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Research Article

Modelling Deforestation Due to Human Population and Its Effect on Farm Fields

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Abstract. In this paper, a nonlinear mathematical model is proposed and analyzed to study the deforestation due to human population and its effect on farm fields. We consider the variables namely, density of human population, density of trees and Farm fields. Both the growth rate and carrying capacity of trees, which follows logistic model, are assumed to be simultaneously depleted by density of human population. Further this affects the growth rate of crops in farm fields. We discretize the model by applying Foward Euler scheme and analyse the stability of the model. Finally, we provide some numerical simulations using MATLAB.

Keywords. Difference equations; Human population; Forest resources; Farm fields

MSC. 34Axx

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1. Introduction

Human ecology has been defined as a type of analysis applied to the relations in human beings that was traditionally applied to plants and animals in ecology. Deforestation, is the removal of a forest or stand of trees where the land is thereafter converted to a non-forest use. Deforestation includes conversion of forestland to farms, ranches, or urban use. Since the industrial age, about half of world's forests have been destroyed and millions of animals and living things have been endangered. Forests cover almost a third of the earth's land surface. Forests provides many environmental benefits including its major role in the hydrologic cycle, soil conservation, prevention of climate change and preservation of biodiversity.

Forest produce vital oxygen and provide homes for people and wildlife. Many of the world's most threatened and endangered animals live in forests, and 1.6 billion people rely on benefits forests offer, including food, fresh water, clothing, traditional medicine and shelter. But forests around the world are under threat from deforestation, jeopardizing these benefits. Deforestation comes in many forms, including fires, clear-cutting for agriculture, ranching and development, unsustainable logging for timber, and degradation due to climate change. Forests in India faces heavy pressure of human and livestock population. The total forest cover in the country is only about 69 million hectares whereas human population is 1210 million, hence per capita forests are as low as 0.06 hectares. About 69 percent of population in India i.e. 833 million live in rural areas and most of them have land based economy and use forest resources one way or the other. It is estimated that about 200 million people live in and around forests, and fully depend for their livelihood on forest resources. Further, of the 530 million livestock population in India, about 190 million fully depends on forests either by direct grazing or by harvesting of fodder causing additional burdens on the forests.

Deforestation can result to watersheds that are no longer able to sustain and regulate water flows from rivers to streams. Trees are highly effective in absorbing water quantities, keeping the amount of water in watersheds to a manageable level. The forest also serves as cover against erosion. Once they are gone, too much water can results to downstream flooding. Forests perform a valuable function by capturing rainwater and releasing it to streams and rivers that provide water for cities and agriculture. Forest soils with a carpet of decomposing leaves absorb rainwater like a sponge, holding the water for gradual release to streams throughout the year. When watersheds lose their forest, the soil can lose its capacity to absorb rainwater as it did before. Rainwater flows quickly off the watershed, causing floods during the rainy season and a diminished supply of water during the dry season.

In addition, the flow of water from the hills irrigate the farm fields, and the quality of the water is worse because deforested hills no longer have trees to protect the ground from heavy rain, so soil erosion is greater, and the irrigation water contains large quantities of mud that settles in irrigation canals and clogs the canals. This decline in the quantity and quality of irrigation water reduces food production even further. The result is poor nutrition and health for people. This chain of effects involving human population growth, deforestation and lower food production is a vicious cycle that is difficult to escape. Forests benefits all who live downstream by reducing erosion and flooding. Floods and erosion will cause severe constraints on food production throughout the world. Farmers in villages depend on the sponge-effect of forests to absorb and slowly release water. Many agricultural exports, are dependent upon forest-generated soils and water. In some cases, water flow, due to rapid runoff kills many high-yielding rice varieties.

