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Research Article

Intuitionistic Fuzzy Soft Near Rings Induced by (T,S)-norms

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Abstract. In this work, we combine the near ring concept and intuitionistic fuzzy soft set. By using t-norm and t-conorm we define intuitionistic fuzzy soft near ring over a given classical near ring and study its characteristic properties.

Keywords. Intuitionistic fuzzy set; Soft set; Intuitionistic fuzzy soft set; Soft near ring; Intuitionistic fuzzy soft near ring

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1. Introduction

By a soft set [8] we mean a pair (F,E), where E is a set interpreted as the set of parameters and the mapping $F : E \to P(X)$ is referred to as the soft structure on X. In [8] Molodtsov pointed out that fuzzy sets were special types of soft sets by using unit interval $[0,1]$ for parameter set. He showed many applications of his soft set theory in various fields. The concept of soft set draw attention both of specialists working in the field of pure mathematics and applied mathematics. This concept is well coordinated with such modern mathematical concepts as a fuzzy set and more general, a many valued set. After Molodtsov's work, some different applications on soft set were studied. Maji *et al*. [7] combined the advantage of soft set and Atanassov's [2] intuitionistic fuzzy set and presented the concept of intuitionistic fuzzy soft set. Many authors have studied algebraic structure of set theories dealing with uncertainties.

The aim of this study is to introduce a basic version of intuitionistic fuzzy soft near ring theory which extends the concept of soft near rings. We define intuitionistic fuzzy soft near ring using (T,S) norm, (λ,β) level set, intuitionistic fuzzy soft near ideals of near rings and investigate some more characterizations of these structures. Furthermore, we give the theorems of homomorphic image and homomorphic preimage.

2. Preliminaries

In this section as a preparation, we will present the basic definitions and notations.

Throughout this paper, let X be an initial universe and E be a set of parameters for the universe X and let I be the closed unit interval, i.e., $I=[0,1]$. Denote the power set of X by $P(X)$.

Definition 1 ([4])**.** By a near ring we mean a non-empty set N with two binary operations '+' and '$\cdot$' satisfying the following axioms:

(1) $(N,+)$ is a group,

(2) $(N,\cdot)$ is a semigroup,

(3) $x\cdot(y+z)=x\cdot y+x\cdot z$, for all $x,y,z\in N$.

It is a left near ring because it satisfies the left distributive law. We will use the word "*near ring*" instead of "*left near ring*". We denote xy instead of $x\cdot y$. Note that $x0=0$ and $x(-y)=-xy$, but $0x\neq 0$ for $x,y\in N$.

Theorem 1. *A non-empty subset M of a near ring N is a subnear ring of N if and only if $x-y$, $xy\in M$ for all $x,y\in M$.*

Definition 2 ([2])**.** An intuitionistic fuzzy set A is defined by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\},$$

where $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$ denote membership and nonmembership functions, respectively. Therefore, $\mu_A(x)$ and $\nu_A(x)$ are membership and nonmembership degree of each element $x\in X$ to the intuitionistic fuzzy set A and $\mu_A(x)+\nu_A(x)\le 1$ for each $x\in X$.

Let $\mathrm{IF}(X)$ denotes the family of all intuitonistic fuzzy sets on X. If $A,B\in\mathrm{IF}(X)$ and $(A_i)_{i\in J}\subseteq\mathrm{IF}(X)$, then some basic set operation for intuitionistic fuzzy sets are given by Atanassov [2] as follows:

(1) $A\subseteq B\Leftrightarrow\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$, for all $x\in X$,

(2) $A=B\Leftrightarrow A\subseteq B,\ B\subseteq A$,

(3) $\bigcup\limits_{i\in J}A_i=\left\{\left\langle x,\bigvee\limits_{i\in J}\mu_{A_i}(x),\bigwedge\limits_{i\in J}\nu_{A_i}(x)\right\rangle : x\in X\right\}$,

(4) $\bigcap\limits_{i\in J}A_i=\left\{\left\langle x,\bigwedge\limits_{i\in J}\mu_{A_i}(x),\bigvee\limits_{i\in J}\nu_{A_i}(x)\right\rangle : x\in X\right\}$,

(5) $A^c=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$,

(6) $\widetilde{1}=\{\langle x,1,0\rangle : x\in X\}$ and $\widetilde{0}=\{\langle x,0,1\rangle : x\in X\}$,

(7) $\Box A=\{\langle x,\mu_A(x),\overline{\mu}_A(x)\rangle : x\in X\}$, where $\overline{\mu}_A(x)=1-\mu_A(x)$,

(8) $\Diamond A=\{\langle x,\overline{\nu}_A(x),\nu_A(x)\rangle : x\in X\}$, where $\overline{\nu}_A(x)=1-\nu_A(x)$.

Definition 3 ([8])**.** A pair (F,E) is called a soft set over X, where $F:E\to P(X)$ is set-valued function.

In other words, the soft set is a parameterized family of subsets of the set X. Every set $F(e)$, for every $e\in E$, from this family may be considered as the set of e-elements of the soft set (F,E), or considered as the set of e-approximate elements of the soft set. According to this manner, we can view a soft set (F,E) as consisting of collection of approximations:

$$(F,E)=\{F(e):e\in E\}.$$

Example 1. A soft set (F,E) describes the attractiveness of the houses which Mr. A is going to buy.

X = The set of houses under consideration.

E = The set of parameters. Each parameter is a word or sentence.

E = {expensive; beautiful; wooden; cheap; in the green surrounding; modern; in good repair; in bad repair}.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. It is worth noting that the sets $F(\varepsilon)$ may be empty for some $\varepsilon\in E$ [8].

