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Research Article

Some Extractions of Fixed Point Theorems Using Various E.A Properties

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Abstract. This paper deals with some fixed point theorems using the notions of *E.A property*, *common E.A property*, *E.A like property* and *common E.A like property* along with *weakly compatible mappings*. Additionally, some examples are also discussed to describe the various E.A properties.

Keywords. Common fixed point; Weakly compatible mappings; E.A property; Common E.A property; E.A like property; Common E.A like property

Mathematics Subject Classification (2020). 47H10; 54H25

1. Introduction

It is well known that the *Fixed Point Theory* is an efficient tool in the branch of mathematics and it has many applications across the different fields of mathematics. The idea of weakly commuting mappings came to existence with the effort of Sessa [12]. Jungck [5] introduced the concept of compatible mappings, which is more general than commuting and weakly commuting maps. Further, Jungck [6] weakened the notion of compatibility by introducing the weakly compatible mappings.

The study of non-compatible mappings of common fixed points in metric space is of great importance and was initiated by Pant [9]. In 2002, Aamri and Moutaakil [1] introduced new

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property for pair of self mappings known to be property E.A, which is a weaker form of the notion of compatible mappings in metric space. Further, Yicheng *et al*. [16] improved property E.A to common property E.A. The concept of E.A like property and common E.A like property in fuzzy metric spaces is investigated [15]. Further different generalizations of fixed point theorems have been noted from [7] to [13].

The aim of this paper is to generalize Jang *et al*. [4] result under different E.A properties namely property E.A, common property E.A, E.A like property and common E.A like property along with weakly compatible mappings.

2. Definitions and Examples

Definition 2.1 ([5])**.** Self mappings $\mathcal{A}$ and $\mathcal{S}$ of a metric space (X,d) are known to be compatible if $\lim_{n\to\infty} d(\mathcal{A}\mathcal{S}\alpha_\eta, \mathcal{S}\mathcal{A}\alpha_\eta) = 0$ holds whenever $\{\alpha_\eta\}$ is emerging as a sequence in X satisfying $\lim_{\eta\to\infty} \mathcal{A}\alpha_\eta = \lim_{n\to\infty} \mathcal{S}\alpha_\eta = \xi$ for some $\xi \in X$.

Definition 2.2 ([6])**.** Two mappings $\mathcal{A}$ and $\mathcal{S}$ of a metric space (X,d) are said to be weakly compatible if they commute at their coincidence points i.e. if $\mathcal{A}u = \mathcal{S}u$ implies $\mathcal{A}\mathcal{S}u = \mathcal{S}\mathcal{A}u$ for some $u \in X$.

Definition 2.3 ([1])**.** Self mappings $\mathcal{A}$ and $\mathcal{S}$ of metric (X,d) is said to satisfy the property (E.A) if there exists a sequence $\{\alpha_\eta\}$ in X such that $\lim_{\eta\to\infty} \mathcal{A}\alpha_\eta = \lim_{\eta\to\infty} \mathcal{S}\alpha_\eta = \xi$ for some $\xi \in X$.

Example 2.1. Let the usual metric space (X,d) with $X = [0,10]$.
Now, the mappings $\mathcal{A},\mathcal{S} : X \times X \to \mathbb{R}$ defined as $\mathcal{A} = \frac{2x}{7}$ and $\mathcal{S} = \frac{x}{7}$ for all $x \in X$. Let the sequence $\{\alpha_\eta\} = \frac{2}{\eta}$, $\eta \in \mathbb{N}$, then the pair $(\mathcal{A},\mathcal{S})$ satisfies property E.A.

Definition 2.4 ([16])**.** The pair of mappings $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ of a metric space (X,d) are said to hold the common property (E.A) whenever $\{\alpha_\eta\}$ and $\{\beta_\eta\}$ are the sequences in X satisfying $\lim_{\eta\to\infty} \mathcal{A}\alpha_\eta = \lim_{\eta\to\infty} S\alpha_\eta = \lim_{\eta\to\infty} T\beta_\eta = \lim_{\eta\to\infty} \mathcal{B}\beta_\eta = \xi$ for some point $\xi \in X$.

Now, we discuss example on this property.

Example 2.2. Let the usual metric space (X,d) with $X = [-10,10]$ and $d(x,y) = |x-y|$. Define the mappings on (X,d) as

$$\mathcal{A}(x) = \begin{cases} \frac{1}{5} & \text{for } x \in \{-10,10\}, \\ \frac{x}{2} & \text{for } x \in (-10,10), \end{cases} \qquad \mathcal{S}(x) = \begin{cases} \frac{1}{4} & \text{for } x = -10, \\ x & \text{for } x \in (-10,10), \\ -\frac{1}{4} & \text{for } x = 10, \end{cases}$$

$$\mathcal{B}(x) = \begin{cases} \frac{1}{5} & \text{for } x \in \{-10,10\}, \\ -\frac{x}{2} & \text{for } x \in (-10,10), \end{cases} \qquad \mathcal{T}(x) = \begin{cases} -\frac{1}{4} & \text{for } x = -10, \\ -x & \text{for } x \in (-10,10), \\ \frac{1}{4} & \text{for } x = 10. \end{cases}$$

Eventually, the pair of mappings $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy the common E.A property for the sequences $\{\alpha_\eta\} = \left\{0 + \frac{1}{\eta}\right\}$ and $\{\beta_\eta\} = \left\{0 - \frac{1}{\eta}\right\}$, where $\eta \in \mathbb{N}$.

