Communications in Mathematics and Applications
Vol. 12, No. 1, pp. 127–143, 2021
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/cma.v12i1.1476

Research Article

Fixed Point Results for $(\alpha\text{-}\beta_k,\phi\text{-}\psi)$ Integral Type Contraction Mappings in Fuzzy Metric Spaces

Rakesh Tiwari and Shraddha Rajput*

Department of Mathematics, Government V.Y.T. Post-Graduate Autonomous College, Durg 491001, Chhattisgarh, India
***Corresponding author:** shraddhasss112@gmail.com

Abstract. In this paper, we introduce the notion of a modified $(\alpha\text{-}\beta_k,\phi\text{-}\psi)$ integral type contraction mappings in fuzzy metric spaces. We study and prove the existence and uniqueness of fixed points theorems in generalized fuzzy contractive mappings of integral type in fuzzy metric spaces. Our main result generalizes the fuzzy Banach contraction theorem and we validate our results by some suitable examples which reveal that our results are proper generalization and modification of some researchers' integral contraction works.

Keywords. Fixed point; Metric spaces; Fuzzy metric spaces; Integral type contraction; Modified $(\alpha\text{-}\beta_k,\phi\text{-}\psi)$ contraction

MSC. 54H25; 47H10

Received: 28 October 2020 **Accepted:** January 6, 2021

1. Introduction

The conviction of fuzzy sets was introduced by Zadeh [15] in 1965. Kramosil and Michalek [9] introduced the concept of fuzzy metric space in 1975, which can be regarded as a generalization of the statistical metric space. Clearly, this work plays an essential role in the erection of fixed point theory in fuzzy metric spaces. Grabiec [5] extended the Banach contraction principle [1] to fuzzy metric spaces in the sense of Kramosil and Michalek [9]. Following Grabiec's work, many authors introduced and generalized different types of fuzzy contractive mappings and investigated some fixed point theorems in fuzzy metric spaces. In 1994, George and Veeramani [4] modified the notion of M-complete fuzzy metric space with the help of continuous t-norms. In 2012, Shen *et al*. [14] introduced the notion of altering distance in fuzzy metric spaces and gave a fixed point results in complete and compact fuzzy metric spaces. Recently, Dosenovic *et al*. [3] introduced and proved fixed point theorems in complete and compact fuzzy metric spaces using altering distance.

The concept of α-admissible for single valued mappings was introduced by Samet *et al*. [13]. Afterwards, Salimi *et al*. [13] introduced β_k-admissible mapping which generalized the concept of α-admissible.

Now, we begin with preliminaries and establish fixed point theorems of integral type in fuzzy metric spaces. We validate our results by suitable examples and figures. Further, we also prove some more fixed point theorems validated by suitable examples. Followed by conclusion on our results.

2. Preliminaries

Now, we begin with some basic concepts.

Definition 2.1 ([4]). An ordered triple $(X,M,*)$ is called fuzzy metric space such that X is a nonempty set, $*$ defined a continuous t-norm and M is a fuzzy set on $X \times X \times (0,\infty)$, satisfying the following conditions, for all $x,y,z \in X$, and $s,t>0$:

(FM-1) $M(x,y,t)>0$.

(FM-2) $M(x,y,t)=1$ iff $x=y$.

(FM-3) $M(x,y,t)=M(y,x,t)$.

(FM-4) $(M(x,y,t)*M(y,z,s)) \le M(x,z,t+s)$.

(FM-5) $M(x,y,*):(0,\infty)\to(0,1]$ is left continuous.

Definition 2.2 ([13]). Let $F:X\to X$ and $\alpha: X\times X\to[0,\infty)$. Then F is an α-admissible mapping, if

$$\alpha(x,y)\ge 1 \;\Rightarrow\; \alpha(F(x),F(y))\ge 1, \quad x,y\in X.$$

Definition 2.3 ([12]). Let $F:X\to X$, $\beta: X\times(0,+\infty)\to[0,+\infty)$ and $k:(0,+\infty)\to(0,1)$. Then F is a β_k-admissible mapping if

$$\beta(x,t)\le\sqrt{k(t)} \;\Rightarrow\; \beta(F(x),t)\le\sqrt{k(t)}, \quad \text{for all } x\in X,\, t>0.$$

Definition 2.4 ([8]). A function $\phi:[0,\infty)\to[0,\infty)$ is a called altering distance function, if $\phi(t)$ is monotonic non-decreasing and continuous and $\phi(t)=0$ if and only if $t=0$.

Definition 2.5 ([8]). An ultra altering distance function is a continuous, non-decreasing mapping $\psi:[0,\infty)\to[0,\infty)$ such that $\psi(t)>0$ for $t>0$ and $\psi(0)\ge 0$.

Definition 2.6 ([6]). Let $(X,M,*)$ be a fuzzy metric space. We say that the mapping $T:X\to X$ is fuzzy contractive if there exists $k\in(0,1)$ such that
$$\frac{1}{M(Tx,Ty,t)}-1\le k\left(\frac{1}{M(x,y,t)}-1\right)$$
for each $x,y\in X$ and $t>0$ (k is called the contractive constant of T).

Theorem 2.7 (Fuzzy Banach Contraction Theorem [10]). *Let $(X,M,*)$ be a complete fuzzy metric space in which fuzzy contractive sequences are Cauchy. Let $T:X\to X$ be a fuzzy contractive mapping being k the contractive constant. Then T has a unique fixed point.*

Branciari [2] generalized the Banach fixed point theorems as following:

Theorem 2.8 ([2]). *Let (X,d) be a complete metric space, $k\in(0,1)$ and let $T:X\to X$ be a mapping such that for each $x,y\in X$,*
$$\int_0^{d(Tx,Ty)}\varphi(s)ds\le k\int_0^{d(x,y)}\varphi(s)ds,$$
where $\varphi:[0,\infty]\to[0,\infty]$ be a Lebesgue-integrable mapping which is summable on each compact subset of $[0,\infty)$, non negative and for each $\epsilon>0$,
$$\int_0^{\epsilon}\varphi(s)ds>0,$$
then T has a unique fixed point $z\in X$ such that for each $z\in X$, $\lim_{n\to\infty}T^nx=z$.

Hussain *et al.* [7] generalized the Fuzzy Banach contraction as follows:

Theorem 2.9 ([7]). *Let $(X,M,*)$ be a complete fuzzy metric space, $k\in(0,1)$, and let $T:X\to X$ be a mapping such that for each $x,y\in X$,*
$$\int_0^{\left(\frac{1}{M(Fx,Fy,t)}-1\right)}\varphi(s)ds\le k\int_0^{\left(\frac{1}{M(x,y,t)}-1\right)}\varphi(s)ds,$$
where $\varphi:[0,\infty]\to[0,\infty]$ be a Lebesgue-integrable mapping which is summable on each compact subset of $[0,\infty)$,non negative and for each $\epsilon>0$,
$$\int_0^{1-\epsilon}\varphi(s)ds>0,$$
then T has a unique fixed point $z\in X$ such that for each $z\in X$, $\lim_{n\to\infty}T^nx=z$.