Definition 4 ([1])**.** Let $A\subseteq E$ and (F,A) be a soft set on X.
The set $\mathrm{Supp}(F,A)=\{a\in A:F(a)\neq\emptyset\}$ is called support of the soft set (F,A).

A soft set is called non-null if its support is not equal to the empty set.

Definition 5 ([7])**.** Let X be an initial universe set, E be a set of parameters and $A\subseteq E$. Then (F,A) is called an intuitionistic fuzzy soft set over X, where F is a mapping given by $F:A\to \mathrm{IF}(X)$.

In general, for every $a\in A$, $F(a)$ is an intuitionistic fuzzy set of X and it is called intuitionistic fuzzy value set of parameter a.

Serkan *et al.* [5] gave a new definition of intuitionistic fuzzy soft set as follows:

Definition 6 ([5])**.** An intuitionistic fuzzy soft set (namely, ifs-set) F on the universe X is a mapping from the parameter set E to $\mathrm{IF}(X)$. It can be written a set of ordered pairs as follows:

$$F=\{(e,\{\langle x,\mu_{F(e)}(x),\nu_{F(e)}(x)\rangle:x\in X\}):e\in E\}.$$

If $F(e)=\widetilde{0}$, then the element $(e,F(e))$ is not appeared in F.
That is $F(e)=(\mu_{F(e)},\nu_{F(e)})$ is an ifs-set over X. For simplicity, we denote $\mu_{F(e)}$, $\nu_{F(e)}$ by f_e and f'_e, respectively.
$\mathrm{IFS}(X,E)$ denotes the set of all intuitonistic fuzzy soft sets over X.

Definition 7. Let $F\in \mathrm{IFS}(X,E)$. The set $\mathrm{Supp}F=\{e\in E:F(e)=(f_e,f'_e)\neq\widetilde{0}\}$ is called support of F.
An ifs-set is called non-null if its support is not equal to empty set.

Definition 8 ([5])**.** Let $F,G \in \text{IFS}(X,E)$. Then

(1) (*Inclusion*) $F \sqsubseteq G \Leftrightarrow F(e) \subseteq G(e)$, $\forall\ e \in E$, i.e., $f_e(x) \le g_e(x)$ and $f'_e(x) \ge g'_e(x)$, $x \in X$.

(2) (*Equality*) $F = G \Leftrightarrow F \sqsubseteq G$, $G \sqsubseteq F$.

(3) (*Intersection*) $H = F \sqcap G \Leftrightarrow H(e) = F(e) \cap G(e)$, $\forall\ e \in E$, i.e., $H(e) = \{\langle x, f_e(x) \wedge g_e(x), f'_e(x) \vee g'_e(x)\rangle : x \in X\}$.

(4) (*Union*) $H = F \sqcup G \Leftrightarrow H(E) = F(e) \cup G(e)$, $\forall\ e \in E$, i.e., $H(e) = \{\langle x, f_e(x) \vee g_e(x), f'_e(x) \wedge g'_e(x) : x \in X\}$.

(5) (*Complement*) $H = F^c \Leftrightarrow H(e) = (F(e))^c$, $\forall\ e \in E$, i.e., $H(e) = \{\langle x, f'_e(x), f_e(x) : x \in X\}$.

(6) (*Null ifs-set*) F is called the null ifs-set, denoted by Φ, if $F(e) = \widetilde{0}$ for all $e \in E$.

(7) (*Universal ifs-set*) F is called universal ifs-set, denoted by $\widetilde{X}$, if $F(e) = \widetilde{1}$, for all $e \in E$.

Clearly, $(\widetilde{X})^c = \Phi$ and $\Phi^c = \widetilde{X}$.

Definition 9 ([10])**.** Let $F \in \text{IFS}(X,E)$.

(1) (*necessity operation*) $\Box F = \{\langle x, f_e(x), \overline{f}_e(x)\rangle : x \in X\}$, where $\overline{f}_e(x) = 1 - f_e(x)$.

(2) (*possibility operation*) $\Diamond F = \{\langle x, \overline{f'}_e(x), f'_e(x)\rangle : x \in X\}$, where $\overline{f'}_e(x) = 1 - f'_e(x)$.

Proposition 1 ([5])**.** *Let $(G_i)_{i\in J} \subseteq \text{IFS}(X,E)$ and $F,G \in \text{IFS}(X,E)$. Then*

(1) $F \sqcap \left(\bigsqcup_{i\in J}(G_i)\right) = \bigsqcup_{i\in J}(F \sqcap (G_i))$, $F \sqcup \left(\bigsqcap_{i\in J}(G_i)\right) = \bigsqcap_{i\in J}(F \sqcup (G_i))$,

(2) $\Phi \sqsubseteq F \sqsubseteq \widetilde{X}$,

(3) $\left(\bigsqcap_{i\in J}(G_i)\right)^c = \bigsqcup_{i\in J}(G_i)^c$, $\left(\bigsqcup_{i\in J}(G_i)\right)^c = \bigsqcap_{i\in J}(G_i)^c$,

(4) $F \sqcup F^c = \widetilde{X}$ *and* $(G^c)^c = G$,

(5) *If* $F \sqsubseteq G$, *then* $(G)^c \sqsubseteq (F)^c$.

