of $G^1_{IVI}\diamondsuit G^2_{IVI}$ is 5.2, then $\gamma_{PIVI} \le \frac{p}{2}$ that is $1.2 < 2.6$

(ii) Let $G^1_{IVI}\diamondsuit G^2_{IVI}$ be a TP of two IVFIGs with

$(\mu^1_K\diamondsuit\mu^2_K)(m_1n_1) = (0.1, 0.5)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_1n_2) = (0.4, 0.7)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_2n_1) = (0.1, 0.4)$,
$(\mu^1_K\diamondsuit\mu^2_K)(m_2n_2) = (0.3, 0.7)$,
$(\mu^1_L\diamondsuit\mu^2_L)(m_1n_1, m_2n_2) = (0.1, 0.7)$,
$(\mu^1_L\diamondsuit\mu^2_L)(m_1n_2, m_2n_1) = (0.1, 0.7)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_1n_1, m_1n_1m_2n_2) = (0.1, 0.7)$,
$(\mu^1_M\diamondsuit\mu^2_M)(m_2n_2, m_1n_1m_2n_2) = (0.1, 0.7)$,

$(\mu_M^1 \diamondsuit \mu_M^2)(m_1n_2, m_1n_2m_2n_1) = (0.1, 0.7),$

$(\mu_M^1 \diamondsuit \mu_M^2)(m_2n_1, m_1n_2m_2n_1) = (0.1, 0.7),$

the incidentally dominates edge are $\{m_1n_1, m_2n_2\}$, $\{m_1n_2, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_1n_2, m_2n_2\}$, $S_{33} = \{m_2n_2, m_2n_1\}$, $S_{44} = \{m_1n_1, m_2n_1\}$. After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.4$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.4$. Here all vertex cardinality of PDS is equal with $\gamma_{PIVI} = 1.4$ and vertex cardinality (p_{IVI}) of $G_{IVI}^1 \diamondsuit G_{IVI}^2$ is 2.8, then $\gamma_{PIVI} = \frac{p}{2}$ that is $1.4 = 1.4$.

Theorem 5.19. *Let $G_{IVI}^1 \times G_{IVI}^2$ be a CP of two IVFIGs and if anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominates edges, then the CP of two IVFIGs containsγ_{PIVI}.*

Proof. Let $G_{IVI}^1 \times G_{IVI}^2$ be a CP of two IVFIGs. If anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominated edges, then the CP of two IVFIG contains γ_{PIVI}.

Conversely, suppose that the CP of two IVFIG containsγ_{PIVI}. To prove that anyone G_{IVI}^1 or G_{IVI}^2 must have incidentally dominates edges. If possible G_{IVI}^1 or G_{IVI}^2 does not have incidentally dominates edges, then $G_{IVI}^1 \times G_{IVI}^2$ dose not havingγ_{PIVI}, which is a contradiction. Hence anyone G_{IVI}^1 or G_{IVI}^2 must having incidentally dominates edges. □

Example 5.20. Let G_{IVI}^1 be a IVFIG with $\mu_K^1(m_1) = (0.4, 0.5)$, $\mu_K^1(m_2) = (0.2, 0.3)$, $\mu_L^1(m_1m_2) = (0.2, 0.5)$, $\mu_M^1(m_1, m_1m_2) = (0.2, 0.5)$, $\mu_M^1(m_2, m_1m_2) = (0.2, 0.5)$ and let G_{IVI}^2 be a IVFIG with $\mu_K^1(n_1) = (0.2, 0.3)$, $\mu_K^1(n_2) = (0.5, 0.6)$, $\mu_L^1(n_1n_2) = (0.2, 0.6)$, $\mu_M^1(n_1, n_1n_2) = (0.1, 0.4)$, $\mu_M^1(n_2, n_1n_2) = (0.1, 0.3)$. Here G_{IVI}^1 having incidentally dominates edge, but G_{IVI}^2 does not have an incidentally dominates edge. Assume $G_{IVI}^1 \times G_{IVI}^2$ is a CP of two IVFIGs with

