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Research Article

Linking R&D Spillovers to Market Structure and Its Impact on Equilibrium Results

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Abstract. We develop an R&D network model by linking R&D spillovers to the degree of substitution of goods. Then, we examine the impact of the new model on the equilibrium outcomes under Cournot competition. The results show that the importance of the spillover restriction appears if the substitution degree of goods is small where in this case, the outcomes are maximized under the restricted spillover. The results also show that as the degree of replacement increases, the structure of socially optimal cooperation under restricted spillover effects is more intense than in the case of free spillover effects.

Keywords. R&D cooperation; R&D Spillover; Market structure

MSC. 91A30; 91A43; 91A80

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1. Introduction

The incentive of companies to invest in R&D lies on reducing the production cost and improving industrial technology ([2, 3, 5, 7]). The investment in R&D is accompanied with existence of R&D spillover, which is defined as an external parameter that allows competitors to take advantage of investor efforts without paying. One way to overcome this problem is to encourage investors to cooperate in R&D that may enhance the level of the economy such as adjustment of outputs and production cost and raise individual and social gains ([6, 8, 14, 15]).

R&D cooperation can be represented through a network of nodes representing companies and links representing R&D relationships ([8, 9, 20, 21]). The formation and development of the cooperation network depend on mutual benefits between cooperating companies. In 2001, Goyal and Moraga-Gonzalez [8] developed a network model structured as a three-stage oligopoly game. In the first stage, companies choose their positions in the R&D network by choosing their partners in R&D. In the second stage, companies choose their R&D level of expenditure. In the third stage, companies decide their output (Cournot competition) in order to maximize their profits.

The main objective of this paper is to develop the R&D network model of Goyal and Moraga-Gonzalez by restricting the R&D spillovers to the market structure. In their model, the spillovers are set free from the structure of network and market. In this paper, we assume that the R&D spillovers are sensitives to the substitution degree of goods; in particular, if the degree is high, the advantage of the R&D spillover decreases and vice versa. Our model differs from many developed models that have linked external parameters to the network formation (e.g., [13, 16, 20]). The contribution of this paper is to study the decision of companies to choose their competitors to cooperate in R&D and its impact on the company level variables (investment, quantity of production, profit, and total welfare).

The results of the paper depend on the size of the market and on the economic variable. In the case of two companies in the market, restricting the spillover to the degree of substitution does not encourage companies to invest in R&D. For other economic variables, the results take a different pattern. When the spillover is limited to the substitution degree, the results are high; especially if the R&D spillover is high.

In the case of three companies in the market, the results are highly sensitive to the degree of substitution of goods. If the degree is small, the investment in R&D under the influence of restricted spillover is high for cooperating companies. As the degree of substitution increases, the investment of companies in R&D is high if the spillover is free from the market structure. For the output and profit, the limitation of the spillover improves the results if the substitution degree is small. If the degree of substitution is large, the results of the cooperating companies will be high under the influence of the free R&D spillover. Although profit is affected by the condition of the spillover, the R&D network that ensures cooperating all companies is profitable in an individual perspective. In the social perspective, the results are affected by the value of the substitution degree. If the degree is small, the total welfare under restricted and free spillovers is consistent. However, with increasing the substitution degree, the structure of socially optimal cooperation under restricted spillover effects is more intense than in the case of free spillover effects.

The paper proceeds as follows. In Section 2, we review some issues related to economics and graph theory. In Section 3, we present our results. In Section 4, we conclude the paper.

2. Background

2.1 The Model

In this paper, we focus on the linear-quadratic function of consumers given by [11]:

$$U = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\beta \sum_{i=1}^{n} q_i^2 + 2\delta \sum_{j \neq i} q_i q_j\right) + I\,. \tag{2.1}$$

The demand parameters $\alpha > 0$ denotes the willingness of consumers to pay and $\alpha > 0$ is the diminishing marginal rate of consumption. To simplify the analysis, we assumed that $\beta = 1$. The parameter q_i is the quantity consumed of good i and I measures the consumer's consumption of another product. The parameter $\delta \in [-1,1]$ captures the marginal rate of differentiation between different goods.

■ **Payoffs.** Let m be a consumer's income. If p_i is the price of good i produced by company i, the money spent to consume q_i of that good is $p_i q_i$ and the balance is $I = m - p_i q_i$. By substituting into (2.1) and calculating $\frac{\partial U}{\partial q_i} = 0$, we determine the optimal consumption of good i:

$$\alpha - q_i - \delta \sum_{j \neq i} q_j - p_i = 0 \;\Rightarrow\; p_i = \alpha - q_i - \delta \sum_{j \neq i} q_j, \quad i = 1, \dots, n.$$

If c_i is the cost of producing good i, the profit of the company i is

$$\pi_i = (p_i - c_i) q_i = \left(\alpha - q_i - \delta \sum_{j \neq i}^{n} q_j - c_i \right) q_i, \tag{2.2}$$

Total welfare is the sum of the industry surplus and the consumer surplus:

$$TW = \underbrace{\frac{1-\delta}{2} \sum_{i=1}^{n} q_i^2 + \frac{\delta}{2} \left(\sum_{i=1}^{n} q_i \right)^2}_{CS} + \underbrace{\sum_{i=1}^{n} \pi_i}_{\Pi}. \tag{2.3}$$