Definition 10 ([10])**.** Let $\text{IFS}(X,E)$ and $\text{IFS}(Y,K)$ be the families of all intuitonistic fuzzy soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two functions. Then the pair (φ,ψ) is called an intuitionistic fuzzy soft mapping from X to Y and denoted by $(\varphi,\psi) : \text{IFS}(X,E) \to \text{IFS}(Y,K)$.

(1) Let $F \in \text{IFS}(X,E)$, then the image of F under (φ,ψ), denoted by $(\varphi,\psi)(F)$ is the ifs-set over Y, where

$$\varphi(f)_k(y) = \begin{cases} \bigvee\limits_{\varphi(x)=y}\ \bigvee\limits_{\psi(e)=k} f_e(x), & \text{if } x \in \varphi^{-1}(y) \\ 0, & \text{otherwise} \end{cases} \qquad \forall\ k \in \psi(E),\ \forall\ y \in Y$$

and

$$\varphi(f')_k(y) = \begin{cases} \bigwedge\limits_{\varphi(x)=y}\ \bigwedge\limits_{\psi(e)=k} f'_e(x), & \text{if } x \in \varphi^{-1}(y) \\ 1, & \text{otherwise} \end{cases} \qquad \forall\ k \in \psi(E),\ \forall\ y \in Y.$$

(2) Let $G \in \text{IFS}(Y,K)$, then the preimage of G under (φ,ψ), denoted by $(\varphi,\psi)^{-1}(G)$, is the ifs-set over X, where

$$\varphi^{-1}(g_e)(x) = g_{\psi}(e)(\varphi(x)) \quad \forall\ e \in \psi^{-1}(K),\ \ \forall\ x \in X$$

and

$$\varphi^{-1}(g'_e)(x) = g'_{\psi}(e)(\varphi(x)) \quad \forall\ e \in \psi^{-1}(K), \quad \forall\ x \in X.$$

If φ and ψ are injective (surjective) then the fuzzy soft mapping (φ,ψ) is said to be injective (surjective).

Definition 11. If $F,G \in \mathrm{IFS}(X,E)$, then F **AND** G is an ifs-set denoted by $F\tilde{\wedge}G$ and defined as $F\tilde{\wedge}G = H$, where $H : E\times E \to \mathrm{IF}(X)$, $H(e,e^*) = F(e)\cap G(e^*)$, $\forall\ (e,e^*) \in E\times E$.

Definition 12 ([3])**.** A mapping $T : I\times I \to I$ is called a *t-norm* defined on $I\times I$, if the following conditions are satisfied:

(1) $T(a,1) = a$, $\forall\ a \in I$,

(2) $T(a,b) = T(b,a)$, $\forall\ a,b \in I$,

(3) $T(a,T(b,c)) = T(T(a,b),c)$, $\forall\ a,b,c \in I$,

(4) If $b \leqslant c$, then $T(a,b) \leqslant T(a,c)$, $\forall\ a,b,c \in I$.

If $T(a,a) = a$ for all $a \in I$, then T is called *an idempotent t-norm*.
T is an idempotent t-norm if and only if $T = \wedge$, i.e., T is minimum t-norm.

Proposition 2. *Let T be a t-norm. Then the following conditions are satisfied:*

(i) $T(a,0) = 0$, $\forall\ a \in I$.

(ii) $T(a,b) \leqslant a\wedge b$, $\forall\ a,b \in I$.

(iii) $T(a,b)\wedge T(a,c) = T(a,b\wedge c)$, $\forall\ a,b,c \in I$.

(iv) $T(a,b)\vee T(a,c) = T(a,b\vee c)$, $\forall\ a,b,c \in I$.

(v) If T is continuous, then $\bigvee_{j\in J} T(a_j,b) = T(\bigvee_{j\in J} a_j,b)$ for each $\{a_j : j \in J\}$.

Proof. It follows from the definition. □

Definition 13 ([6])**.** A mapping $S : I\times I \to I$ is called a *t-conorm* defined on $I\times I$, if the following conditions are satisfied:

(i) $S(a,0) = a$, $S(1,1) = 1$, $\forall\ a \in I$,

(ii) $S(a,b) = S(b,a)$, $\forall\ a,b \in I$,

(iii) $S(a,S(b,c)) = S(S(a,b),c)$, $\forall\ a,b,c \in I$,

(iv) If $a \leqslant c$, $b \leqslant d$, then $S(a,b) \leqslant S(c,d)$, $\forall\ a,b,c,d \in I$.

S is simply called a conorm.
If $S(a,a) = a$ for all $a \in I$, then S is called *an idempotent t-conorm*.
S is an idempotent t-conorm if and only if $S = \vee$, i.e., S is maximum t-conorm.

Proposition 3. *Let S be a t-conorm. Then the following conditions are satisfied:*

(i) $S(a,1) = 1$, $\forall\ a \in I$.

(ii) $S(a,b) \geqslant a\vee b$, $\forall\ a,b \in I$.

(iii) $S(a,b)\wedge S(a,c) = S(a,b\wedge c)$, $\forall\ a,b,c \in I$.

(iv) $S(a,b)\vee S(a,c) = S(a,b\vee c)$, $\forall\ a,b,c \in I$.

(v) If S is continuous, then $\bigwedge_{j\in J} S(a_j,b) = S\left(\bigwedge_{j\in J} a_j, b\right)$ for each $\{a_j : j\in J\}$.

Proof. It follows from the definition. □

Definition 14 ([9]). A mapping $\eta:[0,1]\to[0,1]$ is called a negation if it satisfies followings for all $x\in[0,1]$:

(1) $\eta(0)=1$, $\eta(1)=0$.

(2) η is non-increasing.

(3) $\eta(\eta(x))=x$.