$(\mu_K^1 \times \mu_K^2)(m_1n_1) = (0.2, 0.5)$

$(\mu_K^1 \times \mu_K^2)(m_1n_2) = (0.4, 0.6),$

$(\mu_K^1 \times \mu_K^2)(m_2n_1) = (0.2, 0.3),$

$(\mu_K^1 \times \mu_K^2)(m_2n_2) = (0.2, 0.6),$

$(\mu_L^1 \times \mu_L^2)(m_1n_1, m_1n_2) = (0.2, 0.6),$

$(\mu_L^1 \times \mu_L^2)(m_1n_1, m_2n_1) = (0.2, 0.5),$

$(\mu_L^1 \times \mu_L^2)(m_1n_2, m_2n_2) = (0.2, 0.6),$

$(\mu_L^1 \times \mu_L^2)(m_2n_1, m_2n_2) = (0.2, 0.6),$

$(\mu_M^1 \times \mu_M^2)(m_1n_1, m_1n_1m_1n_2) = (0.1, 0.5),$

$(\mu_M^1 \times \mu_M^2)(m_1n_2, m_1n_1m_1n_2) = (0.1, 0.5),$

$(\mu_M^1 \times \mu_M^2)(m_1n_2, m_1n_2m_2n_2) = (0.2, 0.6),$

$(\mu_M^1 \times \mu_M^2)(m_2n_2, m_1n_2m_2n_2) = (0.2, 0.6),$

$(\mu_M^1 \times \mu_M^2)(m_2n_1, m_2n_1m_2n_2) = (0.2, 0.6),$

$(\mu_M^1 \times \mu_M^2)(m_2n_2, m_2n_1m_2n_2) = (0.1, 0.3),$

$(\mu_M^1 \times \mu_M^2)(m_1n_1, m_1n_1m_2n_1) = (0.2, 0.5)$,

$(\mu_M^1 \times \mu_M^2)(m_2n_1, m_1n_1m_2n_1) = (0.2, 0.5)$.

Here the incidentally dominates edges are $\{m_1n_2, m_2n_2\}$, $\{m_1n_1, m_2n_1\}$ and the PDSs are $S_{11} = \{m_1n_1, m_1n_2\}$, $S_{22} = \{m_2n_2, m_2n_1\}$, $S_{33} = \{m_1n_1, m_2n_2\}$, $S_{44} = \{m_1n_2, m_2n_1\}$.

After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.3$, $|S_{33}| = 1.4$, $|S_{44}| = 1.2$. The vertex cardinality of a minimum PDS is S_{44} and $\gamma_{PIVI} = 1.2$. Therefore $G^1_{IVI} \times G^2_{IVI}$ containsγ_{PIVI}.

Theorem 5.21. *Let $G^1_{IVI} \diamondsuit G^2_{IVI}$ be a TP of two IVFIGs and if G^1_{IVI} and G^2_{IVI}both having incidentally dominates edges, then the TP of two IVFIGs containsγ_{PIVI}.*

Proof. Let $G^1_{IVI} \diamondsuit G^2_{IVI}$ be a TP of two IVFIGs. If G^1_{IVI} and G^2_{IVI} both having incidentally dominates edges, then the TP of two IVFIGs contains γ_{PIVI}.

Conversely, suppose that the TP of two IVFIGs contains γ_{PIVI}. To prove that G^1_{IVI} and G^2_{IVI} both having incidentally dominates edges. If possible G^1_{IVI} does not having incidentally dominant edges, then the TP of two IVFIGs does not contains γ_{PIVI}, which is a contradiction. Hence G^1_{IVI} and G^2_{IVI} must having incidentally dominated edges. □

Example 5.22. In Figure 5 and 6 is a IVFIGs with incidentally dominated edges and Figure 8 contains PDSs are

$$S_{11} = \{m_1n_1, m_1n_2\},\ S_{22} = \{m_1n_2, m_2n_2\},\ S_{33} = \{m_2n_2, m_2n_1\},\ S_{44} = \{m_1n_1, m_2n_1\}.$$

After calculating the vertex cardinality of $S_{11}, \dots S_{44}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.2$, $|S_{33}| = 1.3$, $|S_{44}| = 1.4$. The vertex cardinality of a minimum PDS is $|S_{22}| = 1.2$ and $\gamma_{PIVI} = 1.2$. Therefore $G^1_{IVI} \diamondsuit G^2_{IVI}$ containsγ_{PIVI}.

