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Research Article

On Two Classes of Exponential Diophantine Equations

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Abstract. In this paper, we study on the exponential Diophantine equations: $n^x + 24^y = z^2$, for $n \equiv 5$ or $7 \pmod 8$. We show that $5^x + 24^y = z^2$ has a unique positive integral solution $(2,1,7)$. Further, we show that for $k \in \mathbb{N}$, $(8k+5)^x + 24^y = z^2$ has a unique solution $(0,1,5)$ in non-negative integers. We also show that for a perfect square $8m$, the exponential Diophantine equation $(8m-1)^x + 24^y = z^2$, $m \in \mathbb{N}$ has exactly two non-negative integral solutions $(0,1,5)$ and $(1,0,\sqrt{8m})$. Otherwise, it has a unique solution $(0,1,5)$. Finally, we illustrate our results with some examples and non-examples.

Keywords. Catalan's conjecture, Exponential Diophantine equations, Integer solutions

Mathematics Subject Classification (2020). 11D61, 11D72

1. Introduction

Number theory is one of the oldest field of mathematics. The theory is mainly concerned with the properties of integers. As famously said by G.H. Hardy, it is regarded as the purest branch of pure mathematics. The Diophantine equation is one of the antique branch of Number theory, and its origin can be found in the texts of the ancient Babylonians, Chinese, Indians, Egyptians, and Greeks. The term "Diophantus" comes from Alexander's Hellenistic Mathematician "Diophantus" the father of Algebra, sometimes around 250 AD. Diophantus's work greatly influenced later mathematics. The Diophantine equation can be used in various fields, and there is no universal method to determine whether a given Diophantine equation has a solution. One of the most surprising features of the Diophantine equation is that one can create simpler or more difficult equations by slightly modifying the original equation [7, 19].

Diophantine equations can be linear, or nonlinear. It is widely believed that the Indian mathematician Brahmagupta was the first person to describe linear equations. Individual Diophantine problems have been studied by great mathematicians such as Fermat, Euler, and Gauss. So the general solution is very complicated. The Euclidean algorithm is one of the methods to solve the Diophantine linear equation [7].

Researchers are keen to find solutions to various Diophantine equations because these equations have countless applications in Algebraic topology, Random number generations, Security, Coordinate geometry, Trigonometry, Physics, Chemistry, Memory management, Authentication, Computer science, Applied algebra, and Cryptography etc.[7, 19].

In Number theory, there are various types of Diophantine equations like the polynomial Diophantine equation, Pell's equation, Exponential Diophantine equation, Infinite Diophantine equation, etc. If the Diophantine equation has one or more additional variables as exponents, it is a Diophantine exponent equation similar to the equations of the Fermat-Catalan conjecture and Beal's conjecture, $a^m + b^n = c^k$ with inequality restrictions on the exponents. The general theory of this type of equation is not available; special cases such as Catalan's conjecture have been resolved [20].

The exponential Diophantine equation is an equation in which unknowns can appear in the exponents. The Diophantine problem has fewer equations than unknowns and involves finding integers that solve all equations simultaneously. Many authors have studied the exponential Diophantine equation. In 1844, the great Mathematician, Eugene Charles Catalan created a conjecture that the diophantine equation $a^x - b^y = 1$ where $a, b, x, y \in \mathbb{Z}$ with $\min\{a, b, x, y\} > 1$ has a unique solution $(a, b, x, y) = (3, 2, 2, 3)$ [10]. Le [21], Cohn [13], Terai [31], Cassels [9], Arif and Muriefah [3] etc. have done their research works on the Diophantine equations like $x^2 + b^y = c^z$, $x^2 + c = y^n$, $x^4 - Dy^2 = 1$, $a^x + b^y = c^z$, $x^2 + 2^k = y^n$ etc. in the period of 1993-1997. Famous researchers, Sury [28], Luca [22], Cao [8], Beukers [4] and many others have done considerable works (approx 1995-2001) on various aspects of the Diophantine equations $x^2+2 = y^n$, $x^2+3^m = y^n$, $a^x+b^y = c^z$, $Ax^p+By^q = Cz^r$ etc. The Catalan conjecture was eventually proved by the mathematician P. Mihăilescu in 2002 [23]. In 2002, Arif and Muriefah [3] have done works on the Diophantine equation $x^2 + q^{2k+1} = y^n$. Yuan and Hu [33], and Terai [31], etc. have done extensive research works on Diophantine equations.