Remark 1 ([9]). The T-norm and S-conorm are called dual with respect to the negation $\eta(x)=1-x$, if $S(x,y)=\eta(T(\eta(x),\eta(y)))$.

This holds with respect to η if and only if $T(x,y)=\eta(S(\eta(x),\eta(y)))$.

3. Intuionistic Fuzzy Soft Near Rings

Sezgin *et al.* [11] introduced the notion of soft near rings. In this section we introduce the definition of intuitionistic fuzzy soft near rings and give some fundamental properties of them. From now on all intuitionistic fuzzy soft sets are considered over near ring N. T will be an idempotent t-norm and S will be an idempotent t-conorm.

Definition 15 ([11]). Let (F,A) be a non-null soft set over N. Then (F,A) is called a *soft near ring* over N iff $F(a)$ is a subnear-ring of N for all $a\in \mathrm{Supp}(F,A)$.

Definition 16 ([14]). An intuitonistic fuzzy set $A=(\mu_A(x),\nu_A(x))$ on a near ring N is called an intuitionistic fuzzy subnear ring of N if for all $x,y\in N$

(i) $\mu_A(x-y)\geqslant \min\{\mu_A(x),\mu_A(y)\}$ and $\nu_A(x-y)\leqslant \max\{\nu_A(x),\nu_A(y)\}$.

(ii) $\mu_A(xy)\geqslant \min\{\mu_A(x),\mu_A(y)\}$ and $\nu_A(xy)\leqslant \max\{\nu_A(x),\nu_A(y)\}$.

Definition 17. Let F be a non-null intuitionistic fuzzy soft set over N. Then F is called an *intuitionistic fuzzy soft near ring* (simply, ifs near ring) over N iff for each $e\in E$ and $x,y\in N$

(N1) $f_e(x+y)\geqslant T(f_e(x),f_e(y))$ and $(f_e)'(x+y)\leqslant S((f_e)'(x),(f_e)'(y))$;

(N2) $f_e(-x)\geqslant f_e(x)$ and $(f_e)'(-x)\leqslant (f_e)'(x)$;

(N3) $f_e(xy)\geqslant T(f_e(x),f_e(y))$ and $(f_e)'(xy)\leqslant S((f_e)'(x),(f_e)'(y))$.

That is, for each $e\in E$, $F(e)$ is an intuitionistic fuzzy subnear-ring as in [12] and [14].

Example 2. Let $N=\{0,a,b,c\}$ be a non-empty set with two binary operations '+' and '·' defined by:

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Then $(N,+,\cdot)$ is a near ring.

Let $E=\{e_1,e_2\}$ be the set of parameters. Define an intuitionistic fuzzy soft set F over a near ring N by

$$
\begin{array}{llll}
f_{e_1}(0)=0,2, & f_{e_1}(a)=0,2, & f_{e_1}(b)=0,1, & f_{e_1}(c)=0,1, \\
(f_{e_1})'(0)=0,3, & (f_{e_1})'(a)=0,4, & (f_{e_1})'(b)=0,7, & (f_{e_1})'(c)=0,7. \\
f_{e_2}(0)=0,4, & f_{e_2}(a)=0,4, & f_{e_2}(b)=0,3, & f_{e_2}(c)=0,3, \\
(f_{e_2})'(0)=0,4, & (f_{e_2})'(a)=0,5, & (f_{e_2})'(b)=0,7, & (f_{e_2})'(c)=0,7.
\end{array}
$$

Then the intuitionistic fuzzy soft set F is an ifs near ring over a near ring N.

Theorem 2. *Let F be an intuitonistic fuzzy soft set over a near ring N. Then F is an ifs near ring over N if and only if for each $e\in E$ and $x,y\in N$ the followings are satisfied:*

(i) $f_e(x-y)\geqslant T(f_e(x),f_e(y))$ *and* $(f_e)'(x-y)\leqslant S((f_e)'(x),(f_e)'(y))$.

(ii) $f_e(xy)\geqslant T(f_e(x),f_e(y))$ *and* $(f_e)'(xy)\leqslant S((f_e)'(x),(f_e)'(y))$.

Proof. Let F be an ifs near ring over N. Let $e\in E$ and $x,y\in N$. By monotonicity of T, S, we have

$$
\begin{aligned}
f_e(x-y)&=f_e(x+(-y))\\
&\geqslant T(f_e(x),f_e(-y))\\
&\geqslant T(f_e(x),f_e(y))
\end{aligned}
$$

and

$$
\begin{aligned}
(f_e)'(x-y)&=(f_e)'(x+(-y))\\
&\leqslant S((f_e)'(x),(f_e)'(-y))\\
&\leqslant S((f_e)'(x),(f_e)'(y)).
\end{aligned}
$$

Since F is an ifs near ring over N, the second holds.

Conversely, let F be an intuitionitic fuzzy soft set over N and F satisfies the given two conditions, $\forall\ e\in E$

$$f_e(0)=f_e(x-x)\geqslant T(f_e(x),f_e(x))=f_e(x) \qquad \text{(by ((i)))}.$$

Then, $f_e(0)\geqslant f_e(x)$ for all $x\in N$ and

$$(f_e)'(0)=(f_e)'(x-x)\leqslant S((f_e)'(x),(f_e)'(x))=(f_e)'(x) \qquad \text{(by (i))}.$$

Then, $(f_e)'(0)\leqslant (f_e)'(x)$ for all $x\in N$.

$$f_e(-x)=f_e(0-x)\geqslant T(f_e(0),f_e(x))=f_e(x).$$

Hence, $f_e(-x)\geqslant f_e(x)$ and similarly, $(f_e)'(-x)\leqslant (f_e)'(x)$.