Phiangsungnoen *et al.* [11] proved the following result:

Theorem 2.10 ([11]). *Let $(X,M,*)$ be a complete fuzzy metric space, F are α and β_k-admissible mappings and $\varphi\in\Phi$ such that $\alpha(x,F(x))\alpha(y,F(y))\ge1$ implies that*
$$\varphi(M(F(x),F(y),t))\le\beta(x,t)\beta(y,t)N(x,y,t),$$
where
$$N(x,y,t)=\max\{\varphi(M(x,y,t)),\varphi(M(x,F(x),t)),\varphi(M(y,F(y),t))\}.$$
Suppose that the following conditions hold:

(a) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, F(x_0)) \geq 1$ *and* $\beta(x_0, t) \leq \sqrt{k(t)}$ *for all* $t > 0$,

(b) *if* $\{x_n\}$ *is a sequence such that* $(x_n, x_{n+1}) \geq 1$ *for all* $n \in \mathbb{N}$, *and* $x_n \to x$ *as* $n \to \infty$, *then* $\alpha(x, F(x)) \geq 1$.

Then F *has a unique fixed point on* $x \in X$, *such that* $\alpha(x, F(x)) \geq 1$ *and* $\beta(x, t) < 1$ *for all* $x \in X$ *and* $t > 0$.

The objective of this work is to prove some common fixed point theorems by introducing modified (α-β_k,ϕ-ψ) integral type contraction mappings in fuzzy metric spaces. We prove the existence and uniqueness of fixed points theorems in generalized fuzzy contractive mappings of integral type in fuzzy metric spaces in the sense of George and Veeramani [4]. Our main result generalize the fuzzy Banach contraction theorem of Hussain *et al*. [7] and Phiangsungnoen *et al*. [11]. We validate our results by some suitable examples which revel that our results are proper generalization and modification of Gregori *et al*. [6], Hussain *et al*. [7] and Phiangsungnoen *et al*. [11].

3. Main Results

In this section, we establish fixed point theorems in fuzzy metric spaces.

Theorem 3.1. *Let* $(X, M, *)$ *be a complete fuzzy metric space, and let* ϕ *be an altering distance function and* $\psi : [0,\infty) \to [0,\infty)$ *be a continuous function. Let* F *are* α *and* β_k*-admissible mappings such that*

$$\alpha(x, F(x))\alpha(y, F(y)) \geq 1, \tag{1}$$

implies that

$$\int_0^{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)} \varphi(s)ds \leq \beta(x,t)\beta(y,t)N(x,y,t), \tag{2}$$

where

$$N(x,y,t) = \int_0^{\psi\left(\frac{1}{M(x,y,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x,y,t)}-1\right)} \varphi(s)ds, \tag{3}$$

for every $x, y \in X$ *and* $t \in (0,\infty)$. *Suppose that the following conditions hold:*

(a) *Let* $\varphi : [0,\infty] \to [0,\infty]$ *be a Lebesgue-integrable mapping which is summable on each compact subset of* $[0,\infty)$, *non negative and for each* $\epsilon > 0$,

$$\int_0^{1-\epsilon} \varphi(s)ds > 0, \tag{4}$$

(b) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, F(x_0)) \geq 1$ *and* $\beta(x_0, t) \leq \sqrt{k(t)}$ *for all* $t > 0$,

(c) *if* $\{x_n\}$ *is a sequence such that* $(x_n, x_{n+1}) \geq 1$ *for all* $n \in \mathbb{N}$, *and* $x_n \to x$ *as* $n \to \infty$, *then* $\alpha(x, F(x)) \geq 1$.

Then F *has a unique fixed point on* $z \in X$, *such that* $\alpha(z, F(z)) \geq 1$ *and* $\beta(x, t) < 1$ *for all* $x \in X$ *and* $t > 0$.

Proof. Let $x_0 \in X$ be an arbitrary point such that $\alpha(x_0, F(x_0)) \geq 1$ and and $\beta(x_0,t) \leq \sqrt{k(t)}$ for all $t > 0$ (from condition (2)). We define the sequence $\{x_n\} \subset X$ as $x_n = Fx_{n-1}$, such that

$$x_n = F(x_{n-1}), \quad \text{for all } n \in \mathbb{N}.$$

Since F be a α and β_k-admissible mappings such that $\alpha(x_0,x_1) = \alpha(x_0,F(x_0)) \geq 1$, we conclude that

$$\alpha(x_1,Fx_1) = \alpha(F(x_0),F(x_1)) \geq 1.$$

By continuing this process, we get

$$\alpha(x_{n-1},F(x_{n-1})) \geq 1,$$

for all $n \in \mathbb{N}$. This implies that

$$\alpha(x_{n-1},F(x_{n-1}))\alpha(x_n,F(x_n)) \geq 1,$$

for all $n \in \mathbb{N}$, we deduce that

$$\beta(x_1,t) = \beta(F(x_0),t) \leq \sqrt{k(t)}.$$

Similarly,

$$\beta(x_n,t) \leq \sqrt{k(t)}.$$

Assume that $x_n \neq x_{n+1}$ for all $n \geq 1$. Taking $x = x_{n-1}$ and $y = x_n$ using equation (2) and (3)

$$\begin{aligned}\int_0^{\psi\left(\frac{1}{M(Fx_{n-1},Fx_n,t)}-1\right)} \varphi(s)ds &= \int_0^{\psi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds \\ &\leq \beta(x_{n-1},t)\beta(x_n,t)N(x_{n-1},x_n,t),\end{aligned}$$

where

$$N(x_{n-1},x_n,t) = \int_0^{\psi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds.$$

If $\psi\left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right) \geq \phi\left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right)$, then we have

$$\int_0^{\psi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds \leq \int_0^{\psi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds,$$

this implies that

$$\int_0^{\psi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds = 0 \quad \text{and} \quad \int_0^{\frac{1}{M(x_n,x_{n+1},t)}-1} \varphi(s)ds = 0.$$

This gives $x_n = x_{n+1} = Fx_n$, which contradicts to our assumption. This implies that,

$$\int_0^{\psi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds = \int_0^{\frac{1}{M(x_{n-1},x_n,t)}-1} \varphi(s)ds,$$

$$\begin{aligned}\int_0^{\psi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds &\leq \sqrt{k(t)}\sqrt{k(t)} \int_0^{\psi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds \\ &\leq k(t)\int_0^{\psi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)} \varphi(s)ds.\end{aligned}$$

Because ψ is a non-decreasing function, then

$$\int_0^{\psi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds \le \int_0^{\psi\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)} \varphi(s)ds, \quad \text{for all } n \ge 1.$$