■ **Cost Reduction.** The effective amount of investment in R&D per company is a combination of individual expenditures and other companies' expenditures on the market [6]. The benefit from the expenditure of other companies depends on an external parameter called R&D spillover that captures knowledge flow of non-cooperation companies. In the case of two companies in the industry, the effective investment of the company i is defined as follows:

$$S_i = s_i + \phi s_j, \tag{2.4}$$

where s_i is the amount of investment of the company i in R&D and $\phi \in [0,1)$ is the R&D spillover. The effective investment reduces the marginal production cost of company i. If c_0 is the marginal cost, then the cost function becomes

$$c_i = c_0 - S_i = c_0 - s_i - \phi s_j. \tag{2.5}$$

2.2 Network

A **network**, which is referred to as a graph, is a set of objects (called nodes or vertices) that are connected together by edges or links [17]. Let N be a set of all vertices labeled by numbers or letters $N = \{i, j, k, \dots\}$ and $E = \{ij, jk, \dots\}$ be a set of all edges in the network. Let $\mathcal{G}^n$ be a set of all distinct networks generated from n nodes. Then, $G(N,E) \in \mathcal{G}^n$ refers to a network with nodes N and links E. For simplicity, the network is denoted by G and we assume that each link in the network joins two different vertices and serve both sides (i.e., undirected networks).

A set of **neighbors** of node i consists of all nodes that are linked to it: $N_i = \{j \in N : ij \in E\}$. The length of the neighbors' set of node i is used to refer to the **degree** of that node i.e., for each node $i \in N$, $deg(i) = |N_i|$ where $0 \le deg(i) \le n-1$. Thus, if $|N| = n$ is the number of nodes and $|E| = m$ is the number of links, the density of the network G is $D = 2m/n(n-1)$ where $0 \le D \le 1$.

R&D Network Model. The R&D partnerships between companies can be defined as a network where the companies are represented by nodes and the cooperation by links. We assume that the R&D agreement between any two companies requires the consent and full participation of both companies. This means within the network, each link between any two companies serves both sides. In the network game, we follow Goyal and Moraga-Gonzalez model. Their model consists of three stages as follows:

The first stage: Companies choose their partners in R&D. At the end of this stage, the cooperation network G will be constructed and companies will identify their locations in that network. In practice, the network $G \in \mathscr{G}^n$ is captured by a symmetric $n \times n$ adjacency matrix $A = (a_{ij})$ where $a_{ij} \in \{0,1\}$. If $a_{ij} = 1$, companies i and j are linked (i.e., they cooperate in R&D), and $a_{ij} = 0$ otherwise.

The second stage: Companies choose their amounts of investment (effort) in R&D simultaneously and independently in order to reduce the cost of production. In this stage, we have two models:

Model A. The original model given by Goyal and Moraga-Gonzalez (2001).

In their model, the R&D spillover does not depend on the market structure or on the network architecture. Thus, the effective investment in R&D for each company i in the network G is

$$S_i = s_i + \sum_{j \in N_i} s_j + \phi \sum_{k \notin N_i} s_k, \quad i = 1, \dots, n, \tag{2.6}$$

where s_i denotes R&D investment of company i, N_i is the set of companies participating in a joint venture with company i and $\phi \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from companies not engaged in R&D with company i.

Model B. Linking the spillover to the market structure.

We assume that the spillover varies with the differentiation degree of goods.

$$S_i = s_i + \sum_{j \in N_i} s_j + \Phi \sum_{k \notin N_i} s_k, \quad i = 1, \dots, n, \tag{2.7}$$

$$\Phi = \begin{cases} \phi^{\delta} & : \ \delta \in (0,1]; \\ \phi & : \ \delta \le 0. \end{cases} \tag{2.8}$$

In the new model, the R&D spillover is restricted if goods are substitutes. For independent and complementary goods, the spillover cannot be restricted since the results will not be logical. For example, if we assume that $\delta = -0.1$, then the values of ϕ^{δ} for $\phi > 0$ exceed the original value of the spillover which means that companies in the market benefit from the knowledge flow more than creative companies.

In words, the new model indicates that in a competitive market, the benefit from the knowledge flow between competitors relies on the substitution degree in the sense that as the degree increases, the benefit decreases. Figure 1 shows the opposite relationship between the substitution degree and the spillover in Model B.

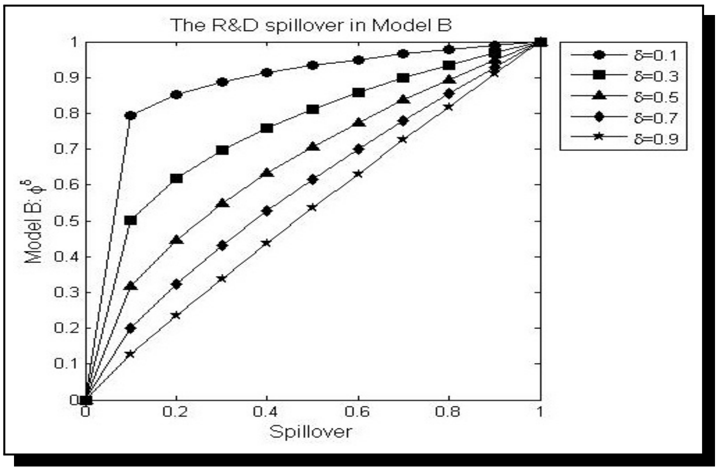

Figure 1. The relationship between the substitution degree and the spillover in Model B

Assume that the marginal cost for company i is constant c_0. The effective R&D investment is cost reducing in the sense that it reduces company i's marginal cost of production:

$$c_i = c_0 - S_i, \tag{2.9}$$

where the effective investment S_i depends on the network structure and the model used.