Thus, (N2) is satisfied.

$$
\begin{aligned}
f_e(x+y)&=f_e(x-(-y))\\
&\geqslant T(f_e(x),f_e(-y))\\
&\geqslant T(f_e(x),f_e(y))
\end{aligned}
$$

and

$$\begin{aligned}(f_e)'(x+y) &= f(f_e)'(x-(-y))\\ &\leqslant S((f_e)'(x),(f_e)'(-y))\\ &\leqslant S((f_e)'(x),(f_e)'(y)).\end{aligned}$$

Therefore, (N1) is satisfied. □

Theorem 3. *Let F be an intuitonistic fuzzy soft near ring over N. Then:*

(i) *$\Box F$ is an intuitonistic fuzzy soft near ring over N.*

(ii) *$\Diamond F$ an intuitonistic fuzzy soft near ring over N.*

Proof. Let F be an intuitonistic fuzzy soft near ring over N.

(i) For any $x,y\in N$, $e\in E$, we have

$$f_e(x-y)\geqslant T(f_e(x),f_e(y)) \quad \text{and} \quad f_e(xy)\geqslant T(f_e(x),f_e(y)).$$

Now, let consider intuitonistic fuzzy soft set $\Box F$:

$$\begin{aligned}\overline{f}_e(x-y) &= 1-f_e(x-y))\\ &\leqslant 1-T(f_e(x),f_e(y))\\ &= S(1-f_e(x),1-f_e(y))\\ &= S(\overline{f}_e(x),\overline{f}_e(y))\end{aligned}$$

and

$$\begin{aligned}\overline{f}_e(xy) &= 1-f_e(xy))\\ &\leqslant 1-T(f_e(x),f_e(y))\\ &= S(1-f_e(x),1-f_e(y))\\ &= S(\overline{f}_e(x),\overline{f}_e(y)).\end{aligned}$$

Hence, $\Box F$ is an intuitonistic fuzzy soft near ring over N.

(ii) The proof is similar with (i). □

Theorem 4. *Let F and G be two intuitionistic fuzzy soft near rings over N. If $F\sqcap G$ is non-null, then it is an intuitionistic fuzzy soft near ring.*

Proof. Let $F\sqcap G=H$, where $H(e)=F(e)\cap G(e)$ for all $e\in E$. Since H is non-null, there exist $e\in E$ such that $h_e(x)\neq 0$ for some $x\in N$. That is, $h_e(x)=f_e(x)\wedge g_e(x)$ and $(h_e)'(x)=f_e)'(x)\vee(g_e)'(x)$. Since F is an intuitionistic fuzzy soft near rings over N, we have

$$f_e(x-y)\geqslant T(f_e(x),f_e(y)),\ \ (f_e)'(x-y)\leqslant S((f_e)'(x),f_e)'(y))$$

$$f_e(xy)\geqslant T(f_e(x),f_e(y)),\ \ (f_e)'(xy)\leqslant S((f_e)'(x),f_e)'(y))$$

and also we have same properties for fuzzy sets g_e and $(g_e)'$.
Now, consider ifs-set H, for any $x,y\in N$, $e\in E$:

$$\begin{aligned}h_e(x-y) &= f_e(x-y)\wedge g_e(x-y)\\ &\geqslant T(f_e(x),f_e(y))\wedge T(g_e(x),g_e(y))\end{aligned}$$

$$
\begin{aligned}
&= T(f_e(x) \wedge g_e(x), f_e(y) \wedge g_e(y)) \\
&= T((f_e \wedge g_e)(x), (f_e \wedge g_e)(y)) \\
&= T(h_e(x), h_e(y))
\end{aligned}
$$

and

$$
\begin{aligned}
h_e(xy) &= f_e(xy) \wedge g_e(xy) \\
&\geqslant T(f_e(x), f_e(y)) \wedge T(g_e(x), g_e(y)) \\
&= T(f_e(x) \wedge g_e(x), f_e(y) \wedge g_e(y)) \\
&= T((f_e \wedge g_e)(x), (f_e \wedge g_e)(y)) \\
&= T(h_e(x), h_e(y)).
\end{aligned}
$$

Similarly, the other inequalities $(h_e)'(x-y) \leqslant S((h_e)'(x),(h_e)'(y))$ and $(h_e)'(xy) \leqslant S((h_e)'(x),(h_e)'(y))$ are satisfied for each $x, y \in N$.

This shows that $F \sqcap G$ is an intuitonistic fuzzy soft near ring over N. □

Theorem 5. *Let F and G be two intuitonistic fuzzy soft near rings over N. If $F \tilde{\wedge} G$ is non-null, then it is an intuitonistic fuzzy soft near ring.*

Proof. The proof is similar to previous theorem. □

Definition 18. Let F be an intuitionistic fuzzy soft set over X. The soft set $F_{(\alpha,\beta)} = \{(F_e)_{(\alpha,\beta)} : e \in E\}$, for each $\alpha \in (0,1]$ and $\beta \in [0,1)$ with $\alpha + \beta \leqslant 1$, is called a (α,β)-level soft set of the intuitionistic fuzzy soft set, where $(F_e)_{(\alpha,\beta)}$ is an (α,β)-level set of the intuitionistic fuzzy set F_e. That is, for each $\alpha \in (0,1]$ and $\beta \in [0,1)$, $F_{(\alpha,\beta)}$ is a soft set in a classical case.