In 2007, Acu [1] proved that the Diophantine equation $2^x + 5^y = z^2$, $x, y, z \in \mathbb{Z}^+$ has only two solutions, viz. $(3, 0, 3)$ and $(2, 1, 3)$. A number of researchers have studied the exponential Diophantine equations in the period 2010-2016 AD. Suvarnamani [29], Suvarnamani *et al*. [30], Sroysang [27], Kishan *et al*. [18], Bravo and Luca [5] etc. have done their extensive works on the different types of Diophantine equations $4^x + 7^y = z^2$, $4^x + 11^y = z^2$, $4^x + 13^y = z^2$, $4^x + 17^y = z^2$, $A^x + B^y = C^z$, $3^x + 5^y = z^2$, $8^x + 19^y = z^2$, $31^x + 32^y = z^2$, $7^x + 8^y = z^2$, $F_n + F_m = 2^a$ etc. There are many other non-linear Diophantine equations. In [24], Rabago discussed the Diophantine equations $3^x+19^y = z^2$, $3^x+91^y = z^2$. Moreover, recently, Burshtein [6], Aggarwal [2], and Kumar *et al*.[20] etc. have done works relating to Diophantine equation. The exponential Diophantine equation $(a^n-1)(b^n-1)=x^2$, $x,n\in\mathbb{N}$ has been studied by a lot of researchers since 2000. After that many authors such as Hajdu and Szalay [15], Ishii [16], Cohn [13], Luca [22], Walsh [32], and Keskin [17], etc. have studied the Diophantine equations. Somanath *et al.* [25, 26] have done their works on the Diophantine equations $x^2=29y^2-7^t$, $t\in\mathbb{N}$, $x^2=9y^2+11z^2$, $\alpha^2-90\beta^2-10\alpha-1260\beta=4401$ in 2020. In [14], Gayo and Bacan, studied and solved the exponential Diophantine equation of the form $M_p^x+(M_q+1)^y=z^2$ for Mersenne primes M_p and M_q and non-negative integers x,y, and z. Chakraborty *et al.* [11] investigated the Diophantine equation $cx^2+p^{2m}=4y^n$.

2. Preliminaries

2.1 The Catalan's Conjecture ([23]). *The unique solution for the Diophantine equation* $a^x-b^y=1$ *where* $a,b,x,y\in\mathbb{Z}$ *with* $\min\{a,b,x,y\}>1$ *is* $(3,2,2,3)$.

2.1 Lemma. *The exponential Diophantine equation* $5^x+1=z^2$ *has no solutions in non-negative integers.*

Proof. $z^2=1+5^x\geq 2\Rightarrow z\geq 2\Rightarrow z^2\geq 4\Rightarrow 5^x=2^2-1\geq 3\Rightarrow x\geq 1$.
Therefore, by Catalan Conjecture, we must have $x=1$. But $z^2=1+5=6$ has no solution in non-negative integers.
Therefore, the given equation has no solution in non-negative integers either. □

2.2 Lemma. *The exponential Diophantine equation* $7^x+1=z^2$ *has no solutions in* $\mathbb{Z}\geq 0$.

Proof. $z^2=1+7^x\geq 2\Rightarrow z\geq 2\Rightarrow z^2\geq 4\Rightarrow 7^x=2^2-1\geq 3\Rightarrow x\geq 1$.
Therefore, by Catalan Conjecture, we must have $x=1$. But $z^2=1+7=8$ is not a perfect square.
Hence, $7^x+1=z^2$ has no solution in non-negative integers. □

We now come to our main result. In the following, we will study all the possible solutions and we will use the Catalan's conjecture and above Lemma's in solving the exponential Diophantine equations of the type $(8m-3)^x+24^y=z^2$ and $(8m-1)^x+24^y=z^2$, $m\in\mathbb{N}$.