The third stage: Companies compete in the product market by setting quantities (Cournot competition). At this stage, companies choose their levels of production in order to maximize their profits.

The investment in R&D is assumed to be costly and the function of the cost is quadratic. Thus, if the company i invest $s_i \in [0, c_0]$, the cost of R&D is $C(s_i) = \mu s_i^2$, where $\mu > 0$ refers to the effectiveness of R&D expenditure [6]. From this, the profit function (2.2) becomes

$$\pi_i = (p_i - c_i)q_i - C(s_i) = \left(\alpha - c_0 - q_i - \beta \sum_{j \neq i}^{n} q_j + S_i\right) q_i - C(s_i), \tag{2.10}$$

where the marginal cost satisfies $\alpha > c_0$.

■ **Stability and efficiency of R&D networks.** The pairwise stability depends on companies' profit functions [13]. It examines the individual incentives in forming and developing the cooperation network. Meaning that when the cooperation network becomes stable, companies do not have incentives to form or delete links.

Definition 1 (Pairwise Stability)**.** A network $G \in \mathscr{G}^n$ is stable if the following two conditions are satisfied for any two companies $i, j \in G$:

(1) If $ij \in G$, $\pi_i(G) \geq \pi_i(G - ij)$ and $\pi_j(G) \geq \pi_j(G - ij)$,

(2) If $ij \notin G$ and if $\pi_i(G) < \pi_i(G + ij)$, then $\pi_j(G) > \pi_j(G + ij)$.

The network $G - ij$ is resulting from deleting the link ij from the network G and the network $G + ij$ is resulting from adding the link ij to the network G.

The efficiency of the cooperation network depends on the total welfare function. It examines the social benefit in forming and developing the cooperation network in the sense that the total welfare in the efficient network is the maximum.

Definition 2 (Network Efficiency)**.** A network $G \in \mathscr{G}^n$ is efficient if $TW(G) > TW(G')$ for all $G' \in \mathscr{G}^n$.

2.3 Nash Equilibria

We assume that the marginal cost is constant and equal for all companies. By using backwards induction, we identify the sub-game perfect Nash equilibrium. From the profit function (2.10), we find the best response function of quantity for good i by calculating $\partial\pi_i/\partial q_i = 0$:

$$q_i = \frac{(\alpha - c_0) + S_i - \delta \sum_{j\neq i} q_j}{2}. \tag{2.11}$$

By substituting the best response functions into each other, we have the symmetric equilibrium output for each good i

$$q_i^* = \frac{(2-\delta)(\alpha - c_0) + (2+(n-2)\delta)S_i + \delta \sum_{j\neq i} S_j}{(2-\delta)((n-1)\delta + 2)}. \tag{2.12}$$

To find the symmetric equilibrium profit, we substitute the equilibrium output (2.12) into the profit function (2.10) which gives

$$\pi_i^* = \left[\frac{(2-\delta)(\alpha - c_0) + (2+(n-2)\delta)S_i + \delta \sum_{j\neq i} S_j}{(2-\delta)((n-1)\delta + 2)}\right]^2 - C(s_i). \tag{2.13}$$

For convenience, the profit function can be expressed in the following form

$$\pi_i^* = {q_i^*}^2 - C(s_i). \tag{2.14}$$

Now the final list of the equilibria depends on the network structure. By knowing the structure, we have the effective investment of each company i. By substituting into the profit function (2.13) and calculating $\partial\pi_i^*/\partial s_i = 0$, we have the best response function of the R&D investment for each company i. By plugging the best response functions into each other, we have the symmetric equilibrium investment s_i^*. Then, we use the backwards induction to have the final equilibria. In Appendix, we provide the final equilibrium equations for the investment and output and for the profit and the total welfare, we need to submit the results into equations (2.14) and (2.3), respectively.

3. Prior Economic Models in R&D Cooperation

The investment and cooperation of companies in R&D have attracted considerable attention from both theoretical and empirical perspectives. This indicates the importance of this subject to the continuity and development of the company in the market. In this section, we provide a brief view of the theoretical studies that focused on cooperation of companies in R&D and its role on other economic features.

D'Aspremont and Jacquemin [6] were among the first to discuss the issue of cooperation of companies in R&D. Under homogeneous Cournot duopoly, they introduced a two-stage game where first, companies choose their investment in R&D, then compete in the product market. In their model, the effective investment of R&D for each company is defined as an individual expenditure in R&D in addition to expenditure of another company where the latter benefit depends on the rate of knowledge spillover (eq. (2.4)). The main finding in the D'Aspremont and Jacquemin paper is that if the spillover exceeded the moderate values, the investment of companies in cooperation case is higher than in non-cooperation case. Also, the quantities of production and profits of companies are higher in cooperation case.

Kamien *et al*. [14] extended the D'Aspremont and Jacquemin's paper by considering an arbitrary number of companies instead of two companies. Also, they studied differentiated products in both cases, quantity competition (Cournot competition) and price competition (Bertrand competition)[1]. The most important extension of the D'Aspremont and Jacquemin model was due to Goyal and Moraga-Gonzalez [8] who presented R&D cooperation as a network. The concept of the author did not affect the basic model of the cooperation of companies in R&D, but they defined R&D agreements as links connecting cooperating companies. They introduced the concept of the network to provide better advantages to some extent simulates the cooperation of companies in R&D in reality. The effective contribution of Goyal and Moraga-Gonzalez is to study the impact of linkages on economic variables and to define the conditions required to determine the profitable structures in individual and social perspectives. For homogeneous goods. They found that the two perspectives are never consistent, because individual interest is always greater with cooperative links; while the maximum social benefits are obtained within a low-density network.