Agarwal and Pathak [5] studied Conservation of Forestry Biomass and wildlife population: A Mathematical Model. Dubey *et al*. [6] analyzed on Modeling the depletion of a renewable resource by population and Industrialization: Effect of Technology on its conservation. In this paper, we have proposed mathematical model to study the effect of deforestation on farm fields. In Section 2, a mathematical model is proposed and discretized. In Section 3, we list the equilibrium points of the model. We analyze the stability of the model in Section 4. Section 5 consists of numerical simulations through MATLAB. Conclusion is given in Section 6.

2. The Mathematical Model

The following assumptions are made in formulating the model on deforestation due to human population.

- The density of forest and human population are governed by logistic type equation.
- It is assumed that the deforestation occurs due to human population.
- It is assumed that human population are dependent on forest resources and crops for their livelihood.
- It is assumed that growth rate of crops in the farm fields are dependent on the irrigation caused by forests.
- The value of all the parameters which are used in the model are positive.

$$\begin{aligned}
\frac{dH}{dt} &= rH\left(1-\frac{H}{K}\right)+r_0HF+r_1HB-r_2H\,,\\
\frac{dB}{dt} &= sB\left(1-\frac{B}{L}\right)-r_1HB-s_1B\,,\\
\frac{dF}{dt} &= (s_2+s_3B)F-r_0HF\,. \qquad (1)
\end{aligned}$$

Applying the Forward Euler Scheme to the system of equations (1) we obtain the following discretized model

$$\begin{aligned}
&H \to H+\delta\left\{rH(1-\frac{H}{K})+r_0HF+r_1HB-r_2H\right\},\\
&B \to B+\delta\left\{sB(1-\frac{B}{L})-r_1HB-s_1B\right\},\\
&F \to F+\delta\{(s_2+s_3B)F-r_0HF\}. \qquad (2)
\end{aligned}$$

where

δ : the step size

H : Density of the Human population

B : Density of the Forest resources

F	:	Density of the crops in the Farm fields
r	:	Intrinsic growth rate the human Population
r_0	:	Rate of consumption of crops by the population
r_1	:	Deforestation rate by the population
r_2	:	Natural death rate of population
s	:	Intrinsic growth rate of the Forest resources
s_1	:	Natural death rate of the Forest resources
s_2	:	Growth rate of crops in farm fields
s_3	:	Rate of irrigation in the Farm fields caused by forests
K	:	Carrying Capacity of the Forest resources
L	:	Carrying Capacity of the population

3. Equilibria

First, we list the possible equilibrium points of our model.
System (2) has the following equilibria, namely,

- $E_1 = (\widehat{H}, 0, 0)$

where $\widehat{H} = K\dfrac{(r-r_2)}{r}$ and $r > r_2$

- $E_2 = (\bar{H}, \bar{B}, 0)$

where

$$\begin{aligned}
\bar{H} &= \frac{K\{Lr_1(s-s_1)+s(r-r_2)\}}{rs+Lr_1{}^2K},\\
\bar{B} &= \frac{L}{s}(s-s_1) - \frac{Lr_1}{s}\left[\frac{K\{Lr_1(s-s_1)+s(r-r_2)\}}{rs+Lr_1{}^2K}\right],\\
s &> s_1 \quad \text{and} \quad (s-s_1) > \left[\frac{Kr_1\{Lr_1(s-s_1)+s(r-r_2)\}}{rs+Lr_1{}^2K}\right]
\end{aligned}$$

- $E_3 = (H^*, B^*, F^*)$

where H^*, B^*, F^* satisfy,

$$\begin{aligned}
&\left\{rH^*\left(1-\frac{H^*}{K}\right) + r_0H^*F^* + r_0H^*B^* - r_2H^*\right\} = 0,\\
&\left\{sB\left(1-\frac{B^*}{L}\right) - r_0H^*B^* - s_1B^*\right\} = 0,\\
&\{(s_2+s_3B^*)F^* - r_0H^*F^*\} = 0.
\end{aligned}$$