Therefore, the sequence $\left\{\int_0^{\psi(\frac{1}{M(x_n,x_{n+1},t)}-1)} \varphi(s)ds > 0\right\}$ is decreasing and converges for $s > 0$. Taking the limits as $n \to \infty$ we obtain

$$\int_0^{\psi(s)} \varphi(s)ds \le k(t)\Big(\int_0^{\psi(s)} \varphi(s)ds - \int_0^{\phi(s)} \varphi(s)ds\Big),$$

this implies that $\int_0^{\phi(s)} \varphi(s)ds = 0$ and hence $s = 0$.

$$\lim_{n\to\infty} \int_0^{\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds = 0,$$

then equation (4) implies that

$$\lim_{n\to\infty}\Big(\frac{1}{M(x_n,x_{n+1},t)} - 1\Big) = 0.$$

Which further implies

$$\lim_{n\to\infty} M(x_n,x_{n+1},t) = 1. \tag{5}$$

Now, we prove that $\{x_n\}$ is a Cauchy sequence. Let $\{x_n\}$ is not a Cauchy sequence. Then there exists $\epsilon > 0$, for which we can find two sub sequences $\{x_{n(i)}\}$ and $\{x_{m(i)}\}$ of $\{x_n\}$ with $n(i) > m(i) > i$ such that

$$\int_0^{\frac{1}{M(x_{m(i)},x_{n(i)},t)}-1} \varphi(s)ds \ge 1-\epsilon, \tag{6}$$

where $n(i)$ is the smallest integer satisfying, for all positive integer i and $t > 0$.

$$\int_0^{\frac{1}{M(x_{m(i)},x_{n(i)-1},t)}-1} \varphi(s)ds \le 1-\epsilon. \tag{7}$$

On applying the triangular inequality, we get

$$\begin{aligned} 1-\epsilon \le \int_0^{\frac{1}{M(x_{m(i)},x_{n(i)},t)}-1} \varphi(s)ds &\le \Big(\int_0^{\frac{1}{M(x_{m(i)},x_{n(i)-1},t)}-1} \varphi(s)ds + \int_0^{\frac{1}{M(x_{n(i)-1},x_{n(i)},t)}-1} \varphi(s)ds\Big) \\ &\le (1-\epsilon) + \int_0^{\frac{1}{M(x_{n(i)-1},x_{n(i)},t)}-1} \varphi(s)ds. \end{aligned}$$

Taking the limit as $i \to \infty$, using (3), we get

$$\int_0^{\frac{1}{M(x_{m(i)},x_{m(i)-1},t)}-1} \varphi(s)ds = \int_0^{\frac{1}{M(x_{n(i)},x_{n(i)-1},t)}-1} \varphi(s)ds = 0.$$

From triangular inequality and equation (6), (7) we have,

$$\begin{aligned} M(x_{n(i)-1},x_{m(i)-1},t) &\ge M\Big(x_{n(i)-1},x_{n(i)},\frac{t}{2}\Big) * M\Big(x_{n(i)},x_{m(i)-1},\frac{t}{2}\Big) \\ &\ge M\Big(x_{n(i)-1},x_{n(i)},\frac{t}{2}\Big) * (1-\epsilon). \end{aligned} \tag{8}$$

Hence

$$\int_0^{\frac{1}{M(x_{n(i)},x_{m(i)},t)}-1} \varphi(s)ds \le \int_0^{\frac{1}{M(x_{n(i)},x_{n(i)-1},t)}-1} \varphi(s)ds - \int_0^{\frac{1}{M(x_{n(i)-1},x_{m(i)},t)}-1} \varphi(s)ds$$

thus

$$1-\epsilon \le \lim_{i\to\infty}\int_0^{\frac{1}{M(x_{n(i)-1},x_{m(i)-1},t)}-1}\varphi(s)ds \le 1-\epsilon, \tag{9}$$

$$\lim_{i\to\infty}\int_0^{\frac{1}{M(x_{n(i)-1},x_{m(i)-1},t)}-1}\varphi(s)ds < \int_0^{\frac{\epsilon}{1-\epsilon}}\varphi(s)ds. \tag{10}$$

Now, we substitute $x = x_{n(i)-1}$ and $y = x_{m(i)-1}$ in (2), (3) which yields

$$\begin{aligned}\int_0^{\frac{\epsilon}{1-\epsilon}}\varphi(s)ds &\le \int_0^{\psi\left(\frac{1}{M(Tx_{n(i)-1},Tx_{m(i)-1},t)}-1\right)}\varphi(s)ds\\ &= \int_0^{\psi\left(\frac{1}{M(Tx_{n(i)},Tx_{m(i)},t)}-1\right)}\varphi(s)ds\\ &\le \beta(x_{n(i)-1},t)\beta(x_{m(i)-1},t)\left(\int_0^{\psi\left(\frac{1}{M(x_{n(i)-1},x_{m(i)-1},t)}-1\right)}\varphi(s)ds-\int_0^{\phi\left(\frac{1}{M(x_{n(i)-1},x_{m(i)-1},t)}-1\right)}\varphi(s)ds\right).\end{aligned}$$

Clearly, as $i\to\infty$ we have $\dfrac{1}{M(x_{n(i)-1},x_{m(i)-1},t)}-1 \to \dfrac{\epsilon}{1-\epsilon}$ and $M(x_{n(i)-1},x_{m(i)-1},t)-1 \to \dfrac{\epsilon}{1-\epsilon}$.
So

$$\begin{aligned}\int_0^{\psi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds &< \sqrt{k(t)}\sqrt{k(t)}\left(\int_0^{\psi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds-\int_0^{\phi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds\right)\\ &< k(t)\left(\int_0^{\psi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds-\int_0^{\phi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds\right),\end{aligned}$$

which implies that $\int_0^{\phi(\frac{\epsilon}{1-\epsilon})}\varphi(s)ds = 0$. But this contradicts our assumption that $\{x_n\}$ is not a Cauchy sequence. Thus, $\{x_n\}$ must be Cauchy sequence. Since $(X,M,*)$ is complete, then $\{x_n\}$ converges to a limit, say $z \in X$ such that $x_n \to z$ as $n\to\infty$, for each $t>0$. Since $x_n \ne x_{n+1}$ for all $n\in\mathbb{N}\cup\{0\}$, we get $0 < M(x_n,x_{n+1},t) < 1$ for all $t>0$. Since, $\alpha(x_{n-1},x_n)\ge 1$ for all $n\in\mathbb{N}$. By condition (3), we have

$$\alpha(x_{n-1},F(x_{n-1}))\alpha(z,F(z)) \ge 1,$$

for all $n\in\mathbb{N}$, $x_n\to z$ in equations (2) and (3), and taking $n\to\infty$. We have

$$\begin{aligned}\int_0^{\psi\left(\frac{1}{M(Fz,F_{x+1},t)}-1\right)}\varphi(s)ds &= \int_0^{\psi\left(\frac{1}{M(z,x_n,t)}-1\right)}\varphi(s)ds\\ &\le \beta(x_n,t)\beta(z,t)N(x_n,z,t)\\ &\le \sqrt{k(t)}\sqrt{k(t)}N(z,x_n,t)\\ &\le k(t)N(z,x_n,t),\end{aligned}$$

since

$$N(x_n,z,t) = \int_0^{\psi\left(\frac{1}{M(x_n,z,t)}-1\right)}\varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x_n,z,t)}-1\right)}\varphi(s)ds = 0,$$

we have $M(Fz,z,t)=1$ as $n\to\infty$. Hence $Fz = z$. Therefore, F has a fixed point.