4. The Results

In our study, we consider two cases in terms of the market size. In the first case, we assume that there are two companies where the R&D cooperation decision is represented as the interaction of the two companies. In the second case, we assume that there are three companies in the market where the cooperation is represented as a network.

4.1 Two-Player Game

In this section, we represent the equilibrium outcomes for two companies invest in R&D and compete by setting their output. The two companies have two options as shown in Figure 2. In the first option, the two companies decided to cooperate in R&D represented by G_1. In the second option, companies separately invest in R&D (G_2). In the latter option, there is an R&D spillover where its process is followed either Model A or Model B. We examine the effect of linking the spillover to the market structure on equilibrium outcomes by comparing the results of the two models.

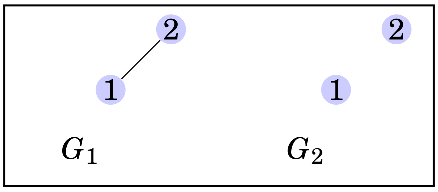

Figure 2. Cooperation cases in R&D for two companies in the market. In the network G_1, the two companies cooperate in R&D; while in the network G_2, they invest separately

Figure 3 displays the equilibrium outcomes under Models A and B for cooperation and non-cooperation case. When focusing on corporate investments in R&D, we found that cooperation

[1]The original work by D'Aspremont and Jacquemin was extended by many authors (e.g., [12, 19, 22]).

always reduces the amount of investment. In the non-cooperation case, the restriction of the spillover is a negative factor on the R&D investment; meaning that with increasing the spillover, the amount of investment under Model B is less than that under Model A. The behavior of other economic variables (production quantity, profit and total welfare) takes the same pattern for all spillover values, but they are high in the cooperation case. When comparing those variables under the two models in the non-cooperation case, we find that spillover restriction always produces higher results.

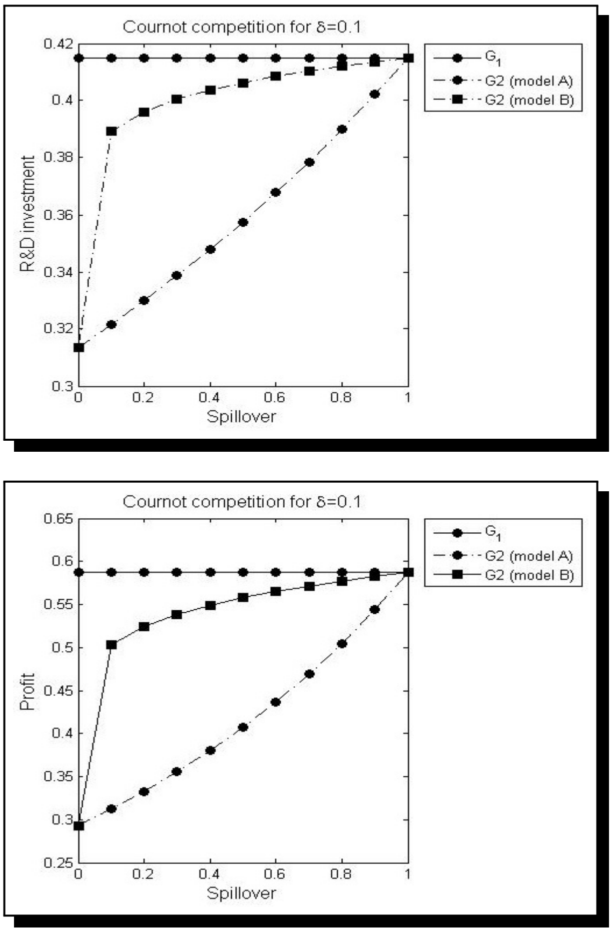

Figure 3. The equilibrium outcomes of the two cases given in Figure 2 under Models A and B. The parameters used to plot the results are $\alpha = 2$, $c_0 = 1$ and $\mu = 1$

4.2 Three-Player Game

In this section, we discuss the equilibrium results of three companies that invest in R&D and compete by determining their output. When the size of the market increases by one company, the possible cooperation structures will increase. For three companies, R&D collaboration can be represented by one of the four networks listed in Figure 4. The first network G_1 is called a complete network such that each two companies in that network are linked. The second network G_2 is the star network, characterized by a company in the center of the network linked to the other two companies called peripheral companies. The third network G_3 is a partial network such that any two companies are linked and the third company stays isolated. The fourth network G_4 is called an empty network that contains companies without links between them.

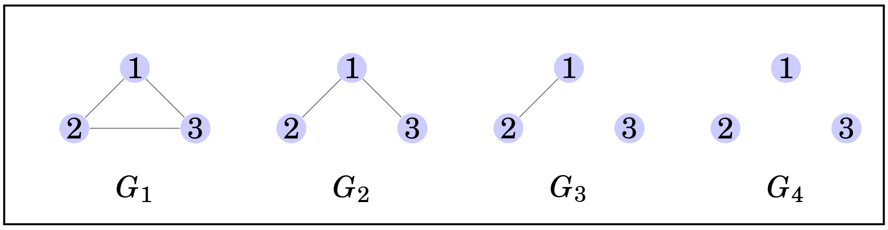

Figure 4. Cooperation cases in R&D for three companies in the market. There are four distinct networks presented in order: a complete network, a star network, a partial network and an empty network

According to Model B, the R&D spillover is not affected by the size or structure of the collaboration network. But it is influenced by the structure of the market so that if the competition or substitution degree of goods increases, the spillover decreases (see Figure 1).