Definition 2.5 ([15])**.** Let $\mathcal{A}$ and $\mathcal{S}$ be mappings of a metric space (X,d). A pair of mappings $(\mathcal{A},\mathcal{S})$ said to hold the property E.A like property if there exists a sequence $\{\alpha_\eta\}$ such that $\lim_{\eta\to\infty}\mathcal{A}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{S}\alpha_\eta=\xi$ for some $\xi\in\mathcal{A}(X)$ or $\xi\in\mathcal{S}(X)$ i.e. $\xi\in\mathcal{A}(X)\cup\mathcal{S}(X)$.

Example 2.3. Let (X,d) be a usual metric space with $X=[-1,1]$. Define the mappings $\mathcal{A}$ and $\mathcal{S}$ as $\mathcal{A}(x)=\frac{x}{2}$, $\mathcal{S}(x)=0$ for all $x\in X$. For a sequence $\{\alpha_\eta\}=\frac{1}{\eta}$, $\eta\in\mathbb{N}$, the pair of mappings $(\mathcal{A},\mathcal{S})$ satisfies E.A like property, since $\lim_{\eta\to\infty}\mathcal{A}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{S}\alpha_\eta=0$ where $0\in\mathcal{A}(X)\cup\mathcal{S}(X)$.

Definition 2.6 ([15])**.** Two pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ of four self mappings of metric space (X,d) are said to satisfy common E.A like property if $\lim_{\eta\to\infty}\mathcal{A}\alpha_n=\lim_{\eta\to\infty}\mathcal{S}\alpha_n=\lim_{\eta\to\infty}\mathcal{T}\beta_\eta=\lim_{n\to\infty}\mathcal{B}\beta_\eta=\xi$ whenever $\exists$ two sequences $\{\alpha_\eta\}$ and $\{\beta_\eta\}$ in X such that $\xi\in\mathcal{S}(X)\cap\mathcal{T}(X)$ or $\xi\in\mathcal{A}(X)\cap\mathcal{B}(X)$.

Now discuss an example.

Example 2.4. Consider $X=[0,1]$ and $d(x,y)=|x-y|$ for all $x\in X$. Define the mappings $\mathcal{A}$, $\mathcal{S}$, $\mathcal{B}$ and $\mathcal{T}$ as $\mathcal{A}(x)=\frac{x}{2}-\frac{1}{8}\mathcal{B}(x)=\frac{x}{4}\mathcal{S}(x)=x-\frac{1}{4}\mathcal{T}(x)=x$.
Define two sequences as $\{\alpha_\eta\}=\frac{1}{4}+\frac{1}{\eta}$ and $\{\beta_\eta\}=\frac{1}{\eta}$, $\eta\in\mathbb{N}$. $\mathcal{A}(X)=[-\frac{1}{8},\frac{3}{8}]$, $\mathcal{B}(X)=[0,\frac{1}{4}]$, $\mathcal{S}(X)=[-\frac{1}{4},\frac{3}{4}]$, $\mathcal{T}(X)=[0,1]$.
Now, $\lim_{\eta\to\infty}\mathcal{A}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{S}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{B}\beta_\eta=\lim_{\eta\to\infty}\mathcal{T}\beta_\eta=0$ where $0\in\mathcal{A}(X)\cap\mathcal{B}(X)$ or $0\in\mathcal{S}(X)\cap\mathcal{T}(X)$.
Hence the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy common E.A property.

Jang et al. [4] proved following the fixed point theorem.

Theorem 2.1. *Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ be mappings form a complete metric space (X,d) into self satisfying the conditions:*

(I) $\mathcal{A}(X)\subset\mathcal{T}(X)$, $\mathcal{B}(X)\subset\mathcal{S}(X)$,

(II) $$d(\mathcal{A}x,\mathcal{B}y)=p\max\left\{d(\mathcal{A}x,\mathcal{S}x),d(\mathcal{B}y,\mathcal{T}y),\frac{1}{2}[d(\mathcal{A}x,\mathcal{T}y)+d(\mathcal{B}y,\mathcal{S}x)],d(\mathcal{S}x,\mathcal{T}y)\right\}$$ $$+q\max\{d(\mathcal{A}x,\mathcal{S}x),d(\mathcal{B}y,\mathcal{T}y)\}+r\max\{(\mathcal{A}x,\mathcal{T}y),d(\mathcal{B}y,\mathcal{S}x)\},$$

for all $x,y\in X$, where $0<p+q+2r<1$, (p,q and r are non negative real numbers).
Suppose that

(III) *one of the $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ are continuous,*

(IV) *the pair of mappings $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ are compatible on X.*

Then the self maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ have a unique common fixed point in X.

Now, we prove weaker form of Theorem 2.1 using different E.A properties on an incomplete metric space.

3. Main Result

Theorem 3.1. *Suppose the four self mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ of a metric space (X,d) satisfying following constraints:*

(C-1) $\mathcal{A}(X)\subseteq\mathcal{T}(X)$ *and* $\mathcal{B}(X)\subseteq\mathcal{S}(X)$, (C-2) $$d(\mathcal{A}x,\mathcal{B}y) \le p\max\left\{d(\mathcal{A}x,\mathcal{S}x), d(\mathcal{B}y,\mathcal{T}y), \frac{1}{2}[d(\mathcal{A}x,\mathcal{T}y)+d(\mathcal{B}y,\mathcal{S}x)], d(\mathcal{S}x,\mathcal{T}y)\right\} + q\max\{d(\mathcal{A}x,\mathcal{S}x), d(\mathcal{B}y,\mathcal{T}y)\} + r\max\{d(\mathcal{A}x,\mathcal{T}y), d(\mathcal{B}y,\mathcal{S}x)\},$$

for all $x,y\in X$, where $0<p+q+2r<1$ (p,q and r are non negative real numbers).