The Jacobian matrix of (2) at the fixed point (H, B, F) is written as

$$J = \begin{pmatrix} 1+\delta\left\{r-\frac{2rH}{K}+r_0F+r_1B-r_2\right\} & \delta r_1H & \delta r_0H \\ -\delta r_1B & 1+\delta\left\{s-\frac{2sB}{L}-r_1H-s_1\right\} & 0 \\ -\delta r_0F & \delta s_3F & 1+\delta\{(s_2+s_3B)-r_0H\} \end{pmatrix} \tag{3}$$

Lemma 1. *Let $F(\lambda) = \lambda^2 + B\lambda + C$. Suppose that $F(1) > 0$; λ_1 and λ_2 are roots of $F(\lambda) = 0$. Then we have:*

(1) $|\lambda_1| < 1$ *and* $|\lambda_2| < 1$ *if and only if* $F(-1) > 0$ *and* $C < 1$.

(2) $|\lambda_1| < 1$ *and* $|\lambda_2| > 1$ (*or* $|\lambda_1| > 1$ *and* $|\lambda_2| < 1$) *if and only if* $F(-1) < 0$.

(3) $|\lambda_1| > 1$ *and* $|\lambda_2| > 1$ *if and only if* $F(-1) > 0$ *and* $C > 1$.

(4) $\lambda_1 = 1$ *and* $\lambda_2 \neq 1$ *if and only if* $F(-1) = 0$ *and* $B \neq 0, 2$.

(5) λ_1 *and* λ_2 *are complex and* $|\lambda_1| = |\lambda_2| = 1$ *if and only if* $B^2 - 4C < 0$ *and* $C = 1$.

4. Stability Analysis of the Model

Stability of the Equilibrium Point $E_1 = (\widehat{H}, 0, 0)$

Theorem 1. *The stability conditions of the equilibrium point $E_1 = (\widehat{H}, 0, 0)$ are given as follows.*

(1) $E_1 = (\widehat{H}, 0, 0)$ *is sink if* $Kr_0(r - r_2) > s_2 r$, *and*

$$0 < \delta < \min\left\{\frac{2}{(r-r_2)}, \frac{2r}{Kr_1(r-r_2)+s_1 r}, \frac{2r}{Kr_0(r-r_2)-s_2 r}\right\}.$$

(2) $E_1 = (\widehat{H}, 0, 0)$ *is source if* $Kr_0(r - r_2) > s_2 r$ *and*

$$\delta > \max\left\{\frac{2}{(r-r_2)}, \frac{2r}{Kr_1(r-r_2)+s_1 r}, \frac{2r}{Kr_0(r-r_2)-s_2 r}\right\}.$$

Proof. The Jacobian matrix of (2) at the fixed point $E_1 = (\widehat{H}, 0, 0)$ is given by

$$J_1 = \begin{pmatrix} 1+\delta\left\{r - \frac{2r\widehat{H}}{K} - r_2\right\} & \delta r_1 \widehat{H} & \delta r_0 \widehat{H} \\ 0 & 1+\delta\{s - r_1\bar{H} - s_1\} & 0 \\ 0 & 0 & 1+\delta\{s_2 - r_0\widehat{H}\} \end{pmatrix} \tag{4}$$

The eigen values of the above matrix are $\lambda_1 = 1 + \delta\left\{r - \frac{2r\widehat{H}}{K} - r_2\right\}$, $\lambda_2 = 1 + \delta\{s - r_1\bar{H} - s_1\}$ and $\lambda_3 = 1 + \delta\{s_2 - r_0\widehat{H}\}$. The equilibrium point $E_1 = (\widehat{H}, 0, 0)$ is stable if $|\lambda_i| < 1$ for $i = 1, 2, 3$. Therefore we can see that the equilibrium point $E_1 = (\widehat{H}, 0, 0)$ is stable if the conditions (1) holds. □