Figure 5 displays the equilibrium outcomes for the four cooperation networks given in Figure 4 under Models A and B. The comparison between the equilibrium outcomes of the two models depends on the degree of substitution between the goods. If the degree is large, the amount of investment in R&D under model B is smaller than that under Model A. But when the degree of substitution is small, the opposite occurs to linked companies like company 1 in networks G_2 and G_3. This result indicates that the restriction of the spillover to the market structure clearly affects the role of the R&D spillover on the R&D investment by companies. Goyal and Moraga-Gonzalez [8] showed that the R&D investment decreases with increasing the spillover for homogeneous goods (perfect substitutes). Therefore, when limiting the spillover between companies by restricting it, the investment in R&D would be better than that when the spillover is free.

For the quantity of production and profit, if the degree of substitution is small, the results will be higher under Model B. If the degree of substitution is high, the results of cooperating companies in R&D under Model B are smaller than those under Model A. This points out that restriction of the spillover improves the production and profit if the substitution degree is small. With increasing the degree, the cooperating companies do not prefer restricted R&D spillover because the advantages will be confined to non-cooperating companies. Although profits are influenced by the type of the model used, the stability of the R&D network under the two models is consistent, as the complete network (G_1) is always preferred by all companies.

For the efficiency of the R&D network, if the substitution degree is small, the results under the two models are consistent. However, with increasing the substitution degree, the density of the efficient network is small if the spillover is restricted compared to the density of the efficient network when the spillover is free. Tables 1 and 2 compare between Models A and B in terms of the efficiency of the R&D networks given in Figure 4.

Table 1. The efficiency of the networks given in Figure 4 under Models A and B

Substitution degree	$0.1 \le \delta \le 0.5$		$\delta = 0.6$		$\delta = 0.7$	
	Network	Spillover	Network	Spillover	Network	Spillover
Model A	G_1	$0 \le \phi < 1$	G_1	$\phi < 0.7$	G_1	$\phi \le 0.1$
			G_2	$0.7 \le \phi < 0.9$	G_2	$0.1 < \phi < 0.6$
			G_3	$0.9 \le \phi \le 1$	G_3	$0.6 \le \phi < 0.8$
					G_4	$0.8 \le \phi \le 1$
Model B	G_1	$0 \le \phi < 1$	G_1	$\phi < 0.5$	G_1	$\phi < 0.1$
			G_2	$0.5 \le \phi < 0.8$	G_2	$0.1 \le \phi < 0.5$
			G_3	$0.8 \le \phi < 0.9$	G_3	$0.5 \le \phi < 0.7$
			G_3	$0.9 \le \phi \le 1$	G_4	$0.7 \le \phi \le 1$

Table 2. The density of the networks given in Figure 4

Network	G_1	G_2	G_3	G_4
D	1	2/3	1/3	0

5. Conclusion

In this paper, we developed an R&D model for companies participate in R&D and compete in production quantity. The aim is to introduce a new model that links the spread of knowledge to the market structure and then to examine the impact of this constraint on economic outcomes.

The results showed that the importance of the spillover restriction appears when the substitution degree of goods is small. For two companies in the market, the results showed that restricting the R&D spillover always improves the quantity of production, profit, and total welfare. For three companies, the results are sensitive to the degree of substitution between the goods. The role of limiting the spillover on the results of equilibrium is shown if the degree of substitution is small. However, with increasing the substitution degree, the results are maximized if the spillover is independent of the market structure. In terms of the stability of the R&D network, we found that linking the spillover to the market does not change the individual preference. In terms of the efficiency, we noted that with increasing the substitution degree, the optimal cooperation structure under restricted spillover effects is more intense than in the case of free spillover.