(C-3) $(\mathcal{A},\mathcal{S})$ *or* $(\mathcal{B},\mathcal{T})$ *satisfies E.A property,*

(C-4) *both the couples* $(\mathcal{A},\mathcal{S})$ *and* $(\mathcal{B},\mathcal{T})$ *are weakly compatible,*

(C-5) *if one of the given four mappings range set is a complete subspace of X then the self mappings $\mathcal{A}$, $\mathcal{B}$, S and $\mathcal{T}$ will be having unique common fixed point.*

Proof. Since the pair of mappings $(\mathcal{B},\mathcal{T})$ follows property E.A then there exists a sequence $\{\alpha_\eta\}$ in X such that $\mathcal{B}\alpha_\eta\to\zeta$, $\mathcal{T}\alpha_\eta\to\zeta$, for some $\zeta\in X$ as $\eta\to\infty$, $\eta\in\mathbb{N}$.
On using $\mathcal{B}(X)\subseteq\mathcal{S}(X)$, there exists a sequence $\{\alpha_\eta\}$ in X such that $\mathcal{B}\alpha_\eta=\mathcal{S}\beta_\eta$.
Hence $\mathcal{S}\beta_\eta\to\zeta$ as $\eta\to\infty$.
Also since $\mathcal{A}(X)\subseteq\mathcal{T}(X)$, there exists a sequence $\{\beta'_\eta\}$ in X such that $\mathcal{A}\beta'_\eta=\mathcal{T}\alpha_\eta$.
Hence $\mathcal{A}\beta'_\eta\to\zeta$ as $\eta\to\infty$.
Suppose $\mathcal{S}(X)$ is a complete subspace of X then $\zeta=\mathcal{S}\mu$ for some $\mu\in X$ and hence the subsequences $\mathcal{A}\beta'_\eta$, $\mathcal{B}\alpha_\eta$, $\mathcal{T}\alpha_\eta$ and $\mathcal{S}\beta_\eta$ converge to ζ $(=\mathcal{S}\mu)$ as $\eta\to\infty$.
Put $x=\mu$, $y=\alpha_\eta$ in contraction condition (C-2) of Theorem 3.1, we get

$$d(\mathcal{A}\mu,\mathcal{B}\alpha_\eta) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\alpha_\eta,\mathcal{T}\alpha_\eta), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\alpha_\eta)+d(\mathcal{B}\alpha_\eta,\mathcal{T}\alpha_\eta)], d(\mathcal{S}\mu,\mathcal{T}\alpha_\eta)\right\} + q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\alpha_\eta,\mathcal{T}\alpha_\eta)\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\alpha_\eta), d(\mathcal{B}\alpha_\eta,\mathcal{S}\mu)\}.$$

Letting $\eta\to\infty$

$$(\mathcal{A}\mu,\mathcal{S}\mu) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{S}\mu,\mathcal{S}\mu), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{S}\mu)+d(\mathcal{S}\mu,\mathcal{S}\mu)], d(\mathcal{S}\mu,\mathcal{S}\mu)\right\} + q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{S}\mu,\mathcal{S}\mu)\} + r\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{S}\mu,\mathcal{S}\mu)\},$$

$$d(\mathcal{A}\mu,\mathcal{S}\mu) \le (p+q+r)d(\mathcal{A}\mu,\mathcal{S}\mu).$$

Since $p+q+r<p+q+2r<1$ giving that

$$\mathcal{A}\mu=\mathcal{S}\mu. \tag{3.1}$$

Now, the weakly compatibility of $\mathcal{A}$ and $\mathcal{S}$ with implies $\mathcal{A}\mathcal{S}\mu=\mathcal{S}\mathcal{A}\mu$ and hence

$$\mathcal{A}\mathcal{A}\mu=\mathcal{A}\mathcal{S}\mu=\mathcal{S}\mathcal{A}\mu=\mathcal{S}\mathcal{S}\mu. \tag{3.2}$$

Since from $(X)\subseteq\mathcal{T}(X)$, there exists $\nu\in X$ such that

$$\mathcal{A}\mu=\mathcal{T}\nu. \tag{3.3}$$

To claim

$$\mathcal{T}\nu=\mathcal{B}\nu.$$

Put $x=\mu y=\nu$ in contraction condition (C-2) of Theorem 3.1 then this gives

$$d(\mathcal{A}\mu,\mathcal{B}\nu) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\nu,\mathcal{T}\nu), \frac{1}{2}[d(\mathcal{A}u,\mathcal{T}\nu)+d(\mathcal{B}\nu,\mathcal{T}\nu)], d(\mathcal{S}\mu,\mathcal{T}\nu)\right\} + q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\nu,\mathcal{T}\nu)\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\nu), d(\mathcal{B}\nu,\mathcal{S}\mu)\}.$$

Using equations (3.1) and (3.3) we get

$$d(\mathcal{T}v,\mathcal{B}v) \leq pd(\mathcal{T}v,\mathcal{B}v) + qd(\mathcal{B}v,\mathcal{T}v) + rd(\mathcal{B}v,\mathcal{T}v)$$

and since $p+q+r<1$ therefore

$$\mathcal{B}v = \mathcal{T}v. \tag{3.4}$$

The equations (3.3) and (3.4) gives $\mathcal{A}\mu = \mathcal{S}\mu = \mathcal{T}v = \mathcal{B}v$.
The weakly compatibility of $\mathcal{B}$ and $\mathcal{T}$ implies

$$\mathcal{B}\mathcal{T}v = \mathcal{T}\mathcal{B}v$$

and hence

$$\mathcal{T}\mathcal{T}v = \mathcal{T}\mathcal{B}v = \mathcal{B}\mathcal{T}v = \mathcal{B}\mathcal{B}v. \tag{3.5}$$