Now, we prove the uniqueness of the fixed point of F. Let us suppose that y and z are two distinct fixed points of F and $M(y,z,t)<1$ such that $\alpha(z,F(z))\ge 1$ and $\beta(y,t)<1$ for all $y,z\in X$

and $t > 0$. Thus, we get

$$\alpha(y,F(y))\alpha(z,F(z)) \geq 1.$$

Consequently, from equations (2) and (3)

$$\begin{aligned}\psi\Big(\frac{1}{M(y,z,t)}-1\Big) = \psi\Big(\frac{1}{M(Fy,Fz,t)}-1\Big)\\ &\leq \beta(y,t)\beta(z,t)N(x,y,t)\\ &\leq \sqrt{k(t)}\,\sqrt{k(t)}N(x,y,t)\\ &\leq k(t)N(x,y,t),\end{aligned}$$

where

$$N(y,z,t) = \psi\Big(\frac{1}{M(y,z,t)}-1\Big) - \phi\Big(\frac{1}{M(y,z,t)}-1\Big).$$

Thus, we have

$$\begin{aligned}\int_0^{\psi\left(\frac{1}{M(y,z,t)}-1\right)} \varphi(s)ds &= \int_0^{\psi\left(\frac{1}{M(Fy,Fz,t)}-1\right)} \varphi(s)ds\\ &\leq k(t)\Big(\int_0^{\psi\left(\frac{1}{M(y,z,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(y,z,t)}-1\right)} \varphi(s)ds\Big).\end{aligned}$$

Since $\phi\Big(\frac{1}{M(y,z,t)}-1\Big) = 0$, we have $M(y,z,t) = 1$ or $y = z$. Hence $y = z$ which complete the proof of the uniqueness. □

Corollary 3.2. *Let $(X,M,*)$ be a complete fuzzy metric space, and let ϕ be an altering distance function and $\psi : [0,\infty) \to [0,\infty)$ be a continuous function. Let F be α and β_k-admissible mappings such that*

$$\alpha(x,F(x))\alpha(y,F(y)) \geq 1,$$

implies that

$$\alpha(x,F(x))\alpha(y,F(y)) \int_0^{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)} \varphi(s)ds \leq \beta(x,t)\beta(y,t)N(x,y,t),$$

where

$$N(x,y,t) = \int_0^{\psi\left(\frac{1}{M(x,y,t)}-1\right)} \varphi(s)ds - \int_0^{\phi\left(\frac{1}{M(x,y,t)}-1\right)} \varphi(s)ds,$$

for every $x, y \in X$, $x \neq y$ and $t \in (0,\infty)$. Suppose that the following conditions hold:

(a) *Let $\varphi : [0,\infty] \to [0,\infty]$ be a Lebesgue-integrable mapping which is summable on each compact subset of $[0,\infty)$, non negative and for each $\epsilon > 0$,*

$$\int_0^{1-\epsilon} \varphi(s)ds > 0,$$

(b) *there exists $x_0 \in X$ such that $\alpha(x_0,F(x_0)) \geq 1$ and $\beta(x_0,t) \leq \sqrt{k(t)}$ for all $t > 0$,*

(c) *if $\{x_n\}$ is a sequence such that $(x_n,x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, and $x_n \to x$ as $n \to \infty$, then $\alpha(x,F(x)) \geq 1$.*

Then F has a unique fixed point on $z \in X$, such that $\alpha(z,F(z)) \geq 1$ and $\beta(x,t) < 1$ for all $x \in X$ and $t > 0$.

On the other hand, by taking $\alpha(x,y)=1$, $\psi(t)=t$ *and* $\phi(t)=0$ *if and only if* $t=0$ *in Corollary* 3.2, *we infer the version of Theorem* 3.1 *in* [7].

We furnish an example to certify our Theorem 3.1.

Example 3.3. Let $X=\left\{\frac{1}{n}: n\in\mathbb{N}\right\}\cup\{0\}$ with fuzzy metric defined by $M(x,y,t)=\frac{1}{1+|x-y|}$ for all $x,y\in X$ and $t>0$. Define a map $F:X\to X$

$$F(x)=\begin{cases}\frac{1}{n+1} & \text{if } x=\frac{1}{n}, n\in\mathbb{N}\\ 0 & \text{if } x=0.\end{cases}$$

Then F is a integral fuzzy contraction with $\varphi(t)=t^{\frac{1}{t}-2}[1-\log t]$ and define functions $\alpha: X\times X\to[0,\infty)$, $\beta: X\times(0,\infty)\to[0,\infty)$, by

$$\alpha(x,y)=1 \ \text{ if } \ x=[0,1] \quad \text{and} \quad \beta(x,t)=\frac{1}{\sqrt{3}} \ \text{ if } \ x\in[0,1].$$

Also, define $\psi(t)=t$, $\phi(t)=\frac{t}{2}$.

Proof. Now, we show that F is an α-admissible mapping. Let $x,y\in X$ with

$$\alpha(x,y)\geq 1,$$

then $x,y\in[0,1]$. On the other hand, for all $x,y\in[0,1]$, and we have $F(x)=\frac{1}{n+1}\leq 1$. It follows that

$$\alpha(F(x),F(y))\geq 1.$$

Hence, F is an α-admissible mapping. In the above arguments, $\alpha(0,F(0))\geq 1$. Let $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1})\geq 1$ for all $n\in\mathbb{N}\cup\{0\}$ and $x_n\to x$ as $n\to\infty$, then $\{x_n\}\in[0,1]$. This implies that $\alpha(x_n,x)\geq 1$ for all $n\in\mathbb{N}$.
Next, we show that F is β_k admissible mapping. Let $x\in X$ with $\beta(x,t)=\frac{1}{\sqrt{3}}\leq\sqrt{k(t)}$ for all $t>0$, and $F(x)=\frac{1}{n+1}<1$, then $x,y\in[0,1]$ and we have

$$\beta(F(x),t)=\frac{1}{\sqrt{3}}\leq\sqrt{k(t)}.$$

So, F is β_k admissible mapping. We will check that the contractive condition of Theorem 3.1 is fulfilled for $x,y\in X$ with $F(x)=\frac{1}{n+1}$ and $F(y)=\frac{1}{m+1}$, we get $\alpha(x,F(x))\alpha(y,F(y))\geq 1$. Here

$$\int_0^u\varphi(t)dt=u^{\frac{1}{u}}.$$

Now, we consider the following two cases:

Case I: Let $m,n\in\mathbb{N}$ with $n<m$ and let $x=\frac{1}{n}$, $y=\frac{1}{m}$, then we have

$$\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)^{\frac{1}{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)}}=\left|\frac{1}{n+1}-\frac{1}{m+1}\right|^{\left(\frac{1}{\left|\frac{1}{n+1}-\frac{1}{m+1}\right|}\right)}=\left(\frac{m-n}{(n+1)(m+1)}\right)^{\left(\frac{(n+1)(m+1)}{m-n}\right)}.$$

On the other hand

$$N\left(\frac{1}{n},\frac{1}{m},t\right)=\psi\left(\frac{1}{M\left(\frac{1}{n},\frac{1}{m},t\right)}-1\right)^{\frac{1}{\psi\left(\frac{1}{M\left(\frac{1}{n},\frac{1}{m},t\right)}-1\right)}}-\phi\left(\frac{1}{M\left(\frac{1}{n},\frac{1}{m},t\right)}-1\right)^{\frac{1}{\phi\left(\frac{1}{M\left(\frac{1}{n},\frac{1}{m},t\right)}-1\right)}}$$
$$=\left(\left|\frac{1}{n}-\frac{1}{m}\right|\right)^{\left(\frac{1}{\left|\frac{1}{n}-\frac{1}{m}\right|}\right)}-\left(\frac{1}{2}\left|\frac{1}{n}-\frac{1}{m}\right|\right)^{\frac{1}{\frac{1}{2}\left|\frac{1}{n}-\frac{1}{m}\right|}}$$
$$=\left(\frac{m-n}{nm}\right)^{\left(\frac{nm}{m-n}\right)}-\frac{1}{2}\left(\frac{m-n}{nm}\right)^{\frac{2nm}{m-n}}.$$

Now, since $nm<(n+1)(m+1)$ and $\dfrac{nm}{(m-n)}>0$ and $\dfrac{2nm}{(m-n)}>0$. We have

$$\left[\left(\frac{nm}{(n+1)(m+1)}\right)^{\left(\frac{nm}{(m-n)}\right)}-\frac{1}{2}\left(\frac{m-n}{nm}\right)^{\frac{2nm}{m-n}}\right]\le 1,$$

since for all $m,n\in\mathbb{N}$, we have $m\le 3n+nm+1$, and so $3(m-n)\le(n+1)(m+1)$, so

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{(n+1)(m+1)}{m-n}}\le\left(\frac{m-n}{nm}\right)^{\left(\frac{nm}{m-n}\right)}-\frac{1}{2}\left(\frac{m-n}{nm}\right)^{\frac{2nm}{m-n}}$$
$$\le\frac{1}{3}\left[\left(\frac{m-n}{nm}\right)^{\left(\frac{nm}{m-n}\right)}-\frac{1}{2}\left(\frac{m-n}{nm}\right)^{\frac{2nm}{m-n}}\right]\tag{11}$$

or

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{(n+m+1)}{m-n}}\left[\left(\frac{nm}{(n+1)(m+1)}\right)^{\left(\frac{nm}{(m-n)}\right)}-\frac{1}{2}\left(\frac{nm}{(n+1)(m+1)}\right)^{\left(\frac{2nm}{(m-n)}\right)}\right]\le\frac{1}{3}$$

or

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{(n+m+1)}{m-n}}\le\frac{1}{3}.$$

Case II: On the other hand, taking $x=\dfrac{1}{n}$ and $y=0$. For each $n\in\mathbb{N}$, we have

$$\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)^{\frac{1}{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)}}=\left(\frac{1}{M(Fx,Fy,t)}-1\right)^{\left(\frac{1}{\left(\frac{1}{M(Fx,Fy,t)}-1\right)}\right)}$$
$$=\left[\frac{1}{n+1}\right]^{(n+1)}$$
$$\le\left(\frac{1}{n}\right)^{n}-\frac{1}{2}\left(\frac{1}{n}\right)^{2n}$$
$$\le\frac{1}{3}\left(\left(\frac{1}{n}\right)^{n}-\frac{1}{2}\left(\frac{1}{n}\right)^{2n}\right)$$
$$\le\frac{1}{3}\left[\psi\left(\frac{1}{M(x,y,t)}-1\right)^{\psi\left(\frac{1}{M(x,y,t)}-1\right)}-\phi\left(\frac{1}{M(x,y,t)}-1\right)^{\phi\left(\frac{1}{M(x,y,t)}-1\right)}\right]\tag{12}$$

or

$$\int_0^{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)}\varphi(t)dt\le\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[\int_0^{\psi\left(\frac{1}{M(x,y,t)}-1\right)}\varphi(t)dt-\int_0^{\phi\left(\frac{1}{M(x,y,t)}-1\right)}\varphi(t)dt\right].$$

Therefore, equation (11) and equation (12) shows that F is integral fuzzy contraction and has unique fixed point 0, such mapping F satisfies condition with $\varphi(t)$, $\phi(t)$ and $\psi(t)$ for $t>0$

but

$$\sup_{\{x,y\in X|x\neq y\}} \frac{\frac{1}{M(Tx,Ty,t)}-1}{\frac{1}{M(x,y,t)}-1}=1.$$

Thus, it is not a fuzzy contraction. □

4. Figures and Table

Figure 1 and 2 manifest comparison of values of L.H.S. with R.H.S. of Example 3.3 Case I and Case II, respectively. Table 1 reveals comparison of values of L.H.S. with R.H.S. of Example 3.3.