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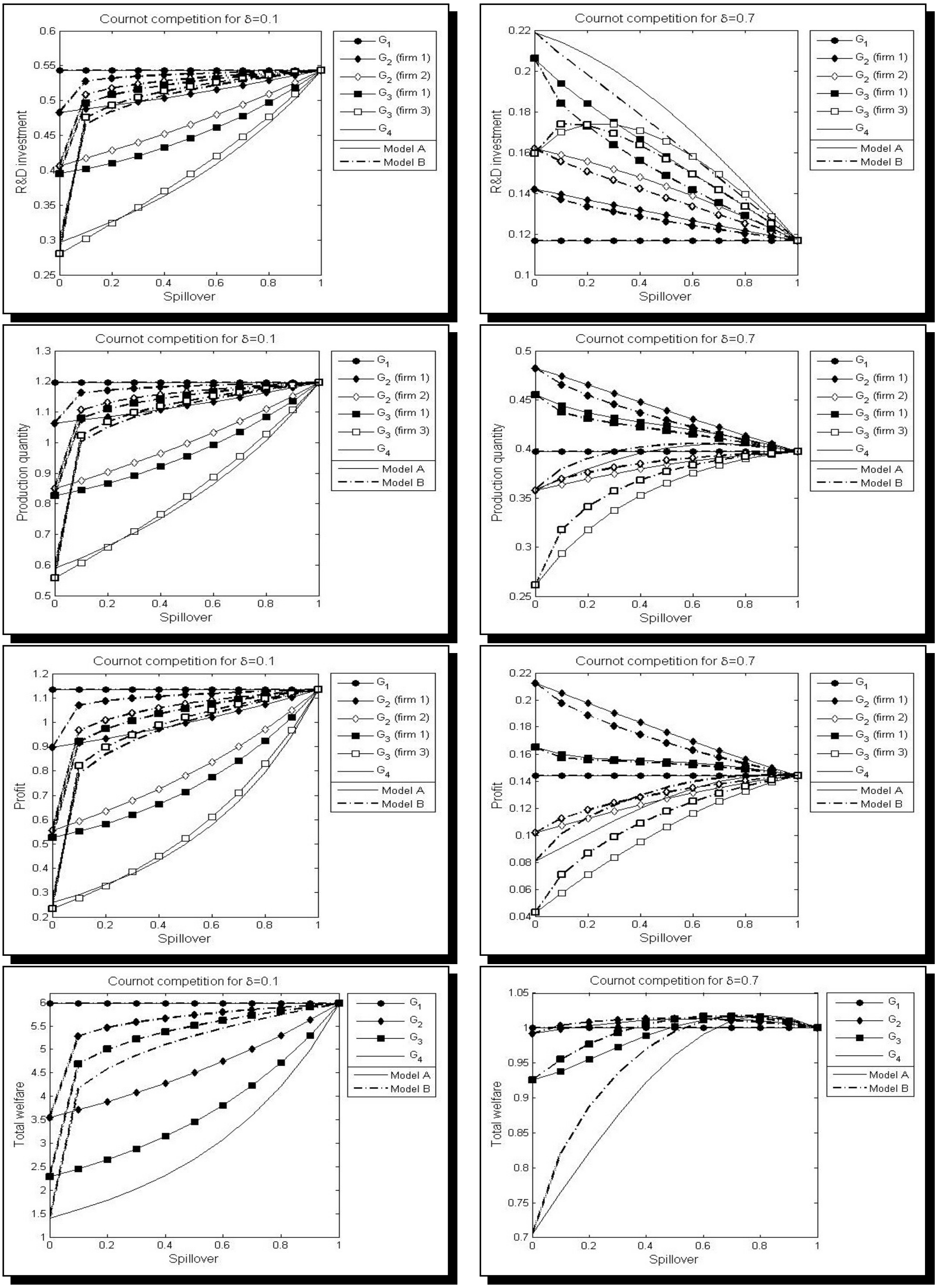

Figure 5. The equilibrium outcomes of the four cooperation networks given in Figure 4 under Models A and B. The parameters used to plot the results are $\alpha = 2$, $c_0 = 1$ and $\mu = 1$

Appendix: Nash Equilibria

Model A:

1. Two-Player Game:

■ Cooperation case:

$$s_{G_1}=\frac{(\alpha-c_0)}{\mu(2+\delta)^2-2}, \tag{A.1a}$$

$$q_{G_1}=\frac{\mu(2+\delta)(\alpha-c_0)}{\mu(2+\delta)^2-2}. \tag{A.1b}$$

■ Non-cooperation case:

$$s_{G_2}=\frac{(\alpha-c_0)(2-\delta\beta)}{\mu(2+\delta)^2(2-\delta)-(1+\beta)(2-\delta\beta)}, \tag{A.2a}$$

$$q_{G_2}=\frac{\mu(4-\delta^2)(\alpha-c_0)}{\mu(2+\delta)^2(2-\delta)-(1+\beta)(2-\delta\beta)}. \tag{A.2b}$$

2. Three-Player Game:

$$s_{G_1}=\frac{(\alpha-c_0)}{((4\delta^2+8\delta+4)\mu-3)}, \tag{A.3a}$$

$$q_{G_1}=\frac{(2\mu(\delta+1)(\alpha-c_0))}{((4\delta^2+8\delta+4)\mu-3)}\ , \tag{A.3b}$$

$$s_{G_2}(\text{company }1)=\frac{(\alpha-c_0)(\beta^2\delta-\beta\delta-2\beta+2\mu\delta^3-6\mu\delta^2+8\mu+2)}{8\mu^2\delta^5-8\mu^2\delta^4-S_1\delta^3+S_2\delta^2+S_3\delta+2(16\mu^2-4(\beta+2)\mu+\beta-1)}, \tag{A.4a}$$

$$q_{G_2}(\text{company }2)=\frac{(2\mu(\alpha-c_0)(\delta+1)(\beta^2\delta-\beta\delta-2\beta+2\mu\delta^3-6\mu\delta^2+8\mu+2))}{8\mu^2\delta^5-8\mu^2\delta^4-S_1\delta^3+S_2\delta^2+S_3\delta+2(16\mu^2-4(\beta+2)\mu+\beta-1)}, \tag{A.4b}$$

$$s_{G_2}(\text{company }1)=\frac{(2\mu(\beta\delta-2)(\delta+1)(\delta-2))(\alpha-c_0)}{8\mu^2\delta^5-8\mu^2\delta^4-S_1\delta^3+S_2\delta^2+S_3\delta+2(16\mu^2-4(\beta+2)\mu+\beta-1)}, \tag{A.4c}$$