Now, we claim $\mathcal{A}\mu$ is common fixed point of the four mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$.
Use contraction condition (C-2) of Theorem 3.1 and put $x = \mathcal{A}\mu$, $y = v$

$$\begin{aligned} d(\mathcal{A}\mathcal{A}\mu,\mathcal{B}v) \leq p\max&\left\{d(\mathcal{A}\mathcal{A}\mu,\mathcal{S}\mathcal{A}\mu), d(\mathcal{B}v,\mathcal{T}v), \frac{1}{2}[d(\mathcal{A}\mathcal{A}\mu,\mathcal{T}v)+d(\mathcal{B}v,\mathcal{T}v)], d(\mathcal{S}\mathcal{A}\mu,\mathcal{T}v)\right\} \\ &+ q\max\{d(\mathcal{A}\mathcal{A}\mu,\mathcal{S}\mathcal{A}\mu), d(\mathcal{B}v,\mathcal{T}v)\} + r\max\{d(\mathcal{A}\mathcal{A}\mu,\mathcal{T}v), d(\mathcal{B}v,\mathcal{S}\mathcal{A}\mu)\}. \end{aligned}$$

Since using (3.2) and (3.4) this implies $\mathcal{A}\mathcal{A}\mu = \mathcal{A}\mu$.
Therefore $\mathcal{A}\mu = \mathcal{A}\mathcal{A}\mu = \mathcal{S}\mathcal{A}\mu$ and consequently $\mathcal{A}\mu$ is common fixed point of $\mathcal{A}$ and $\mathcal{S}$.
Put $x = \mu$ and $y = \mathcal{T}v$ in (C-2) then we get

$$\begin{aligned} d(\mathcal{A}u,\mathcal{B}\mathcal{T}v) \leq p\max&\left\{d(\mathcal{A}u,\mathcal{S}u), d(\mathcal{B}\mathcal{T}v,\mathcal{T}\mathcal{T}v), \frac{1}{2}[d(\mathcal{A}u,\mathcal{T}\mathcal{T}v)+d(\mathcal{B}\mathcal{T}v,\mathcal{T}\mathcal{T}v)], d(\mathcal{S}u,\mathcal{T}\mathcal{T}v)\right\} \\ &+ q\max\{d(\mathcal{A}u,\mathcal{S}u), d(\mathcal{B}\mathcal{T}v,\mathcal{T}\mathcal{T}v)\} + r\max\{d(\mathcal{A}u,\mathcal{T}\mathcal{T}v), d(\mathcal{B}\mathcal{T}v,\mathcal{S}u)\}, \end{aligned}$$

$$d(\mathcal{B}v,\mathcal{B}\mathcal{B}v) \leq pd(\mathcal{B}v,\mathcal{B}\mathcal{B}v) + rd(\mathcal{B}v,\mathcal{B}\mathcal{B}v).$$

Using $p+q+r<1$ and equation (3.5) then this implies

$$\mathcal{B}\mathcal{B}v = \mathcal{T}\mathcal{B}v,$$

which implies mappings $\mathcal{B}$ and $\mathcal{T}$ have a common fixed point $\mathcal{B}v$.
Consequently $\mathcal{A}\mu = \mathcal{S}\mu = \mathcal{B}v = \mathcal{T}v$ implies $\mathcal{S}\mu$ is common fixed point of the maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$.
Since $\mathcal{S}\mu = \zeta$, the mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ are having ζ as common fixed point.
The uniqueness of the fixed-point follows easily.
The result also follows when $\mathcal{T}(X)$ or $\mathcal{A}(X)$ or $\mathcal{B}(X)$ is a complete subspace of X. □

Theorem 3.2. *Let self maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ are defined on metric space (X,d) satisfying the conditions* (C-2), (C-4) *of Theorem* 3.1. *Moreover if*

(F-1) *the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy the common property E.A,*

(F-2) *$\mathcal{S}(X)$ and $\mathcal{T}(X)$ are closed subsets of X.*

Then the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ have coincidence point each and hence $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ have unique common fixed point.

Proof. Since the pair of mappings $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy the common E.A property, then there exists two sequences $\{\alpha_\eta\}$ and $\{\beta_\eta\}$ in X such that $\lim_{\eta\to\infty}\mathcal{A}\alpha_\eta = \lim_{\eta\to\infty}\mathcal{S}\alpha_\eta = \lim_{\eta\to\infty}\mathcal{B}\beta_\eta = \lim_{\eta\to\infty}\mathcal{T}\beta_\eta = \xi$ for some $\xi \in X$.

Since $\mathcal{S}(X)$ is a closed subset of X, therefore $\lim_{\eta\to\infty} \mathcal{S}x_\eta = \xi \in \mathcal{S}(X)$, and then there exists a point $\mu \in X$ such that $\mathcal{S}\mu = \xi$.

Now, we prove that $\mathcal{A}\mu = \mathcal{S}\mu$, to prove this, keep $x = \mu$ and $y = \beta_\eta$ in (C-2) we get

$$d(\mathcal{A}\mu,\mathcal{B}\beta_\eta) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\beta_\eta)+d(\mathcal{B}\beta_\eta,\mathcal{S}\mu)], d(\mathcal{S}\mu,\mathcal{T}y\beta_\eta)\right\}$$
$$+q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta)\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\beta_\eta), d(\mathcal{B}\beta_\eta,\mathcal{S}\mu)\}.$$

Letting $\eta \to \infty$ this implies

$$d(\mathcal{A}\mu,\xi) \le (p+q+r)d(\mathcal{A}\mu,\xi)$$

since $p+q+r<1$ gives $\mathcal{A}\mu = \xi$. Hence

$$\mathcal{A}\mu = \mathcal{S}\mu \tag{3.6}$$

this implies that the point μ is a coincidence point of the pair $(\mathcal{A},\mathcal{S})$.