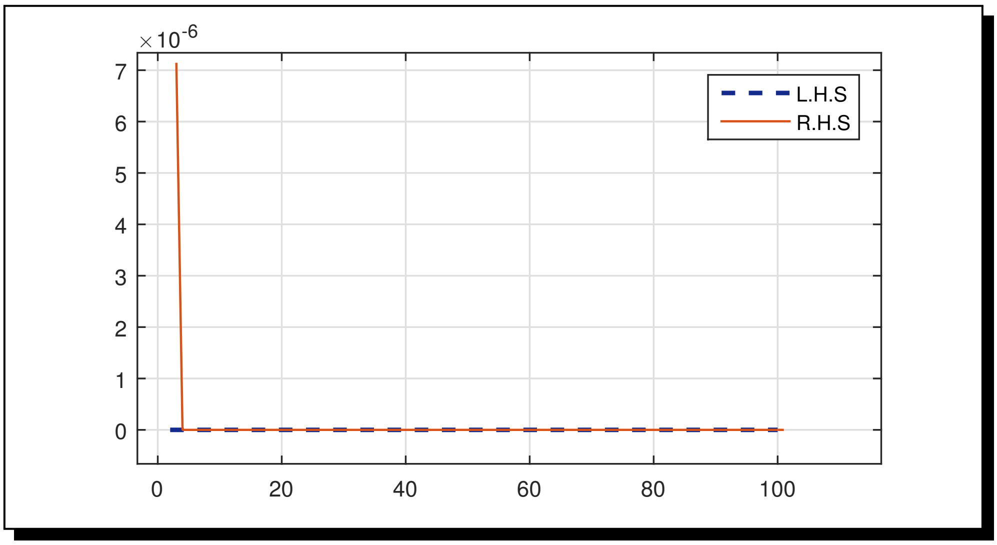

Figure 1

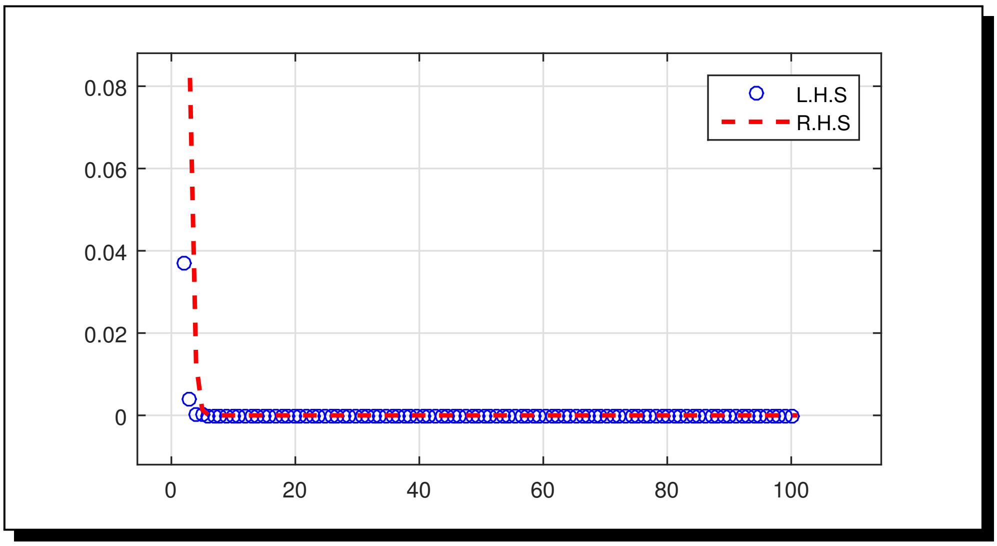

Figure 2

Table 1

Value of n	Value of m	L.H.S ≤ R.H.S. (Case I)	L.H.S ≤ R.H.S. (Case II)
2	3	1.1216e-13≤ 7.1445e-06	0.0370≤0.0820
	1000	0.0363≤ 0.0815	0.0370≤0.0820
20	50	3.7132e-56≤5.7578e-52	1.7116e-28≤3.1789e-27
	100	1.8229e-38 ≤3.7530e-36	1.7116e-28 ≤ 3.1789e-27
100	500	1.2001e-266≤ 2.5652e-263	3.6605e-203≤3.3333e-201
⋮	⋮	⋮	⋮

5. Some Fixed Point Theorems

Now, we furnish our second result.

Theorem 5.1. *Let $(X,M,*)$ be a complete fuzzy metric space and let ϕ be an altering distance function and $\phi:[0,\infty)\to[0,\infty)$ be a continuous function with $\phi(t)<t$ for each $t>0$. Let F be α and β_k-admissible mappings such that*

$$\alpha(x,F(x))\alpha(y,F(y))\geq 1, \tag{13}$$

implies that

$$\int_0^{\left(\frac{1}{M(Fx,Fy,t)}-1\right)}\varphi(s)ds\leq\beta(x,t)\beta(y,t)N(x,y,t), \tag{14}$$

where

$$N(x,y,t)=\int_0^{\phi\left(\max\left\{\frac{1}{M(x,Fx,t)}-1\right),\left(\frac{1}{M(y,Fy,t)}-1\right),\left(\frac{1}{M(x,y,t)}-1\right\}\right)}\varphi(s)ds \tag{15}$$

for every $x,y\in X$ and $t\in(0,\infty)$. Suppose that the following conditions hold:

(a) *Let $\varphi:[0,\infty]\to[0,\infty]$ be a Lebesgue-integrable mapping which is summable on each compact subset of $[0,\infty)$, non negative and for each $\epsilon>0$,*

$$\int_0^{1-\epsilon}\varphi(s)ds>0, \tag{16}$$

(b) *there exists $x_0\in X$ such that $\alpha(x_0,F(x_0))\geq 1$ and $\beta(x_0,t)\leq\sqrt{k(t)}$ for all $t>0$,*

(c) *if $\{x_n\}$ is a sequence such that $(x_n,x_{n+1})\geq 1$ for all $n\in\mathbb{N}$, and $x_n\to x$ as $n\to\infty$, then $\alpha(x,F(x))\geq 1$.*

Then F has a unique fixed point on $z\in X$, such that $\alpha(z,F(z))\geq 1$ and $\beta(x,t)<1$ for all $x\in X$ and $t>0$.

Proof. By Theorem 3.1, assume that $x_n\neq x_{n+1}$. Now take $x=x_{n-1}$ and $y=x_n$ in equation (12) and (13)

$$\int_0^{\left(\frac{1}{M(Fx_{n-1},Fx_n,t)}-1\right)}\varphi(s)ds\leq\beta(x_{n-1},t)\beta(x_n,t)N(x_{n-1},x_n,t), \tag{17}$$

where

$$N(x_{n-1},x_n,t)=\int_0^{\phi\left(\max\left\{\frac{1}{M(x_{n-1},Fx_{n-1},t)}-1\right),\left(\frac{1}{M(x_n,Fx_n,t)}-1\right),\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\right\}\right)}\varphi(s)ds$$

$$=\int_0^{\phi\left(\max\left\{\frac{1}{M(x_{n-1},x_n,t)}-1\right),\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right),\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\right\}\right)}\varphi(s)ds$$

$$=\int_0^{\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)}\varphi(s)ds.$$

Thus

$$\int_0^{\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)}\varphi(s)ds\le k(t)\int_0^{\phi\left\{\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\right\}}\varphi(s)ds$$

$$<\int_0^{\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)}\varphi(s)ds$$

a contradiction. Therefore, $x_n=x_{n+1}=z$ is a common fixed point of F. □

We provide an example to certify our Theorem 5.1.

Example 5.2. Let $X=\left\{\frac{1}{n}:n\in\mathbb{N}\right\}\cup\{0\}$ with fuzzy metric defined by $M(x,y,t)=\dfrac{1}{1+|x-y|}$ for all $x,y\in X$ and $t>0$. Define a map $F:X\to X$

$$F(x)=\begin{cases}\dfrac{1}{n+1} & \text{if } x=\dfrac{1}{n},\ n\in\mathbb{N}\\ 0 & \text{if } x=0.\end{cases}$$

Then F is a integral fuzzy contraction with $\varphi(t)=t^{\frac{1}{t}-2}[1-\log t]$ and define the function $\alpha:X\times X\to[0,\infty),\beta:X\times(0,\infty)\to[0,\infty)$, by

$$\alpha(x,y)=1\ \text{ if }\ x=[0,1]\quad\text{and}\quad\beta(x,t)=\frac{1}{\sqrt{3}}\ \text{ if }\ x\in[0,1].$$

Also, $\phi:[0,\infty)\to[0,\infty)$ be a continuous function with $\phi(t)=t$ for each $t>0$.