$$q_{G_2}(\text{company }2)=\frac{4\mu^2(\alpha-c_0)(\delta-\delta^2+2)^2}{8\mu^2\delta^5-8\mu^2\delta^4-S_1\delta^3+S_2\delta^2+S_3\delta+2(16\mu^2-4(\beta+2)\mu+\beta-1)}\ , \tag{A.4d}$$

$$s_{G_3}(\text{company }1)=\frac{(\beta\delta-2)(\alpha-c_0)(2\beta^2\delta-3\beta\delta-2\beta-2\mu\delta^3+6\mu\delta^2+\delta-8\mu+2)}{2(-4\mu^2\delta^6+12\mu^2\delta^5+S_4\delta^4+S_5\delta^3+S_6\delta^2+S_7\delta+4(8\mu^2-6\mu-\beta^2+1))}, \tag{A.5a}$$

$$q_{G_3}(\text{company }1)=\frac{(\mu(\alpha-c_0)(\delta-\delta^2+2)(3\beta\delta-2\beta^2\delta+2\beta+2\mu\delta^3-6\mu\delta^2-\delta+8\mu-2))}{-4\mu^2\delta^6+12\mu^2\delta^5+S_4\delta^4+S_5\delta^3+S_6\delta^2+S_7\delta+4(8\mu^2-6\mu-\beta^2+1)}, \tag{A.5b}$$

$$s_{G_3}(\text{company }3)=\frac{(\alpha-c_0)(\delta-2\beta\delta+2)(\beta\delta-\beta^2\delta+2\beta+\mu\delta^3-3\mu\delta^2+4\mu-2)}{-4\mu^2\delta^6+12\mu^2\delta^5+S_4\delta^4+S_5\delta^3+S_6\delta^2+S_7\delta+4(8\mu^2-6\mu-\beta^2+1)}, \tag{A.5c}$$

$$q_{G_3}(\text{company }3)=\frac{(2\mu(\alpha-c_0)(\delta-\delta^2+2)(\beta\delta-\beta^2\delta+2\beta+\mu\delta^3-3\mu\delta^2+4\mu-2))}{-4\mu^2\delta^6+12\mu^2\delta^5+S_4\delta^4+S_5\delta^3+S_6\delta^2+S_7\delta+4(8\mu^2-6\mu-\beta^2+1)}\ , \tag{A.5d}$$

where

$$S_1=2(20\mu^2+(2\phi+1)\mu),$$

$$S_2=2(4\mu^2+(2\phi^2+7)\mu),$$

$$S_3 = 64\mu^2 + 4\phi(\phi-1)(4\mu-1),$$
$$S_4 = 12\mu^2 + (6\phi^2 - 4\phi + 1)\mu,$$
$$S_5 = -44\mu^2 - (6\phi^2 + 12\phi - 3)\mu,$$
$$S_6 = (6 + 24\phi - 12\phi^2)\mu - 24\mu^2 - \phi(\phi^2-1)(2\phi-1),$$
$$S_7 = 2(\phi(3\phi^2 - \phi - 3) + 24\mu^2 - (10 - 16\phi)\mu + 1).$$

$$s_{G_4} = \frac{(\alpha - c_0)(\delta(2\beta-1)-2)}{2 + 4\beta - 8\mu + (1 - 12\mu - 4\beta^2)\delta + 4\mu\delta^3}, \tag{A.6a}$$
$$q_{G_4} = \frac{(2\mu(\alpha - c_0)(\delta^2 - \delta - 2))}{2 + 4\beta - 8\mu + (1 - 12\mu - 4\beta^2)\delta + 4\mu\delta^3}, \tag{A.6b}$$

Model B:

1. Two-Player Game:

■ Cooperation case:

$$s_{G_1} = \frac{(\alpha - c_0)}{\mu(2+\delta)^2 - 2}, \tag{B.1a}$$
$$q_{G_1} = \frac{\mu(2+\delta)(\alpha - c_0)}{\mu(2+\delta)^2 - 2}. \tag{B.1b}$$

■ Non-cooperation case:

$$s_{G_2} = \frac{-((\phi^\delta\delta - 2)(\alpha - c_0))}{(8\mu - 2\phi^\delta + 4\mu\delta + \phi^{2\delta}\delta - 2\mu\delta^2 - \mu\delta^3 + \phi^\delta\delta - 2)}, \tag{B.2a}$$
$$q_{G_2} = \frac{-(\mu(\delta^2 - 4)(\alpha - c_0))}{(8\mu - 2\phi^\delta + 4\mu\delta + \phi^{2\delta}\delta - 2\mu\delta^2 - \mu\delta^3 + \phi^\delta\delta - 2)}. \tag{B.2b}$$