Stability of the Equilibrium Point $E_2 = (\bar{H}, \bar{B}, 0)$

The Jacobian matrix of (2) at the fixed point $(\bar{H}, \bar{B}, 0)$ is given by

$$J_2 = \begin{pmatrix} 1+\delta\left\{r - \frac{2r\bar{H}}{K} + r_1\bar{B} - r_2\right\} & \delta r_1 \bar{H} & \delta r_0 \bar{H} \\ -\delta r_1 \bar{B} & 1+\delta\left\{s - \frac{2s\bar{B}}{L} - r_1\bar{H} - s_1\right\} & 0 \\ 0 & 0 & 1+\delta\{(s_2 + s_3\bar{B}) - r_0\bar{H}\} \end{pmatrix} \tag{5}$$

The characteristic equation of the above Jacobian matrix is given by:

$$\varphi(\lambda) = \left\{1 + \delta\left\{(s_2 + s_3\bar{B}) - r_0\bar{H}\right\} - \lambda\right\}\left\{\lambda^2 - \lambda(2 + G\delta) + \left(1 + G\delta + H\delta^2\right)\right\} = 0. \tag{6}$$

Therefore, we get

$$\lambda = 1 + \delta\left\{(s_2 + s_3\bar{B}) - r_0\bar{H}\right\}$$

or

$$\{\lambda^2-\lambda(2+G\delta)+\left(1+G\delta+H\delta^2\right)\}=0,$$

where

$$G=r+s-\left(\frac{2r\bar{H}}{K}+\frac{2s\bar{B}}{L}\right)+r_1(\bar{B}-\bar{H})-r_2-s_1,$$

$$H=\left[r-\frac{2r\bar{H}}{K}+r_1\bar{B}-r_2\right]\left[s-\frac{2s\bar{B}}{L}-r_1\bar{H}-s_1\right]+{r_1}^2\bar{H}\bar{B},$$

$$F(\lambda)=\lambda^2-\lambda(2+G\delta)+\left(1+G\delta+H\delta^2\right)=0,$$

$$F(-1)=4+2G\delta+H\delta^2.$$

By using Lemma 1 we obtain the following theorem.

Theorem 2. *The stability conditions of the equilibrium point* $E_2=(\bar{H},\bar{B},0)$ *is given as follows.*

(1) $E_2=(\bar{H},\bar{B},0)$ *is a sink if either* (1.1) *or* (1.2) *holds.*

(1.1) $G^2-4H\ge 0$ *and* $0<\delta<\dfrac{-G-\sqrt{G^2-4H}}{H}$,

(1.2) $G^2-4H<0$ *and* $\delta<\dfrac{-G}{H}$.

(2) $E_2=(\bar{H},\bar{B},0)$ *is a source if either* (2.1) *or* (2.2) *holds.*

(2.1) $G^2-4H\ge 0$ *and* $\delta>\dfrac{-G-\sqrt{G^2-4H}}{H}$,

(2.2) $G^2-4H<0$ *and* $\delta>\dfrac{-G}{H}$.

Stability of the Equilibrium Point $E_3=(H^*,B^*,F^*)$

Theorem 3. *The equilibrium point* E_3 *is locally asymptotically stable provided that the following conditions are satisfied*

$$\delta\left(\frac{2rH^*}{K}+r_2-r-r_1B^*-r_0F^*\right)>1, \tag{7}$$

$$\delta\left(\frac{2sB^*}{L}+r_1H^*+s_1-s\right)>1, \tag{8}$$

$$\delta(r_0H^*-s_3B^*-s_2)>1, \tag{9}$$

$$\Omega_3<1, \tag{10}$$

otherwise unstable.