Also, since $\mathcal{T}(X)$ is closed subset of X, this gives $\lim_{\eta\to\infty} \mathcal{T}\beta_n = \xi \in \mathcal{T}(X)$ and consequently there exists a point $\omega \in X$ such that $\mathcal{T}\omega = \xi$.

Now, we show that $\mathcal{B}\omega = \mathcal{T}\omega$.

To accomplish this, using contraction condition (C-2) with $x = \mu$, $y = \omega$ we get

$$d(\mathcal{A}\mu,\mathcal{B}\omega) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\omega,\mathcal{T}\omega), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\omega)+d(\mathcal{B}\omega,\mathcal{S}\mu)], d(\mathcal{S}\mu,\mathcal{T}\omega)\right\}$$
$$+q\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\omega,\mathcal{T}\omega)\right\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\omega), d(\mathcal{B}\omega,\mathcal{S}\mu)\},$$
$$d(\xi,\mathcal{B}\omega) \le p\max\left\{d(\xi,\xi), d(\mathcal{B}\omega,\xi), \frac{1}{2}[d(\xi,\mathcal{B}\omega)+d(\mathcal{B}\omega,\xi)], d(\xi,\mathcal{B}\omega)\right\}$$
$$+q\max\{d(\xi,\xi), d(\mathcal{B}\omega,\mathcal{B}\omega)\} + r\max\{d(\xi,\mathcal{B}\omega), d(\mathcal{B}\omega,\omega)\}.$$

On using $\mathcal{T}\omega = \xi$ and (3.6) this imply $\mathcal{B}\omega = \xi$ and hence

$$\mathcal{T}\omega = \mathcal{B}\omega. \tag{3.7}$$

This shows that ω coincidence point of the pair $(\mathcal{B},\mathcal{T})$.

Now in view of the weakly compatible mappings of the pair $(\mathcal{A},\mathcal{S})$ with (3.6) gives

$$\mathcal{A}\xi = \mathcal{A}\mathcal{S}\mu = \mathcal{S}\mathcal{A}\mu = \mathcal{S}\xi$$

this implies

$$\mathcal{A}\xi = \mathcal{S}\xi. \tag{3.8}$$

Now on using contraction condition with $x = \mu$ and $y = \omega$ in (C-2), we get

$$d(\mathcal{A}\mu,\mathcal{B}\omega) \le p\max\left\{d(\mathcal{A}\xi,\mathcal{S}\xi), d(\mathcal{B}\omega,\mathcal{T}\omega), \frac{1}{2}[d(\mathcal{A}\xi,\mathcal{T}\omega)+d(\mathcal{B}\omega,\mathcal{S}\xi)], d(\mathcal{S}\xi,\mathcal{T}\omega)\right\}$$
$$+p\max\{d(\mathcal{A}\xi,\mathcal{S}\xi), d(\mathcal{B}\omega,\mathcal{T}\omega)\} + r\max\{d(\mathcal{A}\xi,\mathcal{T}\omega), d(\mathcal{B}\omega,\mathcal{S}\xi)\}.$$

By using $\mathcal{B}\omega = \mathcal{T}\omega = \xi$ and (3.8) we get $\mathcal{A}\xi = \xi$ and implies

$$\mathcal{A}\xi = \mathcal{S}\xi = \xi. \tag{3.9}$$

Showing that the pair $(\mathcal{A},\mathcal{S})$ is having ξ as fixed point.

Again using the weakly compatible mappings of the pair $(\mathcal{B},\mathcal{T})$ with (3.7), we get

$$\mathcal{B}\xi = \mathcal{B}\mathcal{T}\omega = \mathcal{T}\mathcal{B}\omega = \mathcal{T}\xi.$$

This gives

$$\mathcal{B}\xi = \mathcal{T}\xi. \tag{3.10}$$

Next, we establish that ξ is also common-fixed point of the pair of mappings $(\mathcal{B},\mathcal{T})$.
To ascertain this, using the contraction condition (C-2) with $x = \mu$, $y = \xi$ we get

$$d(\mathcal{A}\mu,\mathcal{B}\xi) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\xi,\mathcal{T}\xi), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\xi)+d(\mathcal{B}\xi,\mathcal{S}\mu)], d(\mathcal{S}\mu,\mathcal{T}\xi)\right\}$$
$$+ q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\xi,\mathcal{T}\xi)\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\xi), d(\mathcal{B}\xi,\mathcal{S}\mu)\}.$$

On using $\mathcal{A}\mu = \xi$ and (x) we get $\mathcal{B}\xi = \xi$ this implies

$$\mathcal{B}\xi = \mathcal{T}\xi = \xi \tag{3.11}$$

which shows ξ is common fixed point of the pair $(\mathcal{B},\mathcal{T})$.
Hence from (3.9) and (3.11) we get ξ is common fixed point for all four maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$.
The uniqueness of the fixed point can be proved easily.
This completes the proof. □

Theorem 3.3. *Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ be four self mappings on metric space (X,d) satisfying the following conditions with* (C-2) *and* (C-4) *of Theorem 3.1*

(G-1) $\mathcal{A}(X) \subseteq \mathcal{T}(X)$ *or* $\mathcal{B}(X) \subseteq \mathcal{S}(X)$,

(G-2) *two pairs* $(\mathcal{A},\mathcal{S})$ *and* $(\mathcal{B},\mathcal{T})$ *satisfy E.A likely property.*

Then the mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ will be having unique common fixed point in X.