3. Main Results

3.1 Theorem. *The exponential Diophantine equation* $5^x+24^y=z^2$ *has exactly two solutions* $(0,1,5)$ *and* $(2,1,7)$ *in non-negative integers.*

Proof. The equation is

$$5^x+24^y=z^2. \tag{3.1}$$

If $y=0$, from Lemma 2.1, we get that equation (3.1) has no solution in non-negative integers.
Therefore, $y\neq 0$ for equation (3.1), to have any solution.
Thus z is odd from equation (3.1). Taking $\bmod 8$ of equation (3.1), we get

$$(-3)^x\equiv 1 \pmod 8$$

$\Rightarrow x$ is even.

Let $x = 2k, k \geq 0$ is an integer.
Therefore, equation (3.1)
$\Rightarrow 5^{2k} + 24^y = z^2 \Rightarrow 24^y = (z - 5^k)(z + 5^k)$

Two cases arise:

Case 1. $z - 5^k = 2^y, z + 5^k = 12^y$
$\Rightarrow 2.5^k = 2^y(6^y - 1) \Rightarrow y = 1, 5^k = 5 \Rightarrow k = 1 \Rightarrow x = 2 \Rightarrow z^2 = 5^2 + 24 = 7^2 \Rightarrow z = 7$
Therefore $(x, y, z) = (2, 1, 7)$.

Case 2. $z - 5^k = 4^y$, $z + 5^k = 6^y$
$\Rightarrow 2.5^k = 2^y(3^y - 2^y) \Rightarrow y = 1, 5^k = 3 - 2 = 1 \Rightarrow k = 0 \Rightarrow x = 0 \Rightarrow z = 4^1 + 5^0 = 5$
Therefore $(x, y, z) = (0, 1, 5)$.
This completes the proof. □

We now present our second theorem, which extends the above theorem.

3.2 Theorem. *The exponential Diophantine equation* $(8k+5)^x + 24^y = z^2$*,* $k \in \mathbb{N}$ *which has a unique solution* $(0, 1, 5)$ *in non-negative integers.*

Proof. Given equation is

$$(8k+5)^x + 24^y = z^2, \quad k \in \mathbb{N} \tag{3.2}$$

If $y = 0$, eq. (3.2) $\Rightarrow z^2 - (8k+5)^x = 1$.
Therefore, by Catalan conjecture, $\min\{z, x, 8k+5\} \leq 1$.
Because eq. (3.2) $\Rightarrow z \geq 2$ and $k \in \mathbb{N} \Rightarrow 8k+5 \geq 13$.
Therefore, we must have $x = 0$ or 1.
$x = 0 \Rightarrow z^2 = 2 \Rightarrow$ no integral solution and $x = 1 \Rightarrow z^2 = 8k+6 \equiv 6 \pmod 8$.
But, any square is either congruent to $0, 1$ or $4 \pmod 8 \Rightarrow$ no integral solution.
Therefore, $y \neq 0$, and z is odd, eq. (3.2) $\Rightarrow 5^x \equiv z^2 \equiv 1 \pmod 8 \Rightarrow x$ is even.
Let $x = 2l$, $l \geq 0$.
Therefore, eq. (3.2) $\Rightarrow 24^y = z^2 - (8k+5)^{2l} = (z - (8k+5)^l)(z + (8k+5)^l)$.

Two cases arise:

Case 1. $z - (8k+5)^l = 2^y, z + (8k+5)^l = 12^y$.
$\Rightarrow 2.(8k+5)^l = 2^y(6^y - 1)$
$\Rightarrow y = 1, (8k+5)^l = 5$
But, $k \in \mathbb{N} \Rightarrow 8k+5 > 5 \Rightarrow (8k+5)^l \neq 5$.
Therefore, no solution exists in this case.