Proof. The Jacobian matrix of the system (2) at $E_3=(H^*,B^*,F^*)$ is given by

$$J_3=\begin{pmatrix} 1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\} & \delta r_1H^* & \delta r_0H^* \\ -\delta r_1B^* & 1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\} & 0 \\ -\delta r_0F^* & \delta s_3F^* & 1+\delta\{(s_2+s_3B^*)-r_0H^*\} \end{pmatrix} \tag{11}$$

The characteristic equation of the above Jacobian matrix is given by:

$$\phi(\lambda) = \lambda^3 + \Omega_1\lambda^2 + \Omega_2\lambda + \Omega_3 = 0, \tag{12}$$

where

$$\begin{aligned}\Omega_1 &= -[a_{11} + a_{22} + a_{33}]\\ &= -\left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right] - \left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right]\\ &\quad - [1+\delta\{(s_2+s_3B^*) - r_0H^*\}],\end{aligned} \tag{13}$$

$$\begin{aligned}\Omega_2 &= a_{11}a_{22} - a_{12}a_{21} + a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{33}\\ &= \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right]\left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right]\\ &\quad + \left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right][1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\\ &\quad + \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right][1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\\ &\quad + \delta^2{r_1}^2H^*B^* + \delta^2{r_0}^2H^*F^*,\end{aligned} \tag{14}$$

$$\Omega_3 = a_{12}a_{21}a_{33} - a_{13}a_{21}a_{32} - a_{11}a_{22}a_{33} + a_{13}a_{31}a_{22} \tag{15}$$

$$\begin{aligned}&= \delta^3 r_0r_1s_3H^*B^*F^* - \delta^2r_0^2H^*F^*\left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right]\\ &\quad - \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right]\left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right]\\ &\quad \times [1+\delta\{(s_2+s_3B^*) - r_0H^*\}] - \delta^2r_1^2H^*B^*[1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\end{aligned} \tag{16}$$

It follows from the Jury's conditions, that the modulus of all the roots of the above characteristic equation is less than 1, if and only if the conditions $\phi(1) > 0$, $\phi(-1) < 0$ and $|\det J_3| < 1$ hold.

$$\phi(1) = 1 + \Omega_1 + \Omega_2 + \Omega_3 \tag{17}$$

$$\begin{aligned}&= 1 - \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right]\\ &\quad - \left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right] - [1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\\ &\quad + \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right]\left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right]\\ &\quad + \left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right][1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\\ &\quad + \left[1+\delta\left\{r - \frac{2rH^*}{K} + r_0F^* + r_1B^* - r_2\right\}\right][1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\\ &\quad + \delta^2{r_1}^2H^*B^* + \delta^2{r_0}^2H^*F^* + \delta^3r_0r_1s_3H^*B^*F^*\\ &\quad - \delta^2r_0^2H^*F^*\left[1+\delta\left\{s - \frac{2sB^*}{L} - r_1H^* - s_1\right\}\right] - \delta^2r_1^2H^*F^*[1+\delta\{(s_2+s_3B^*) - r_0H^*\}]\end{aligned}$$

$$-\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]$$
$$\times\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right] \tag{18}$$

Assume the conditions (6), (7) and (8) holds then obviously $\phi(1)>0$.

$$\phi(-1)=-1+\Omega_1-\Omega_2+\Omega_3 \tag{19}$$
$$\begin{aligned}&=-1-\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\\
&\quad-\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]-\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\\
&\quad-\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]\\
&\quad-\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\\
&\quad-\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\\
&\quad-\delta^2{r_1}^2H^*B^*-\delta^2{r_0}^2H^*F^*+\delta^3r_0r_1s_3H^*B^*F^*\\
&\quad-\delta^2r_0^2H^*F^*\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]-\delta^2r_1^2H^*F^*\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\\
&\quad-\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]\\
&\quad\times\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\end{aligned} \tag{20}$$

We can see that $\phi(-1)<0$ iff $\Omega_3<1-\Omega_1+\Omega_2$.

$$\begin{aligned}\det(J_3)&=\delta^2r_0^2H^*F^*\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]+\delta^2r_1^2H^*B^*\\
&\quad\times\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]+\left[1+\delta\left\{r-\frac{2rH^*}{K}+r_0F^*+r_1B^*-r_2\right\}\right]\\
&\quad\times\left[1+\delta\left\{s-\frac{2sB^*}{L}-r_1H^*-s_1\right\}\right]\left[1+\delta\left\{(s_2+s_3B^*)-r_0H^*\right\}\right]\\
&\quad-\delta^3r_0r_1s_3H^*B^*F^*.\end{aligned} \tag{21}$$

We can see that $|\det J_3|<1$ iff $\Omega_3<1$.