Theorem 6. *F is an ifs near ring over N iff for all $e \in E$ and for arbitrary $\alpha \in (0,1]$, $\beta \in [0,1)$ with $\alpha + \beta \leqslant 1$ and $(F_e)_{(\alpha,\beta)} \neq \emptyset$, the (α,β)-level soft set $F_{(\alpha,\beta)}$ is a soft near ring over N.*

Proof. Let F be an ifs near ring over N. Then for each $e \in E$, $F_e = (f_e, (f_e)')$ is an intuitionistic fuzzy subnear ring of N. Let $x, y \in (F_e)_{(\alpha,\beta)}$ for arbitrary $\alpha \in (0,1]$, $\beta \in [0,)$ with $\alpha + \beta \leqslant 1$ and $(F_e)_{(\alpha,\beta)} \neq \emptyset$.

Then $f_e(x) \geqslant \alpha$, $(f_e)'(x) \leqslant \beta$, $f_e(y) \geqslant \alpha$ and $(f_e)'(y) \leqslant \beta$.

Therefore, $f_e(xy) \geqslant T(f_e(x), f_e(y)) \geqslant T(\alpha,\alpha) = \alpha$.

$$(f_e)')(xy) \leqslant S((f_e)'(x),(f_e)'(y)) \leqslant S(\beta,\beta) = \beta.$$

Hence, $xy \in (F_e)_{(\alpha,\beta)}$.

Furthermore, $f_e(x-y) \geqslant T(f_e(x), f_e(y)) \geqslant \alpha$.

$$(f_e)'(x-y) \leqslant S((f_e)'(x),(f_e)'(y)) \leqslant \beta.$$

Hence, $x - y \in (F_e)_{(\alpha,\beta)}$.

We obtain that $(F_e)_{(\alpha,\beta)}$ is a sub near ring of N, $\forall e \in E$. Consequently, $F_{(\alpha,\beta)}$ is a soft near ring over N in classical case.

Conversely, let assume that F is not an ifs near ring over N. Then there exist $e \in E$ such that $F(a)$ is not an intuitionistic fuzzy subnear ring of N. Then there exist $x_0, y_0 \in N$ such that

$f_e(x_0-y_0)<T(f_e(x_0),f_e(y_0))$ or $(f_e)')(x_0-y_0)>S((f_e)'(x_0),(f_e)'(y_0))$.

Let assume that $f_e(x_0-y_0)<T(f_e(x_0),f_e(y_0))$. Let $f_e(x_0-y_0)=\lambda$, $f_e(x_0)=\gamma$, $f_e(y_0)=\delta$. We have $\lambda<T(\gamma,\delta)$.

Let $\alpha=\frac{\lambda+T(\gamma,\delta)}{2}$, then $\lambda<\alpha<T(\gamma,\delta)$. So we have $f_e(x_0-y_0)=\lambda<\alpha$. Hence, $x_0-y_0\notin F_{(\alpha,\beta)}$. But, since $\gamma>T(\gamma,\delta)>\alpha$ and $\delta>T(\gamma,\delta)>\alpha$, then we have $f_e(x_0)>\alpha$ and $f_e(y_0)>\alpha$ and for $(f_e)')(x_0)<\beta$, $(f_e)')(y_0)<\beta$, we obtain $x_0,y_0\in F_{(\alpha,\beta)}$. This contradicts with the fact that $F_{(\alpha,\beta)}$ is soft near ring over N.

For the case $(f_e)')(x_0-y_0)>S((f_e)'(x_0),(f_e)'(y_0))$, we can proof by similar way. □

4. Idealistic Intuitionistic Fuzzy Soft Near Rings

Definition 19. Let F be an ifs near ring over N. Then F is called an idealistic intuitionistic fuzzy soft near ring if $F(e)$ is an intuitionistic fuzzy ideal of N for all $e\in \mathrm{Supp}F$, i.e.,

(i) $f_e(x+y)\geqslant T(f_e(x),f_e(y))$, $(f_e)'(x+y)\leqslant S((f_e)'(x),(f_e)'(y))$, for all $x,y\in N$.

(ii) $f_e(-x)\geqslant f_e(x)$, $(f_e)'(-x)\leqslant (f_e)'(x)$, for all $x\in N$.

(iii) $f_e(x)=f_e(y+x-y)$, $(f_e)'(x)=(f_e)'(y+x-y)$, for all $x,y\in N$.

(iv) $f_e(xy)\geqslant f_e(y)$, $(f_e)'(xy)\leqslant (f_e)'(y)$, for all $x,y\in N$.

(v) $f_e[(x+i)y-xy]\geqslant f_e(i)$, $(f_e)'[(x+i)y-xy]\leqslant (f_e)'(i)$, for all $x,y,i\in N$.

If f_e, $(f_e)'$ satisfies (i), (ii), (iii) and (iv), then it is called left idealistic intuitionistic fuzzy soft near ring of N and if they satisfies (i), (ii), (iii) and (v) then it is called right idealistic intuitionistic fuzzy soft near ring of N.

Example 3. Let $N=\{a,b,c,d\}$ be a non-empty set with two binary operations '+' and '·' defined by:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(N,+,\cdot)$ is a near ring.