Case 2. $z - (8k+5)^l = 4^y, z + (8k+5)^l = 6^y$
$\Rightarrow 2.(8k+5)^l = 2^y(3^y - 2^y)$
$\Rightarrow y = 1$, $(8k+5)^l = 3^1 - 2^1 = 1$
$\Rightarrow l = 0$ for any $k \in \mathbb{N} \Rightarrow x = 0$ and $z = 4 + 1 = 5$

i.e. $(0,1,5)$ is a solution of eq. (3.2), $\forall\ k \in \mathbb{N}$.
This completes the theorem. □

3.1 Example. The exponential Diophantine equation $13^x + 24^y = z^2$, $21^x + 24^y = z^2$, $101^x + 24^y = z^2$ has a unique solution $(0,1,5)$ in non-negative integers.

3.1 Corollary. *The equation* $(8k-3)^x + 24^y = w^4$, $k \in \mathbb{N}$ *has no solution in non-negative integers.*

Proof. If (x,y,w) is a solution then by Theorem 3.1 and Theorem 3.2, we must have

$z = w^2 = 5$ or 7 which is a contradiction. □

3.2 Example. The exponential Diophantine equation $5^x + 24^y = w^4$, $13^x + 24^y = w^4$, $101^x + 24^y = w^4$ have a no solution in non-negative integers.

3.2 Corollary. *The equation* $(8k-3)^x + 24^y = w^{2m}$, $k \in \mathbb{N}$, $1 < m \in \mathbb{N}$ *has no solution in non-negative integers.*

Proof. If (x,y,w) were a solution then by Theorem 3.1 and Theorem 3.2, we must have $z = w^m = 5$ or 7. Since $m > 1$.
This is not possible. Hence no solution exists. □

3.3 Example. The exponential Diophantine equation $5^x + 24^y = w^6$, $21^x + 24^y = w^8$, $37^x + 24^y = w^{10}$ have a no solution in non-negative integers.

Now, we come to another class of equations.

3.3 Theorem. *The exponential Diophantine equation* $7^x + 24^y = z^2$ *has a unique solution* $(0,1,5)$ *in non-negative integers.*

Proof. $y = 0 \Rightarrow 7^x + 1 = z^2$, and by Lemma 2.2 this has no solution in non-negative integers.
Therefore, $y \neq 0$.
Thus, z is odd. Now, given equation is

$$7^x + 24^y = z^2. \tag{3.3}$$

Taking '(mod 8)' from eq. (3.3), we get $(-1)^x \equiv 1 \pmod 8 \Rightarrow x$ is even.
Let $x = 2u$, $u \geq 0$, $u \in \mathbb{Z}$.
Equation (3.3) $\Rightarrow 7^{2u} + 24^y = z^2 \Rightarrow 24^y = (z - 7^u)(z + 7^u)$.

Two cases arise:

Case 1. $z - 7^u = 2^y$, $z + 7^u = 12^y$
$\Rightarrow 2.7^u = 2^y(6^y - 1)$
$\Rightarrow y = 1, 7^u = 5$
No solutions in this case.

Case 2. $z - 7^u = 4^y$, $z + 7^u = 6^y$
$\Rightarrow 2.7^u = 2^y(3^y - 2^y)$

$\Rightarrow y = 1, 7^u = 3^1 - 2^1 = 1$
$\Rightarrow u = 0$
$\Rightarrow x = 0$
$\Rightarrow z = 4^1 + 7^0 = 5$
Therefore, $(x, y, z) = (0, 1, 5)$ is a solution of eq. (3.3).
This completes the proof. □

Below, we give a generalised version of the above theorem.

3.4 Theorem. *The exponential Diophantine equation* $(8m-1)^x + 24^y = z^2$, $m \in \mathbb{N}$ *has*

(1) *Exactly two solutions if '8m' is a perfect square, the solutions are given by* $(0,1,5)$ *and* $(1,0,\sqrt{8m})$.