Proof. If the pair $(\mathcal{A},\mathcal{S})$ satisfies E.A property then there exists a sequence $\{\alpha_\eta\}$ in X such that

$$\lim_{\eta\to\infty} \mathcal{A}\alpha_\eta = \lim_{\eta\to\infty} \mathcal{S}\alpha_\eta = \zeta \quad \text{for some } \zeta \in \mathcal{A}(X)\cup\mathcal{S}(X).$$

By assuming $\mathcal{A}(X) \subseteq \mathcal{T}(X)$ there exists a sequence $\{\beta_\eta\} \in X$ such that $\mathcal{A}\alpha_\eta = \mathcal{T}\beta_\eta$ and letting $\eta \to \infty$ gives

$$\lim_{\eta\to\infty} \mathcal{A}\alpha_\eta = \lim_{\eta\to\infty} \mathcal{T}\beta_\eta = \lim_{\eta\to\infty} \mathcal{S}\alpha_\eta = \zeta.$$

To prove $\lim_{\eta\to\infty} \mathcal{B}\beta_\eta = \zeta$ put $x = \alpha_\eta$, $y = \beta_\eta$ in contraction condition (C-2) then we get

$$d(\mathcal{A}\alpha_\eta,\mathcal{B}\beta_\eta) \le p\max\left\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta), \frac{1}{2}[d(\mathcal{A}\alpha_\eta,\mathcal{T}\beta_\eta)+d(\mathcal{B}\beta_\eta,\mathcal{S}\beta_\eta)], d(\mathcal{S}\alpha_\eta,\mathcal{T}\beta_\eta)\right\}$$
$$+ q\max\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta)\} + r\max\{d(\mathcal{A}\alpha_\eta,\mathcal{T}\beta_\eta), d(\mathcal{B}\beta_\eta,\mathcal{S}\alpha_\eta)\}.$$

Letting $\eta \to \infty$ gives

$$d(\zeta,\mathcal{B}\beta_n) \le (p+q+r)d(\mathcal{B}\beta_\eta,\zeta)$$

this implies $\lim_{\eta\to\infty} \mathcal{B}\beta_\eta = \zeta$ since $p+q+r<1$.
Suppose $\zeta \in \mathcal{S}(X)$ then there exists a $\mu \in X$ such that $\zeta = \mathcal{S}u$.
Put $x = \mu$, $y = \beta_\eta$ in contraction condition (C-2), then we get

$$dt(\mathcal{A}\mu,\mathcal{B}\beta_\eta) \le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta), \frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\beta_\eta)+d(\mathcal{B}\beta_\eta,\mathcal{S}\mu)], d(\mathcal{S}\mu,\mathcal{T}y_n)\right\}$$
$$+ q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu), d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta)\} + r\max\{d(\mathcal{A}\mu,\mathcal{T}\beta_\eta), d(\mathcal{B}\beta_\eta,\mathcal{S}\mu)\}.$$

Letting $\eta \to \infty$ and using $\zeta = \mathcal{S}\mu$ this implies $\mathcal{A}\mu = \zeta$.

Therefore

$$\mathcal{A}\mu = \mathcal{S}\mu = \zeta .$$

Since the pair $(\mathcal{A},\mathcal{S})$ is weakly compatible with $\mathcal{A}\mu = \mathcal{S}\mu$ implies

$$\mathcal{A}\zeta = \mathcal{A}\mathcal{S}\mu = \mathcal{S}\mathcal{A}\mu = \mathcal{S}\zeta$$

and this on using above relation

$$\mathcal{A}\zeta = \mathcal{S}\zeta . \tag{3.12}$$

Since the pair $(\mathcal{B},\mathcal{T})$ satisfies E.A likely property then $\exists$ sequence $\{\beta'_\eta\} \in X$ such that

$$\lim_{\eta\to\infty} \mathcal{B}\beta'_\eta = \lim_{\eta\to\infty} \mathcal{T}\beta'_\eta = \zeta \quad \text{for some } \zeta \in \mathcal{B}(X)\cup\mathcal{T}(X).$$

If $\zeta \in \mathcal{T}(X)$ then there exists $\nu \in X$ such that $\mathcal{T}\nu = \zeta$.
Now, we claim $\mathcal{B}\nu = \zeta$.
Put $x = \alpha_\eta$, $y = \nu$ in (C-2) then we get

$$d(\mathcal{A}\alpha_\eta,\mathcal{B}\nu) \le p\max\left\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), d(\mathcal{B}\nu,\mathcal{T}\nu), \frac{1}{2}[d(\mathcal{A}\alpha_\eta,\mathcal{T}\nu)+d(\mathcal{B}\nu,\mathcal{S}\alpha_\eta)], d(\mathcal{S}\alpha_\eta,\mathcal{T}\nu)\right\}$$
$$+ q\max\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), d(\mathcal{B}\nu,\mathcal{T}\nu)\} + r\max\{d(\mathcal{A}\alpha_\eta,\mathcal{T}\nu), d(\mathcal{B}\nu,\mathcal{S}\alpha_\eta)\}.$$