Proof. As shown in Example 3.3, F is an α and β_k admissible mapping. Here

$$\int_0^u\varphi(t)dt=u^{\frac{1}{u}}.$$

Now, we consider the following two cases.

Case I: Let $m,n\in\mathbb{N}$ with $n<m$ and let $x=\frac{1}{n}$, $y=\frac{1}{m}$, then we have

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\left(\frac{(n+1)(m+1)}{m-n}\right)}\le\left(\phi\left(\frac{m-n}{mn}\right)\right)^{\frac{1}{\left(\phi\left(\frac{m-n}{mn}\right)\right)}}$$

$$\le\left(\frac{m-n}{mn}\right)^{\frac{mn}{m-n}}. \tag{18}$$

Since $nm<(n+1)(m+1)$ and $\dfrac{nm}{(m-n)}>0$, we have $\left(\dfrac{nm}{(n+1)(m+1)}\right)^{\frac{(nm)}{m-n}}\le 1$. In addition to, since for all $m,n\in\mathbb{N}$, we have $m\le 3n+nm+1$, and so $3(m-n)\le(n+1)(m+1)$, we have

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{(n+m+1)}{m-n}}\left(\frac{nm}{(n+1)(m+1)}\right)^{\frac{(nm)}{m-n}}\le\frac{1}{3}$$

or

$$\left(\frac{m-n}{(n+1)(m+1)}\right)^{\frac{(n+m+1)}{m-n}} \le \frac{1}{3}.$$

Case II: On the other hand, taking $x = n$ and $y = 0$. For each $n \in \mathbb{N}$, we have

$$\begin{aligned}\left(\frac{1}{M(Fx,Fy,t)}-1\right)^{\frac{1}{\left(\frac{1}{M(Fx,Fy,t)}-1\right)}} &= \left(\frac{1}{n+1}\right)^{n+1}\\ &\le \phi\left(\max\left\{\frac{1}{n(n+1)},0,\frac{1}{n}\right\}\right)^{\frac{1}{\phi\left(\max\left(\frac{1}{n(n+1)},0,\frac{1}{n}\right)\right)}}\\ &\le \phi\left(\frac{1}{n}\right)^{\frac{1}{\phi\left(\frac{1}{n}\right)}}\\ &\le \left(\frac{1}{n}\right)^{n}\end{aligned}$$

or

$$\int_0^{\psi\left(\frac{1}{M(Fx,Fy,t)}-1\right)} \varphi(s)ds \le \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[\int_0^{\phi\left(\max\left\{\left(\frac{1}{M(x,Fx,t)}-1\right),\left(\frac{1}{M(y,Fy,t)}-1\right),\left(\frac{1}{M(x,y,t)}-1\right)\right\}\right)} \varphi(s)ds\right]. \tag{19}$$

Therefore, equation (18) and equation (19) shows that F is integral fuzzy contraction and has unique fixed point 0, such mapping F satisfies Theorem 5.1 but it is not a fuzzy contraction. □

We present our last result as follows.

Theorem 5.3. *Let $(X,M,*)$ be a complete fuzzy metric space and let ϕ be an altering distance function and $\phi : [0,\infty) \to [0,\infty)$ be a continuous function such that $\phi(t) = t$ if and only if $t = 0$. Let F be α and β_k-admissible mappings such that*

$$\alpha(x,F(x))\alpha(y,F(y)) \ge 1, \tag{20}$$

implies that

$$\int_0^{\phi\left(\frac{1}{M(Fx,Fy,t)}-1\right)} \varphi(s)ds \le \beta(x,t)\beta(y,t)N(x,y,t), \tag{21}$$

where

$$N(x,y,t) = \int_0^{\psi\left(\max\left\{\phi\left(\frac{1}{M(x,Fx,t)}-1\right),\phi\left(\frac{1}{M(y,Fy,t)}-1\right),\phi\left(\frac{1}{M(x,y,t)}-1\right)\right\}\right)} \varphi(s)ds \tag{22}$$

for every $x,y \in X$ and $t \in (0,\infty)$, where $\psi : [0,\infty) \to [0,\infty)$ such that $\psi(t) < t$ for each $t > 0$. Suppose that the following conditions hold:

(a) *Let $\varphi : [0,\infty] \to [0,\infty]$ be a Lebesgue-integrable mapping which is summable on each compact subset of $[0,\infty)$, non negative and for each $\epsilon > 0$,*

$$\int_0^{1-\epsilon} \varphi(s)ds > 0, \tag{23}$$

(b) *there exists $x_0 \in X$ such that $\alpha(x_0,F(x_0)) \ge 1$ and $\beta(x_0,t) \le \sqrt{k(t)}$ for all $t > 0$,*

(c) *if $\{x_n\}$ is a sequence such that $(x_n,x_{n+1}) \ge 1$ for all $n \in \mathbb{N}$, and $x_n \to x$ as $n \to \infty$, then $\alpha(x,F(x)) \ge 1$.*

Then F has a unique fixed point on $z \in X$, such that $\alpha(z,F(z)) \geq 1$ and $\beta(x,t) < 1$ for all $x \in X$ and $t > 0$.

Proof. By Theorem 3.1, assume that $x_n \neq x_{n+1}$. Now take $x = x_{n-1}$ and $y = x_n$ in equation (17) and (18)

$$\int_0^{\phi\left(\frac{1}{M(Fx_{n-1},Fx_n,t)}-1\right)} \varphi(s)ds \leq \beta(x_{n-1},t)\beta(x_n,t)N(x_{n-1},x_n,t), \tag{24}$$

where

$$N(x_{n-1},x_n,t) = \int_0^{\psi\left(\max\left\{\phi\left(\frac{1}{M(x_{n-1},Fx_{n-1},t)}-1\right),\phi\left(\frac{1}{M(x_n,Fx_n,t)}-1\right),\phi\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\right\}\right)} \varphi(s)ds$$
$$= \int_0^{\psi\left(\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\right)} \varphi(s)ds$$

we have

$$\int_0^{\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds \leq k(t)\int_0^{\psi\left(\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\right)} \varphi(s)ds$$
$$< \int_0^{\phi\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)} \varphi(s)ds,$$

a contradiction. Therefore $\phi\left(\dfrac{1}{M(x_n,x_{n+1},t)}-1\right) = 0$, which implies that $\dfrac{1}{M(x_n,x_{n+1},t)} - 1 = 0$, i.e. $M(x_n,x_{n+1},t) = 1$. Hence $x_n = x_{n+1} = z$ is a common fixed point of F. □

Example 5.4. Let $X = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ with fuzzy metric defined by $M(x,y,t) = \dfrac{1}{1+|x-y|}$ for all $x,y \in X$ and $t > 0$. Define a map $F : X \to X$

$$F(x) = \begin{cases} \dfrac{1}{n+1} & \text{if } x = \dfrac{1}{n},\ n \in \mathbb{N} \\ 0 & \text{if } x = 0. \end{cases}$$

Then F is a integral fuzzy contraction with $\varphi(t) = t^{\frac{1}{t}-2}[1-\log t]$ and define the function $\alpha : X \times X \to [0,\infty)$, $\beta : X \times (0,\infty) \to [0,\infty)$, by

$$\alpha(x,y) = 1 \ \text{ if } \ x = [0,1] \quad \text{and} \quad \beta(x,t) = \frac{1}{\sqrt{3}} \ \text{ if } \ x \in [0,1].$$

Also, define $\psi(t) = t$ for each $t > 0$.