(2) *A unique solution* $(0,1,5)$ *otherwise.*

Proof. The equation is

$$(8m-1)^x + 24^y = z^2, \quad m \in \mathbb{N} \tag{3.4}$$

$y = 0 \Rightarrow z^2 - (8m-1)^x = 1$, $z > 1$, $m \in \mathbb{N} \Rightarrow x = 0, 1$, by Catalan conjecture.
$x = 0 \Rightarrow z^2 = 2 \Rightarrow$ no solution exists and $x = 1 \Rightarrow z^2 = 8m$.
Thus, if and only if '$8m$' is a perfect square, $(x, y, z) = (1, 0, \sqrt{8m})$ is a solution.
Otherwise, $y \neq 0$, this implies z is odd. Equation (3.4) $\Rightarrow (-1)^x \equiv 1 (\text{mod } 8) \Rightarrow x$ is even. Let $x = 2v$, $v \geq 0$.
Therefore, eq. (3.4) $\Rightarrow (8m-1)^{2v} + 24^y = z^2$, $m \in \mathbb{N} \Rightarrow 24^y = (z - (8m-1)^v)(z + (8m-1)^v)$.

Two cases arise:

Case 1. $z - (8m-1)^v = 2^y$, $z + (8m-1)^v = 12^y \Rightarrow 2.(8m-1)^v = 2^y(6^y - 1) \Rightarrow y = 1$, $(8m-1)^v = 5$ $\Rightarrow$ no solution exists.

Case 2. $z - (8m-1)^v = 4^y, z + (8m-1)^v = 6^y \Rightarrow 2.(8m-1)^v = 2^y(3^y - 2^y) \Rightarrow y = 1$, $(8m-1)^v = 1 \Rightarrow$ $v = 0 \Rightarrow x = 0 \Rightarrow z = 4^1 + 1 = 5$
i.e., $(0, 1, 5)$ is a solution of eq. (3.4).
This completes the proof. □

We conclude with some examples and corollaries as follows.

3.4 Example. (1) The exponential Diophantine equation $23^x + 24^y = z^2$, $31^x + 24^y = z^2$, $55^x + 24^y = z^2$ has a unique solution $(0,1,5)$ in non-negative integers.

(2) The exponential Diophantine equation $15^x + 24^y = z^2$, has exactly two solutions in non-negative integers: $(0,1,5)$ and $(1,0,4)$.

(3) The exponential Diophantine equation $63^x + 24^y = z^2$, has exactly two solutions in $\mathbb{Z} \geq 0$: $(0,1,5)$ and $(1,0,8)$.

3.3 Corollary. *The equation* $(8m-1)^x + 24^y = w^4$ *has no solution in positive integers.*

Proof. If (x,y,w) were a solution then by theorem above $x=0$ or $y=0$, which is a contradiction. □

3.5 Example. The exponential Diophantine equation $47^x+24^y=w^4$, $71^x+24^y=w^4$, has no solution in positive integers.

3.4 Corollary. *The equation* $(8m-1)^x+24^y=w^{2n}$, $n\in\mathbb{N}$ *has no solution in positive integers.*

Proof. If (x,y,w) were a solution, then $(x,y,z=w^n)$ is a solution of eq. (3.4).
This means $x=0$ or $y=0$, by Theorem 3.4, which is a contradiction. □

3.6 Example. The exponential Diophantine equation $79^x+24^y=w^6$, $87^x+24^y=w^8$, $95^x+24^y=w^{10}$ has no solution in positive integers.

4. Conclusion

Many linear and non-linear Diophantine equations with a finite or an infinite number of variables can be solved. In this article, we have shown that the exponential Diophantine equation $(8k+5)^x+24^y=z^2$ has a unique solution $(0,1,5)$ in non-negative integers. We also showed that for $k=0$, this has exactly two solutions: $(0,1,5)$ and $(2,1,7)$. Finally, we showed that the exponential Diophantine equation $(8m-1)^x+24^y=z^2$, $m\in\mathbb{N}$ has exactly two solutions if "$8m$" is a perfect square, the solutions being given by $(0,1,5)$ and $(1,0,\sqrt{8m})$; and if $8m$ is not a square,it has a unique solution $(0,1,5)$.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] D. Acu, On a Diophantine equation, *General Mathematics* **15** (2007), 145 – 148, URL: https://www.emis.de/journals/GM/vol15nr4/acu/acu.pdf.