Let $E=\{e_1,e_2\}$ be the set of parameters. Define an intuitionistic fuzzy soft set F over a near ring N by

$$f_{e_1}(a)=0,8,\quad f_{e_1}(b)=0,6,\quad f_{e_1}(c)=0,3,\quad f_{e_1}(d)=0,3,$$
$$(f_{e_1})'(0)=0,2,\quad (f_{e_1})'(a)=0,3,\quad (f_{e_1})'(b)=0,7,\quad (f_{e_1})'(c)=0,7.$$
$$f_{e_2}(0)=0,4,\quad f_{e_2}(a)=0,3,\quad f_{e_2}(b)=0,3,\quad f_{e_2}(c)=0,3,$$
$$(f_{e_2})'(0)=0,9,\quad (f_{e_2})'(a)=0,5,\quad (f_{e_2})'(b)=0,5,\quad (f_{e_2})'(c)=0,5.$$

Then the intuitionistic fuzzy soft set F is an idealistic intuitonistic fuzzy soft near over a near ring N.

Theorem 7. *Let F and G be two idealistic intuitonistic fuzzy soft near rings over N. Then $F \sqcap G$ is an idealistic intuitonistic fuzzy soft near ring over N if it is non-null.*

Proof. Let $H = F \sqcap G$, for all $e \in E$ $H(e) = F(e) \cap G(e)$. Suppose H is non-null, so there exist $e \in \mathrm{Supp}\, H$ such that $H(e) = F(e) \cap G(e) \neq 0_N$.
That is, $h_e(x) = f_e(x) \wedge g_e(x)$ and $(h_e)'(x) = (f_e)'(x) \vee (g_e)'(x)$, for all $x \in X$.
Since F is idealistic ifs near ring over N, we have

$$\begin{array}{ll} f_e(x+y) \geqslant T(f_e(x), f_e(y)), & (f_e)'(x+y) \leqslant S((f_e)'(x), (f_e)'(y)), \\ f_e(-x) \geqslant f_e(x), & (f_e)'(-x) \leqslant (f_e)'(x), \\ f_e(x) = f_e(y+x-y), & (f_e)'(x) = (f_e)'(y+x-y) \\ f_e(xy) \geqslant f_e(y), & (f_e)'(xy) \leqslant (f_e)'(y), \\ f_e[(x+iy)-y] \geqslant f_e(i), & (f_e)'[(x+iy)-y] \leqslant (f_e)'(i) \end{array}$$

and also we have same properties for fuzzy sets g_e and $(g_e)'$.
Then we obtain,

$$\begin{aligned} h_e(x+y) &= (f_e \wedge g_e)(x+y) \\ &= f_e(x+y) \wedge g_e(x+y)) \\ &\geqslant T(f_e(x), f_e(y)) \wedge T(g_e(x), g_e(y)) \\ &= T(f_e(x) \wedge g_e(x), f_e(y) \wedge g_e(y)) \\ &= T((f_e \wedge g_e)(x), (f_e \wedge g_e)(y)) \\ &= T(h_e(x), h_e(y)). \end{aligned}$$

Similarly, we get $(h_e)'(x+y) = ((f_e)' \vee (g_e)')(x+y) \leqslant S(((f_e)' \vee (g_e)')(x), ((f_e)' \vee (g_e)')(y) = S((h_e)'(x), (h_e)'(y))$.
Now, let prove $h_e[(x+iy)-xy] \geqslant h_e(i)$ and $(h_e)'[(x+iy)-xy] \geqslant (h_e)'(i)$,

$$\begin{aligned} h_e[(x+iy)-xy] &= f_e[(x+iy)-xy] \wedge g_e[(x+iy)-xy] \\ &\geqslant f_e(i) \wedge g_e(i) \\ &= h_e(i) \end{aligned}$$

and

$$\begin{aligned} (h_e)'[(x+iy)-xy] &= (f_e)'[(x+iy)-xy] \vee (g_e)'[(x+iy)-xy] \\ &\leqslant (f_e)'(i) \vee (g_e)'(i) \\ &= (h_e)'(i). \end{aligned}$$

The other equalities are similarly proved for each $x, y \in N$. Hence, $F \sqcap G$ is an idealistic intuitonistic fuzzy soft near ring over N, as desired. □

Theorem 8. *Let F and G be two idealistic intuitonistic fuzzy soft near rings over N. Then $F \tilde{\wedge} G$ is an idealistic intuitonistic fuzzy soft near ring.*

Proof. Let $F \tilde{\wedge} G = H$, where $H(e, e^*) = F(e) \cap G(e^*)$ for all $(e, e^*) \in E \times E$. Let $(e, e^*) \in \mathrm{Supp}\, H$, then $H(e, e^*) = F(e) \cap G(e^*) \neq 0_N$.

For simplicity, we only show that $h_{(e,e^*)}(xy) \geqslant h_{(e,e^*)}(y)$ and $(h_{(e,e^*)})'(xy) \leqslant (h_{(e,e^*)})'(y)$

$$\begin{aligned} h_{(e,e^*)}(xy) &= f_e(xy) \wedge g_{e^*}(xy) \\ &\geqslant f_e(y) \wedge g_{e^*}(y) \\ &= h_{(e,e^*)}(y) \end{aligned}$$

and

$$\begin{aligned} (h_{(e,e^*)})'(xy) &= (f_e)'(xy) \vee (g_{e^*})'(xy) \\ &\leqslant (f_e)'(y) \vee (g_{e^*})'(y) \\ &= (h_{(e,e^*)})'(y). \end{aligned}$$

The other equalities are satisfied easily. This shows that $F \tilde{\wedge} G$ is an idealistic intuitonistic fuzzy soft near ring over N. □

Theorem 9. *F is an idealistic intuitonistic fuzzy soft near ring over N iff for all $e \in E$ and for arbitrary $\alpha \in (0,1]$, $\beta \in [0,1)$ with $\alpha + \beta \leqslant 1$ and $(F_e)_{(\alpha,\beta)} \neq \emptyset$, the (α,β)-level soft set $F_{(\alpha,\beta)}$ is an idealistic soft near ring over N as Sezgin's sense* [11].