Proof. As shown in Example 3.3, F is an α and β_k admissible mapping. Here

$$\int_0^u \varphi(s)ds = u^{\frac{1}{u}}$$

Now we consider the following two cases.

Case I: Let $m,n \in \mathbb{N}$ with $n < m$ and let $x = \dfrac{1}{n}$, $y = \dfrac{1}{m}$,. Thus, we have

$$\left(\phi\left(\frac{m-n}{(n+1)(m+1)}\right)\right)^{\phi\left(\frac{(n+1)(m+1)}{m-n}\right)} \leq \left(\psi\left(\phi\left(\frac{m-n}{mn}\right)\right)\right)^{\frac{1}{\psi\left(\phi\left(\frac{m-n}{mn}\right)\right)}}$$
$$\leq \frac{1}{3}\left(\phi\left(\frac{m-n}{mn}\right)\right)^{\phi\left(\frac{mn}{(m-n)}\right)} \tag{25}$$

or

$$\left(\phi\left(\frac{m-n}{(n+1)(m+1)}\right)\right)^{\phi\left(\frac{(n+m+1)}{m-n}\right)} \le \frac{1}{3}.$$

Case II: On the other hand, taking $x=\frac{1}{n}$ and $y=0$. For each $n\in\mathbb{N}$, we have

$$\begin{aligned}\left(\phi\left(\frac{1}{M(Fx,Fy,t)}-1\right)\right)^{\frac{1}{\left(\frac{1}{M(Fx,Fy,t)}-1\right)}} &= \phi\left(\frac{1}{n+1}\right)^{\phi(n+1)}\\ &\le \psi\left(\phi\left(\frac{1}{n}\right)\right)^{\frac{1}{\psi\left(\phi\left(\frac{1}{n}\right)\right)}}\\ &\le \frac{1}{3}(\phi(n))^{\frac{1}{(\phi(n))}}\end{aligned}$$

or

$$\int_0^{\phi\left(\frac{1}{M(Fx,Fy,t)}-1\right)}\varphi(s)ds \le \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[\int_0^{\psi\left(\max\left\{\phi\left(\frac{1}{M(x,Fx,t)}-1\right),\phi\left(\frac{1}{M(y,Fy,t)}-1\right),\phi\left(\frac{1}{M(x,y,t)}-1\right)\right\}\right)}\varphi(s)ds\right]. \tag{26}$$

Therefore, equation (25) and equation (26) shows that F is integral fuzzy contraction and has unique fixed point '0', such mapping F satisfies all the conditions of Theorem 5.3 but it is not a fuzzy contraction. □

6. Conclusions

We prove the existence and uniqueness of fixed points theorems by introducing modified (α-β_k,ϕ-ψ) integral type contraction mappings in fuzzy metric spaces. Our investigation and results obtained are supported by some suitable examples with graphs and table, which provides new path for researchers in the concerned field.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] S. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations integrals, *Fundamenta Mathematicae* **3** (1922), 133 – 181, URL: http://matwbn.icm.edu.pl/ksiazki/or/or2/or215.pdf.

[2] A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, *International Journal of Mathematics and Mathematical Sciences* **29**(9) (2002), 531 – 536, DOI: 10.1155/S0161171202007524.

[3] T. Dosenovic, D. Rakic and M. Brdara, Fixed point theorem in fuzzy metric spaces using altering distance, *Filomat* **28**(7) (2014), 1517 – 1524, DOI: 10.2298/FIL1407517D.

[4] A. George and P. Veeramani, On some results in fuzzy metric spaces, *Fuzzy Sets and Systems* **64** (1994), 395 – 399, DOI: 10.1016/0165-0114(94)90162-7.

[5] M. Grabiec, Fixed points in fuzzy metric spaces, *Fuzzy Sets and Systems* **27** (1988), 385 – 389, DOI: 10.1016/0165-0114(88)90064-4.

[6] V. Gregori and A. Sapena, On fixed-point theorems in fuzzy metric spaces, *Fuzzy Sets and Systems* **125** (2002), 245 – 252, DOI: 10.1016/S0165-0114(00)00088-9.

[7] S. Hussain and M. Samreen, A fixed point theorem satisfying integral type contraction in fuzzy metric space, *Results in Fixed Point and Applications*, (2018), DOI: 10.30697/rfpta-2018-013.

[8] M. S. Khan, M. Swaleh and S. Sessa, Fixed point theorems by altering distance between the points, *Bulletin of the Australian Mathematical Society* **30** (1984), 1 – 9, DOI: 10.1017/S0004972700001659.

[9] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, *Kybernetika* **11** (1975), 326 – 334, URL: `http://dml.cz/dmlcz/125556`.

[10] D. Mihet, A Banach contraction theorem in fuzzy metric spaces, *Fuzzy Sets and Systems* **144**(3) (2004), 431 – 439, DOI: 10.1016/S0165-0114(03)00305-1.

[11] S. Phiangsungnoen, P. Thounthong and P. Kumam, Fixed point results in fuzzy metric spaces via α and β_k-admissible mappings with application to integral types *Journal of Intelligent and Fuzzy Systems* **34** (2018), 467 – 475, DOI: 10.3233/JIFS-17350.

[12] P. Salimi, C. Vetro and P. Vetro, Some new fixed point results in non-Archimedean fuzzy metric spaces, *Nonlinear Analysis Modelling and Control* **18**(3) (2013), 344 – 358, DOI: 10.15388/NA.18.3.14014.

[13] B. Samet, C. Vetro and P. Vetro, Fixed point theorems for α-ψ-contractive type mappings, *Nonlinear Analysis* **75** (2012), 2154 – 2165, DOI: 10.1016/j.na.2011.10.014.

[14] Y. Shen, D. Qiu and W. Chen, Fixed point theorems in fuzzy metric spaces, *Applied Mathematics Letters* **25** (2012), 138 – 141, DOI: 10.1016/j.aml.2011.08.002.

[15] L. A. Zadeh, Fuzzy sets, *Inform and Control* **8** (1965), 338 – 353, DOI: 10.1016/S0019-9958(65)90241-X.