6. Application

We incorporate a genuine use of perfect domination number in a matter of education policies among various countries. As an outline case, consider an network $G^1_{IVI} \times G^2_{IVI}$ of four vertices addressing four distinct countries $C_1(m_1n_1)$, $C_2(m_1n_2)$, $C_3(m_2n_2)$ and $C_4(m_2n_1)$ as displayed in Figure 5. The MS value of the vertices shows the percentage of people who are educated and the NMS value of the vertices demonstrates the percentage of those people who are uneducated. The MS value of the edges communicates the cooperation of one country with another country to enhance the percentage of educated people and the NMS value indicates the non cooperation with one another. The MS value of the incidence pair means the education policies among these countries and the NMS value of the incidence pair indicates the un education policies among these countries. With the assistance of the perfect domination number, we will want to discover which country (countries) have the best education policies.

In Figure 5, the PDSs are $S_{11} = \{C_1, C_2\}$, $S_{22} = \{C_2, C_3\}$, $S_{33} = \{C_3, C_4\}$, $S_{44} = \{C_1, C_4\}$, $S_{55} = \{C_1, C_3\}$, $S_{66} = \{C_2, C_4\}$.

After calculating the vertex cardinality of $S_{11}, S_{22}, \dots S_{66}$, we obtain $|S_{11}| = 1.3$, $|S_{22}| = 1.4$, $|S_{33}| = 1.4$, $|S_{44}| = 1.3$, $|S_{55}| = 1.5$, $|S_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|S_{66}| = 1.2$ and $\gamma_{PIVI} = 1.2$.

It is obvious that S_{66} has the minimum PDS between other PDSs, hence we conclude that C_2 and C_4 countries have best education policies among all other countries.

7. Comparative Analysis

In Figure 5 a $G^1_{IVI} \times G^2_{IVI}$ indicating four different countries C_1, C_2, C_3 and C_4 and we get minimum PDS $S_{66} = \{C_2, C_4\}$ with$\gamma_{PIVI} = 1.2$. But in Figure 5 if we remove all the incidence pairs we get IVFG. In the case of IVFG, we find the all PDSs. All possible PDSs of the IVFG are $W_{11} = \{C_1, C_2\}$, $W_{22} = \{C_2, C_3\}$, $W_{33} = \{C_3, C_4\}$, $W_{44} = \{C_1, C_4\}$, $W_{55} = \{C_1, C_3\}$, $W_{66} = \{C_2, C_4\}$ with vertex cardinality $|W_{11}| = 1.3$, $|W_{22}| = 1.4$, $|W_{33}| = 1.4$, $|W_{44}| = 1.3$, $|W_{55}| = 1.5$, $|W_{66}| = 1.2$. The vertex cardinality of a minimum PDS is $|W_{66}| = 1.2$ with $\gamma_{PIV} = 1.2$. By applying the model on the $G^1_{IVI} \diamondsuit G^2_{IVI}$ given in Figure 8, we get minimum PDS $S_{22} = \{C_2, C_3\}$ with$\gamma_{PIVI} = 1.2$. But in figure 8 if we remove all the incidence pairs we get IVFG. In the case of IVFG, we find the all PDSs. All possible PDSs of the IVFG are $M_{11} = \{C_1, C_2\}$, $M_{22} = \{C_2, C_3\}$, $M_{33} = \{C_3, C_4\}$, $M_{44} = \{C_1, C_4\}$ with vertex cardinality $|M_{11}| = 1.3$, $|M_{22}| = 1.2$, $|M_{33}| = 1.3$, $|M_{44}| = 1.3$. The vertex cardinality of a minimum PDS is $|M_{22}| = 1.2$ with $\gamma_{PIV} = 1.2$. Here $G^1_{IVI} \times G^2_{IVI}$ and $G^1_{IVI} \diamondsuit G^2_{IVI}$ both the models $\gamma_{PIV} = \gamma_{PIVI}$, however, on account of IVFG, we can not discuss best education policies because of the non-accessibility of incidence pairs. IVFGs can show the relationship among various countries yet quiet to discuss education policies among various countries. In this way, IVFIGs are more advantageous and compelling IVFGs.

8. Conclusion

In this exploration article, CP and TP in IVFIGs are presented and we inspected the DG of the vertices of the IVFIGs $G^1_{IVI} \times G^2_{IVI}$and $G^1_{IVI} \diamondsuit G^2_{IVI}$ under specific agreements and showed them with different models. We additionally settled some new outcomes on the DG of a vertex as far as hypotheses. The idea of perfect domination in IVFIGs utilizing incidence pairs is additionally considered. The perfect domination number of IVFIGs is determined. It is also possible to use PDN in the context of education policies in different countries. We plan to expand our research into Vague FIGs, Hamiltonian FIGs, and Intuitionistic FIGs in the future. In the near future, more work on these ideas will be presented in articles.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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