Letting $\eta\to\infty$ and using

$$d(\zeta,\mathcal{B}\nu) \le (p+p+r)d(\zeta,\mathcal{B}\nu)$$

implies $\mathcal{B}\nu = \zeta$.
Since the pair $(\mathcal{B},\mathcal{T})$ is weakly compatible then we have $\mathcal{B}\zeta = \mathcal{B}\mathcal{T}\nu = \mathcal{T}\mathcal{B}\nu = \mathcal{T}\zeta$.
Now, we claim $\mathcal{B}\zeta = \zeta$. For this put $x = \alpha_\eta$, $y = \zeta$ in (C-2) then we get

$$\mathcal{T}\nu = \zeta .$$

Letting n tends infinity

$$d(\mathcal{A}\alpha_\eta,\mathcal{B}\zeta) \le p\max\left\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), (\mathcal{B}\zeta,\mathcal{T}\zeta), \frac{1}{2}[d(\mathcal{A}\alpha_\eta,\mathcal{T}z)+d(\mathcal{B}\zeta,\mathcal{S}\alpha_\eta)], d(\mathcal{S}\alpha_\eta,\mathcal{T}\zeta)\right\}$$
$$+ q\max\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta), d(\mathcal{B}\zeta,\mathcal{T}\zeta)\} + r\max\{d(\mathcal{A}\alpha_\eta,\mathcal{T}\zeta), d(\mathcal{B}\zeta,\mathcal{S}\alpha_\eta)\},$$
$$d(\zeta,\mathcal{B}\zeta) \le (p+q+r)d(\mathcal{B}\zeta,\zeta)$$

this implies $\mathcal{B}\zeta = \zeta$.
This gives

$$\mathcal{B}\zeta = \mathcal{T}\zeta = \zeta . \tag{3.13}$$

Put $x = \zeta$, $y = \zeta$ in (C-2) then this implies

$$d(\mathcal{A}\zeta,\mathcal{B}\zeta) \le p\max\left\{d(\mathcal{A}\zeta,\mathcal{S}\zeta), d(\mathcal{B}\zeta,\mathcal{T}\zeta), \frac{1}{2}[d(\mathcal{A}\zeta,\mathcal{T}\zeta)+d(\mathcal{B}\zeta,\mathcal{S}\zeta)], d(\mathcal{S}\zeta,\mathcal{T}\zeta)\right\}$$
$$+ q\max\{d(\mathcal{A}\zeta,\mathcal{S}\zeta), d(\mathcal{B}\zeta,\mathcal{T}\zeta)\} + r\max\{d(\mathcal{A}\zeta,\mathcal{T}\zeta), d(\mathcal{B}\zeta,\mathcal{S}\zeta)\}$$

this implies

$$\mathcal{A}\zeta = \mathcal{B}\zeta . \tag{3.14}$$

Therefore from (3.12), (3.13) and (3.14) we conclude that $\mathcal{A}\zeta = \mathcal{B}\zeta = \mathcal{T}\zeta = \mathcal{S}\zeta = \zeta$. This gives ζ is common fixed point of the maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$.

Uniqueness of the fixed point can be easily verified. □

Finally, we prove a theorem on

Theorem 3.4. *Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ be self maps of a metric space (X,d) satisfying the conditions* (C-2), (C-4) *of Theorem* 3.1 *and assuming*

(H-1) *the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy the common (E.A) like property.*

Then the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ have a coincidence point each and hence the maps $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ have unique common fixed point.

Proof. Since the two pairs of mappings $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ satisfy the common E.A like property, we find two sequences $\{\alpha_\eta\}$ and $\{\beta_\eta\}$ in X such that

$$\lim_{\eta\to\infty}\mathcal{A}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{S}\alpha_\eta=\lim_{\eta\to\infty}\mathcal{B}\beta_\eta=\lim_{\eta\to\infty}\mathcal{T}\beta_\eta=\zeta,$$

where $\zeta\in\mathcal{S}(X)\cap\mathcal{T}(X)$ or $\zeta\in\mathcal{A}(X)\cap\mathcal{B}(X)$.
Suppose $\zeta\in\mathcal{S}(X)\cap\mathcal{T}(X)$.
Now $\zeta\in\mathcal{S}(X)$ there exists $\mu\in X$ such that $\mathcal{S}\mu=\zeta$.
Now, we assert that $\mathcal{A}\mu=\mathcal{S}\mu$, using inequality (C-2) with $x=\mu$, $y=\beta_\eta$, we get

$$\begin{aligned}d(\mathcal{A}\mu,\mathcal{B}\beta_\eta)\le p\max&\left\{d(\mathcal{A}\mu,\mathcal{S}\mu),d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta),\frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\beta_\eta)+d(\mathcal{B}\beta_\eta,\mathcal{S}\mu)],d(\mathcal{S}\mu,\mathcal{T}\beta_\eta)\right\}\\&+q\max\{d(\mathcal{A}\mu,\mathcal{S}\mu),d(\mathcal{B}\beta_\eta,\mathcal{T}\beta_\eta)\}+r\max\{d(\mathcal{A}\mu,\mathcal{T}\beta_\eta),d(\mathcal{B}\beta y_\eta,\mathcal{S}\mu)\}.\end{aligned}$$

Letting $\eta\to\infty$

$$d(\mathcal{A}\mu,\zeta)\le(p+q+r)d(\mathcal{A}\mu,\zeta)\text{ implies }\mathcal{A}\mu=\zeta.$$