Clearly, $\phi(1)>0$, $\phi(-1)<0$ and $|\det J_3|<1$ if the conditions (**??**), (**??**), (**??**) and (**??**) hold. □

5. Numerical Simulations

Numerical simulations have been carried out to investigate the dynamics of the proposed model. The simulations have been performed using MATLAB to explain our theoretical results.

Case 1

Taking the following set of parametric values:

$$r=1.5,\ s=1.05,\ K=5,\ L=5,\ r_1=0.4,\ r_2=0.07,\ s_1=0.01,\ s_2=0.6,\ s_3=0.03.$$

We assume different values of r_0 to investigate the effects of farming (Figure 1 and Figure 2).

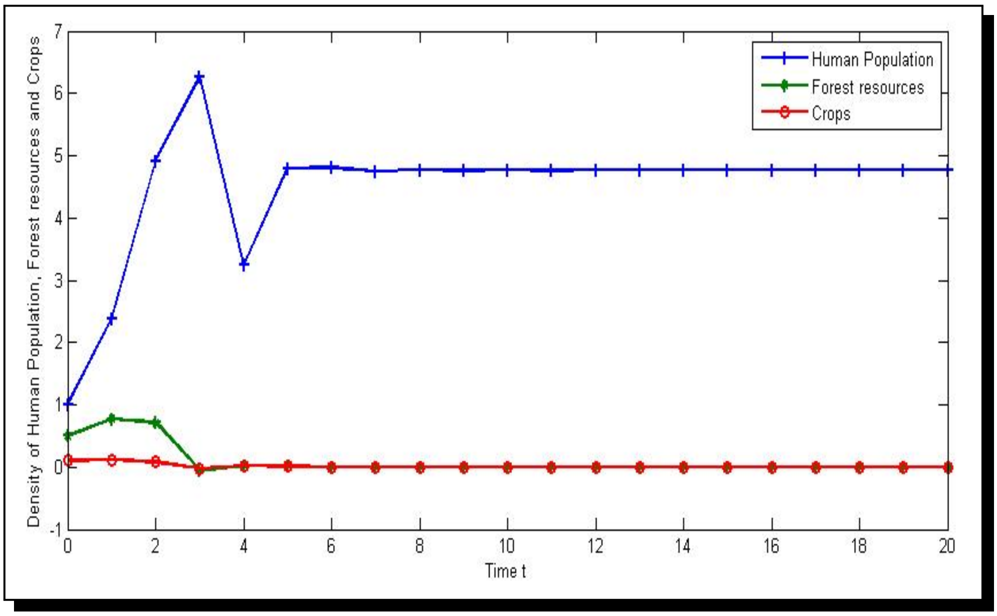