[2] S. Aggarwal, On the exponential Diophantine equation $(2^{2m+1}-1)+(13)^n=z^2$, *Engineering and Applied Science Letter* **4**(1) (2021), 77 – 79, DOI: 10.30538/psrp-eas12021.0064.

[3] S.A. Arif and F.S.A. Muriefah, On the Diophantine equation $x^2+q^{2k+1}=y^n$, *Journal of Number Theory* **95** (2002), 95 – 100, DOI: 10.1006/jnth.2001.2750.

[4] F. Beukers, The Diophantine equation $Ax^p+By^q+Cz^r$, *Duke Mathematical Journal* **91**(1) (1998), 61 – 88, DOI: 10.1215/S0012-7094-98-09105-0.

[5] J.J. Bravo and F. Luca, On the Diophantine equation $F_n+F_m=2^a$, *Quaestiones Mathematiae* **39**(3) (2016), 391 – 400, DOI: 10.2989/16073606.2015.1070377.

[6] N. Burshtein, On the Diophantine equation $2^{2x+1}+7^y=z^2$, *Annals of Pure and Applied Mathematics* **16**(1) (2018), 177 – 179, DOI: 10.22457/apam.v16n1a19.

[7] D.M. Burton, *Elementary Number Theory*, 6th edition, McGraw-Hill Education (2007), URL: https://books.google.co.in/books?hl=en&lr=&id=XMQjuoTqqRMC&oi=fnd&pg=PR9&dq=D.+M.+Burton.

[8] Z. Cao, A note on the Diophantine equation $a^x+n^y=c^z$, *Acta Arthmetica* **XCI**(1) (1999), 85 – 93, URL: http://matwbn.icm.edu.pl/ksiazki/aa/aa91/aa9115.pdf.

[9] J. Cassels, A Diophantine equation, *Glasgow Mathematical Journal* **27** (1985), 11 – 18, DOI: 10.1017/S0017089500006030.

[10] E. Catalan, Note extraite d'une lettre adressée à l'éditeur par Mr. E. Catalan, Répétiteur à l'école polytechnique de Paris, *Journal für die reine und angewandte Mathematik* **1844**(27) (1844), p. 192, DOI: 10.1515/crll.1844.27.192.

[11] K. Chakraborty, A. Hoque and K. Srinivas, On the Diophantine equation $cx^2+p^{2m}=4y^n$, *Results in Mathematics* **76** (2021), Article number: 57, 10.1007/s00025-021-01366-w.

[12] J.H.E. Cohn, On the Diophantine system $x^2-6y^2=-5$, $x=2z^2-1$, *Mathematica Scandinavica* **82**(2) (1998), 161 – 164, DOI: 10.7146/math.scand.a-13830.

[13] J.H.E. Cohn, The Diophantine equation $x^2+C=y^n$, *Acta Arithmetica* **LXV**(4) (1993), 387 – 381, URL: http://matwbn.icm.edu.pl/ksiazki/aa/aa65/aa6546.pdf.

[14] W.S. Gayo and J.B. Bacan, On the Diophantine equation $M_p^x+(M_q+1)^y=z^2$, *European Journal of Pure and Applied Mathematics* **14**(2) (2021), 396 – 403, DOI: 10.29020/nybg.ejpam.v14i2.3948.

[15] L. Hajdu and L. Szalay, On the Diophantine equations $(2^n-1)(6^n-1)=x^2$ and $(a^n-1)(a^{kn}-1)=x^2$, *Periodica Mathematica Hungarica* **40** (2000), 141 – 145, DOI: 10.1023/A:1010335509489.

[16] K. Ishii, On the exponential Diophantine equation $(a^x-1)(b^n-1)=x^2$, *Publicationes Mathematicae Debrecen* **89**(1-2) (2016), 253 – 256, URL: https://publi.math.unideb.hu/load_jpg.php?p=2098.