Hence $\mathcal{A}\mu=\zeta=\mathcal{S}\mu$ this gives μ is seen as a coincidence point of the pair of mappings $(\mathcal{A},\mathcal{S})$.
Again $\zeta\in\mathcal{T}(X)$, we have $\zeta=\mathcal{T}\nu$ for some $\nu\in X$.
We show that $\mathcal{B}\nu=\mathcal{T}\nu$, using the contraction condition (C-2) with $x=\beta_\eta$, $y=\nu$

$$\begin{aligned}d(\mathcal{A}\alpha_\eta,\mathcal{B}\nu)\le p\max&\left\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta),d(\mathcal{B}\nu,\mathcal{T}\nu),\frac{1}{2}[d(\mathcal{A}\alpha_\eta,\mathcal{T}\nu)+d(\mathcal{B}\nu,\mathcal{S}\alpha_\eta)],d(\mathcal{S}\alpha_\eta,\mathcal{T}\nu)\right\}\\&+q\max\{d(\mathcal{A}\alpha_\eta,\mathcal{S}\alpha_\eta),d(\mathcal{B}\nu,\mathcal{T}\nu)+r\max\{d(\mathcal{A}\alpha_\eta,\mathcal{T}\nu),d(\mathcal{B}\nu,\mathcal{S}\alpha_\eta)\}.\end{aligned}$$

Letting $\zeta\to\infty$

$$d(\zeta,\mathcal{B}\zeta)\le pd(\mathcal{B}\nu,\zeta)+qd(\mathcal{B}\nu,\zeta)+rd(\mathcal{B}\nu,\zeta),$$

$$d(\mathcal{B}\nu,\zeta)\le(p+q+r)d(\mathcal{B}\nu,\zeta)$$

implies $\mathcal{B}\nu=\zeta t$ is gives $\mathcal{B}\nu=\mathcal{T}\nu=\zeta$.
By using the weakly compatible nature of the pairs $(\mathcal{A},\mathcal{S})$ and $(\mathcal{B},\mathcal{T})$ and $\mathcal{A}\mu=\mathcal{S}\mu$, $\mathcal{B}\nu=\mathcal{T}\nu$, therefore $\mathcal{A}\mu=\mathcal{A}\mathcal{S}\mu=\mathcal{S}\mathcal{A}\mu=\mathcal{S}\mu$ and $\mathcal{B}\zeta=\mathcal{B}\mathcal{T}\nu=\mathcal{T}\mathcal{B}\nu=\mathcal{T}\zeta$.
Now, we establish that ζ is a common fixed point of $\mathcal{A}$ and $\mathcal{S}$.
On using contraction condition (C-2) with $x=\zeta$, $y=\nu$ implies

$$d(\mathcal{A}\zeta,\mathcal{B}\nu)\le p\max\left\{d(\mathcal{A}\zeta,\mathcal{S}\zeta),d(\mathcal{B}\nu,\mathcal{T}\nu),\frac{1}{2}[d(\mathcal{A}\zeta,\mathcal{T}\nu)+d(\mathcal{B}\nu,\mathcal{S}\zeta)],d(\mathcal{S}\zeta,\mathcal{T}\zeta)\right\}$$

$$+q\max\{d(\mathcal{A}\zeta,\mathcal{S}\zeta),d(\mathcal{B}v,\mathcal{T}v)\}+r\max\{d(\mathcal{A}\zeta,\mathcal{T}v),d(\mathcal{B}v,\mathcal{S}\zeta)\},$$

$$d(\mathcal{A}\zeta,\zeta)\le(p+r)d(\mathcal{A}\zeta,\zeta)$$

implies $\mathcal{A}\zeta=\zeta$ which implies

$$\mathcal{A}\zeta=\mathcal{S}\zeta=\zeta. \tag{3.15}$$

Now, we show that the pair $(\mathcal{B},\mathcal{T})$ has common fixed point ζ.
Using contraction condition (C-2) with $x=\mu$, $y=\zeta$ we get

$$d(\mathcal{A}\mu,\mathcal{B}\zeta)\le p\max\left\{d(\mathcal{A}\mu,\mathcal{S}\mu),d(\mathcal{B}\zeta,\mathcal{T}\zeta),\frac{1}{2}[d(\mathcal{A}\mu,\mathcal{T}\zeta)+d(\mathcal{B}\zeta,\mathcal{S}\mu)],d(\mathcal{S}\mu,\mathcal{T}\zeta)\right\}$$
$$+p\max\{d(\mathcal{A}\mu,\mathcal{S}\mu),d(\mathcal{B}\zeta,\mathcal{T}\zeta)\}+r\max\{d(\mathcal{A}\mu,\mathcal{T}\zeta),d(\mathcal{B}\zeta,\mathcal{S}\mu)\}$$

$$d(\zeta,\mathcal{B}\zeta)\le(p+r)d(\zeta,\mathcal{B}\zeta)$$

implies $\mathcal{B}\zeta=\zeta$ is a contradiction implies

$$\mathcal{B}\zeta=\zeta=\mathcal{T}\zeta. \tag{3.16}$$

Thus from (3.15) and (3.16), mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{S}$ and $\mathcal{T}$ have ζ as a common fixed point.
Similarly, the theorems hold $\zeta\in\mathcal{A}(X)\cap\mathcal{B}(X)$.
Uniqueness the of common fixed point can be easy verified. □

4. Conclusion

In this paper we established the generalizations of Theorem 2.1 in different forms by using the conditions, by assuming one of the pair of mappings satisfying E.A property along with the assumption of range of one of the mappings is complete sub space of X in Theorem 3.1, by assuming common E.A property for one pair of mappings along with the assumption of ranges of $\mathcal{S}$ and $\mathcal{T}$ being closed subsets of X in Theorem 3.2, assuming E.A like property for both the pairs of mappings with minimal contained condition in Theorem 3.3, assuming both of the pair of mappings satisfying Common E.A like property without using the contained inequality in Theorem 3.4, respectively. Moreover, another pair of mappings is assumed to be weakly compatible mappings which is eventually weaker then compatible mappings assumed in Theorem 2.1. Further, the completeness and continuity conditions are being relaxed in all the above theorems. Hence we claim that all our theorems generalize Theorem 2.1.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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