Proof. The proof is similar with Theorem 6. □

5. Homomorphism of Fuzzy Soft Rings

In this section we show that the homomorphic image and pre-image of an intuitionistic fuzzy soft near ring are also intuitonistic fuzzy soft near ring.

Definition 20. Let N_1 and N_2 be two classical near rings and (φ,ψ) be an intuitionstic fuzzy soft function from N_1 to N_2. If φ is a homomorphism from N_1 to N_2 then (φ,ψ) is said to be intuitionistic fuzzy soft homomorphism. If φ is a isomorphism from X to Y and ψ is one-to-one mapping from E onto K then (φ,ψ) is said to be fuzzy soft isomorphism.

Theorem 10. *Let F be an ifs near ring over N_1 and (φ,ψ) be an intuitionistic fuzzy soft homomorphism from N_1 to N_2. Then $(\varphi,\psi)(F)$ is an ifs near ring N_2.*

Proof. Let $k \in \psi(E)$ and $y_1, y_2 \in N_2$. If $\varphi^{-1}(y_1) = \emptyset$ or $\varphi^{-1}(y_2) = \emptyset$ the proof is straightforward. Let assume that there exist $x_1, x_2 \in N_1$ such that $\varphi(x_1) = y_1, \varphi(x_2) = y_2$.

$$\begin{aligned} \varphi(f)_k(y_1 - y_2) &= \bigvee_{\varphi(t) = y_1 - y_2} \bigvee_{\psi(e)=k} f_e(t) \\ &\geqslant \bigvee_{\psi(e)=k} f_e(x_1 - x_2) \\ &\geqslant \bigvee_{\psi(e)=k} T(f_e(x_1), f_e(x_2)) \\ &= T\Big(\bigvee_{\psi(e)=k} f_e(x_1), \bigvee_{\psi(e)=k} f_e(x_2) \Big). \end{aligned}$$

This inequality is satisfied for each $x_1, x_2 \in N_1$, which satisfy $\varphi(x_1) = y_1, \varphi(x_2) = y_2$.

Then we have

$$\begin{aligned}\varphi(f)_k(y_1-y_2) &\geqslant T\Big(\bigvee_{\varphi(t_1)=y_1}\bigvee_{\psi(e)=k} f_e(t_1), \bigvee_{\varphi(t_2)=y_2}\bigvee_{\psi(e)=k} f_e(t_2)\Big)\\ &= T(\varphi(f)_k(y_1),\varphi(f)_k(y_2)).\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}\varphi(f')_k(y_1-y_2) &\geqslant S\Big(\bigwedge_{\varphi(t_1)=y_1}\bigwedge_{\psi(e)=k} (f_e)'(t_1), \bigwedge_{\varphi(t_2)=y_2}\bigwedge_{\psi(e)=k} (f_e)'(t_2)\Big)\\ &= S(\varphi(f')_k(y_1),\varphi(f')_k(y_2))\end{aligned}$$

and similarly, we have

$$\begin{aligned}\varphi(f)_k(y_1y_2) &= \bigvee_{\varphi(t)=y_1y_2}\ \bigvee_{\psi(e)=k} f_e(t)\\ &\geqslant \bigvee_{\psi(e)=k} f_e(x_1x_2)\\ &\geqslant \bigvee_{\psi(e)=k} T(f_e(x_1),f_e(x_2)).\end{aligned}$$

Thus, we conclude that $(\varphi,\psi)(F)$ is an ifs near ring over N_2. □

Theorem 11. *Let G be an ifs near ring over N_2 and (φ,ψ) be an intuitionistic fuzzy soft homomorphism from N_1 to N_2. Then $(\varphi,\psi)^{-1}(G,B)$ is an ifs near ring over N_1.*

Proof. Let $e \in \psi^{-1}(B)$ and $x_1,x_2 \in N_1$.

$$\begin{aligned}\varphi^{-1}(g)_e(x_1-x_2) &= g_{\psi(e)}(\varphi(x_1-x_2))\\ &= g_{\psi(e)}(\varphi(x_1)-\varphi(x_2))\\ &\geqslant T(g_{\psi(e)}(\varphi(x_1)),g_{\psi(e)}(\varphi(x_2)))\\ &= T(\varphi^{-1}(g)_e(x_1),\varphi^{-1}(g)_e(x_2))\end{aligned}$$

and similarly, we have $\varphi^{-1}(g')_e(x_1-x_2) \leqslant S(\varphi^{-1}(g')_e(x_1),\varphi^{-1}(g')_e(x_2))$. So, $(\varphi,\psi)^{-1}(G,B)$ is a intuitionistic fuzzy soft near ring over N_1. □

6. Conclusion

The concept of a near ring is one of the fundamental structure in algebra. With this paper, we decided to combine the near ring concept and intuitionistic fuzzy soft set. We defined intuitionistic fuzzy soft near ring over a given classical near ring and studied its characteristic properties. Further, the notion of idealistic fuzzy soft ring is introduced. The soft homomorphism between intuitionistic fuzzy soft near rings are defined and they are proved that image and preimage of intuitionistic fuzzy soft near rings are intuitionistic fuzzy soft near rings. To extend this study, one could study the properties of intuitionistic fuzzy soft sets in other algebraic structures.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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