2. Three-Player Game:

$$s_{G_1} = \frac{(\alpha - c_0)}{((4\delta^2 + 8\delta + 4)\mu - 3)}, \tag{B.3a}$$
$$q_{G_1} = \frac{(2\mu(\delta+1)(\alpha - c_0))}{((4\delta^2 + 8\delta + 4)\mu - 3)}, \tag{B.3b}$$

$$s_{G_2}(\text{company } 1) = \frac{-((\alpha - c_0)(8\mu - 2\phi^\delta + \phi^{2\delta}\delta - 6\mu\delta^2 + 2\mu\delta^3 - \phi^\delta\delta + 2))}{16\mu - 32\mu^2 + \delta Y_1 + \delta^2 Y_2 + \delta^3 Y_3 + 8\mu^2\delta^4 - 8\mu^2\delta^5 + \phi^\delta(8\mu - 2) + 2}, \tag{B.4a}$$
$$q_{G_2}(\text{company } 1) = \frac{-(2\mu(\alpha - c_0)(\delta+1)(8\mu - 2\phi^\delta + \phi^{2\delta}\delta - 6\mu\delta^2 + 2\mu\delta^3 - \phi^\delta\delta + 2))}{16\mu - 32\mu^2 + \delta Y_1 + \delta^2 Y_2 + \delta^3 Y_3 + 8\mu^2\delta^4 - 8\mu^2\delta^5 + \phi^\delta(8\mu - 2) + 2}, \tag{B.4b}$$
$$s_{G_2}(\text{company } 2) = \frac{-(2\mu(\phi^\delta\delta - 2)(\alpha - c_0)(\delta+1)(\delta-2))}{16\mu - 32\mu^2 + \delta Y_1 + \delta^2 Y_2 + \delta^3 Y_3 + 8\mu^2\delta^4 - 8\mu^2\delta^5 + \phi^\delta(8\mu - 2) + 2}, \tag{B.4c}$$
$$q_{G_2}(\text{company } 2) = \frac{-(4\mu^2(\alpha - c_0)(-\delta^2 + \delta + 2)^2)}{16\mu - 32\mu^2 + \delta Y_1 + \delta^2 Y_2 + \delta^3 Y_3 + 8\mu^2\delta^4 - 8\mu^2\delta^5 + \phi^\delta(8\mu - 2) + 2}. \tag{B.4d}$$

where

$$Y_1 = \phi^{2\delta} - \phi^\delta + 4\phi^\delta\mu - 4\phi^{2\delta}\mu - 64\mu^2,$$

$$Y_2 = -14\mu - 8\mu^2 - 4\phi^{2\delta}\mu,$$
$$Y_3 = 2\mu + 40\mu^2 + 4\phi^{\delta}\mu.$$

$$s_{G_3}(\text{company } 1) = \frac{-((\phi^\delta\delta-2)(\alpha-c_0)(8\mu-\delta+2\phi^\delta-2\phi^{2\delta}\delta-6\mu\delta^2+2\mu\delta^3+3\phi^\delta\delta-2))}{(2(-24\mu-4\phi^{2\delta}+\delta Y_4+\delta^2Y_5+\delta^3Y_6+\delta^4Y_7+32\mu^2+12\mu^2\delta^5-4\mu^2\delta^6+4))}, \tag{B.5a}$$

$$q_{G_3}(\text{company } 1) = \frac{(\mu(\alpha-c_0)(-\delta^2+\delta+2)(8\mu-\delta+2\phi^\delta-2\phi^{2\delta}\delta-6\mu\delta^2+2\mu\delta^3+3\phi^\delta\delta-2))}{(-24\mu-4\phi^{2\delta}+\delta Y_4+\delta^2Y_5+\delta^3Y_6+\delta^4Y_7+32\mu^2+12\mu^2\delta^5-4\mu^2\delta^6+4)}, \tag{B.5b}$$

$$s_{G_3}(\text{company } 3) = \frac{((\alpha-c_0)(\delta-2\phi^\delta\delta+2)(4\mu+2\phi^\delta-\phi^{2\delta}\delta-3\mu\delta^2+\mu\delta^3+\phi^\delta\delta-2))}{(-24\mu-4\phi^{2\delta}+\delta Y_4+\delta^2Y_5+\delta^3Y_6+\delta^4Y_7+32\mu^2+12\mu^2\delta^5-4\mu^2\delta^6+4)}, \tag{B.5c}$$

$$q_{G_3}(\text{company } 3) = \frac{(2\mu(\alpha-c_0)(-\delta^2+\delta+2)(4\mu+2\phi^\delta-\phi^{2\delta}\delta-3\mu\delta^2+\mu\delta^3+\phi^\delta\delta-2))}{(-24\mu-4\phi^{2\delta}+\delta Y_4+\delta^2Y_5+\delta^3Y_6+\delta^4Y_7+32\mu^2+12\mu^2\delta^5-4\mu^2\delta^6+4)}. \tag{B.5d}$$

where

$$Y_4 = (2-20\mu-2\phi^{2\delta}+6\phi^{3\delta}+48\mu^2-6\phi^\delta+32\phi^\delta\mu),$$
$$Y_5 = (-\phi^\delta+6\mu+2\phi^{2\delta}+\phi^{3\delta}-2\phi^{4\delta}-24\mu^2-12\phi^{2\delta}\mu+24\phi^\delta\mu),$$
$$Y_6 = (3\mu-44\mu^2-6\phi^{2\delta}\mu-12\phi^\delta\mu)$$
$$Y_7 = (\mu+12\mu^2+6\phi^{2\delta}\mu-4\phi^\delta\mu).$$

$$s_{G_4} = \frac{-((\alpha-c_0)(\delta-2\phi^\delta\delta+2))}{(\delta-8\mu+4\phi^\delta-12\mu\delta-4\phi^{2\delta}\delta+4\mu\delta^3+2)}, \tag{B.6a}$$

$$q_{G_4} = \frac{-(2\mu(\alpha-c_0)(-\delta^2+\delta+2))}{(\delta-8\mu+4\phi^\delta-12\mu\delta-4\phi^{2\delta}\delta+4\mu\delta^3+2)}. \tag{B.6b}$$

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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