[17] R. Keskin, A note on exponential Diophantine equation $(a^n-1)(b^n-1)=x^2$, *Proceedings - Mathematical Sciences* **129** (2019), Article number 69, DOI: 10.1007/s12044-019-0520-x.

[18] H. Kishan, M. Ravi and S. Sarita, On the Diophantine equation, *Journal of Advances in Mathematics* **11**(9) (2016), DOI: 10.24297/jam.v11i9.815.

[19] J. Klaska, Real-world applications of Number theory, *South Bohemia Mathematical Letters* **25**(1) (2017), 39 – 47, URL: http://home.pf.jcu.cz/~sbml/wp-content/uploads/Klaska.pdf.

[20] S. Kumar, S. Gupta and H. Kishan, On the Non-linear Diophantine equation $61^x+67^y=z^2$ and $67^x+73^y=z^2$, *Annals of Pure and Applied Mathematics* **18**(1) (2018), 91 – 94, DOI: 10.22457/apam.v18n1a13.

[21] M. Le (Zhanjiang), A note on the Diophantine equation $x^2+b^y=c^z$, *Acta Arithmetica* **LXXI**(3) (1995), 253 – 257, URL: http://matwbn.icm.edu.pl/ksiazki/aa/aa71/aa7134.pdf.

[22] F. Luca, On a Diophantine equation, *Bulletin of the Australian Mathematical Society* **61** (2000), 241 – 246, DOI: 10.1017/S0004972700022231.

[23] P. Mihăilescu, Primary cycolotomic units and a proof of Catalan's conjecture, *Journal für die reine und angewandte Mathematik* **27** (2004), 167 – 195, DOI: 10.1515/crll.2004.048.

[24] J.F.T. Rabago, On two Diophantine equations $3^x+19^y=z^2$ and $3^x+91^y=z^2$, *International Journal of Mathematics and Scientific Computing* **3**(1) (2013), 28 – 29, URL: https://www.veltech.edu.in/wp-content/uploads/2016/04/Paper-08-13.pdf.

[25] M. Somanath, K. Raja, J. Kannan and A. Akila, Integral solutions of an infinite elliptic cone $x^2 = 9y^2 + 11z^2$, *Advances and Applications in Mathematical Sciences* **19**(11) (2020), 1119 – 1124.

[26] M. Somanath, K. Raja, J. Kannan and S. Nivetha, Exponential Diophantine equation in three unknowns, *Advances and Applications in Mathematical Sciences* **19**(11) (2020), 1113 – 1118.

[27] B. Sroysang, On the Diophantine equation $7^x + 8^y = z^2$, *International Journal of Pure and Applied Mathematics* **84**(1) (2013), 111 – 114, DOI: 10.12732/ijpam.v84i1.8.

[28] B. Sury, On the Diophantine equation $x^2 + 2 = y^n$, *Archiv der Mathemctik* **74** (2000), 350 – 355, DOI: 10.1007/s000130050454.

[29] A. Suvarnamani, Solution of the Diophantine equation $p^x + q^y = z^2$, *International Journal of Pure and Applied Mathematics* **94**(4) (2014), 457 – 460, DOI: 10.12732/ijpam.v94i4.1.

[30] A. Suvarnamani, A. Singta and S. Chotchaisthit, On two Diophantine equations $4^x + y^y = z^2$ and $4^x + 11^y = z^2$, *Science and Technology RMUTT Journal* **1**(1) (2011), 25 – 28.

[31] N. Terai, The Diophantine equation $a^x + b^y = c^z$, *Proceedings of the Japan Academy, Series A, Mathematical Sciences* **70**(1) (1994), 22 – 26, DOI: 10.3792/pjaa.70.22.

[32] P.G. Walsh, On Diophantine equation of the form, *Tatra Mountains Mathematical Publications* **20**(1) (2000), 87 – 89.

[33] P. Yuan and Y. Hu, On the Diophantine equation $x^2 + D^m = p^n$, *Journal of Number Theory* **111**(1) (2005), 144 – 153, DOI: 10.1016/j.jnt.2004.11.005.