Figure 1. Dynamics of the fixed point $(\widehat{H}, 0, 0)$ with $r_0 = 0.4$

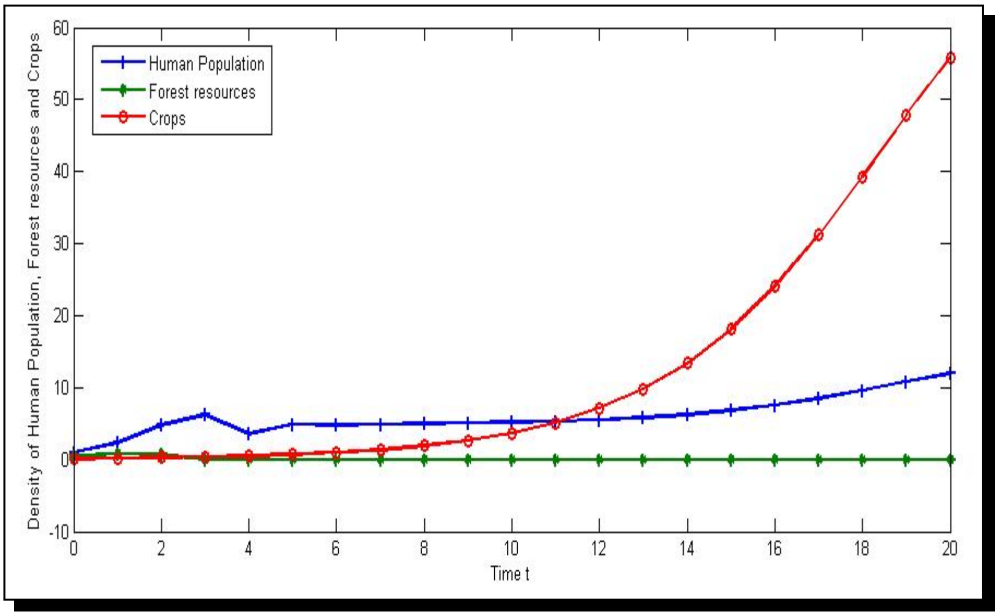

Figure 2. Dynamics of the fixed point $(\widehat{H}, 0, 0)$ with $r_0 = 0.04$

Case 2

Taking the following set of parametric values:

$$r = 1.5,\ s = 1.05,\ K = 5,\ L = 5,\ r_1 = 0.09,\ r_2 = 0.07,\ s_1 = 0.01,\ s_2 = 0.6,\ s_3 = 0.3\,.$$

We assume different values of r_0 to investigate the effects of farming (Figure 3 and Figure 4).

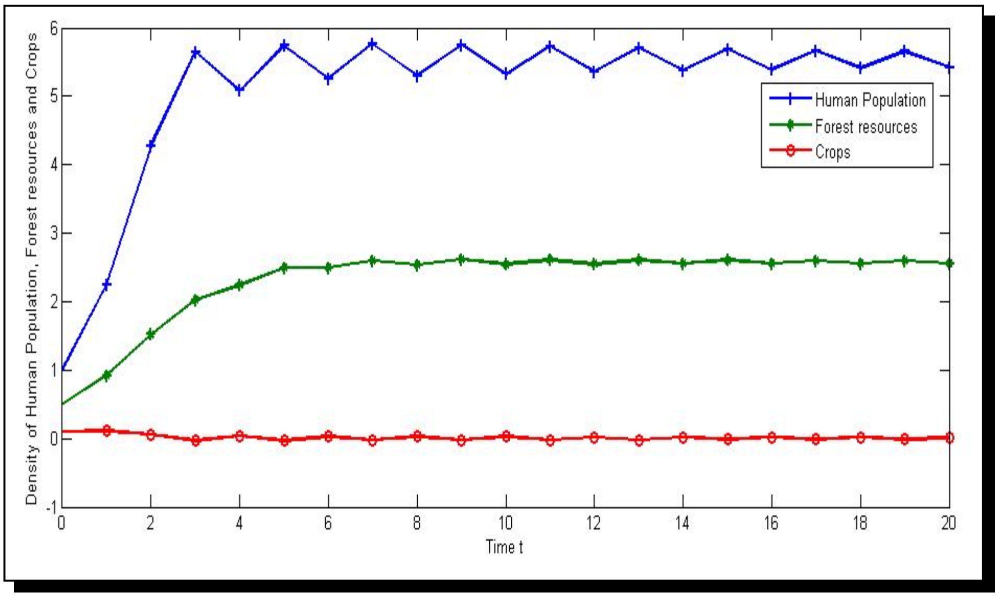

Figure 3. Dynamics of the fixed point $(\bar{H},\bar{B},0)$ with $r_0 = 0.4$

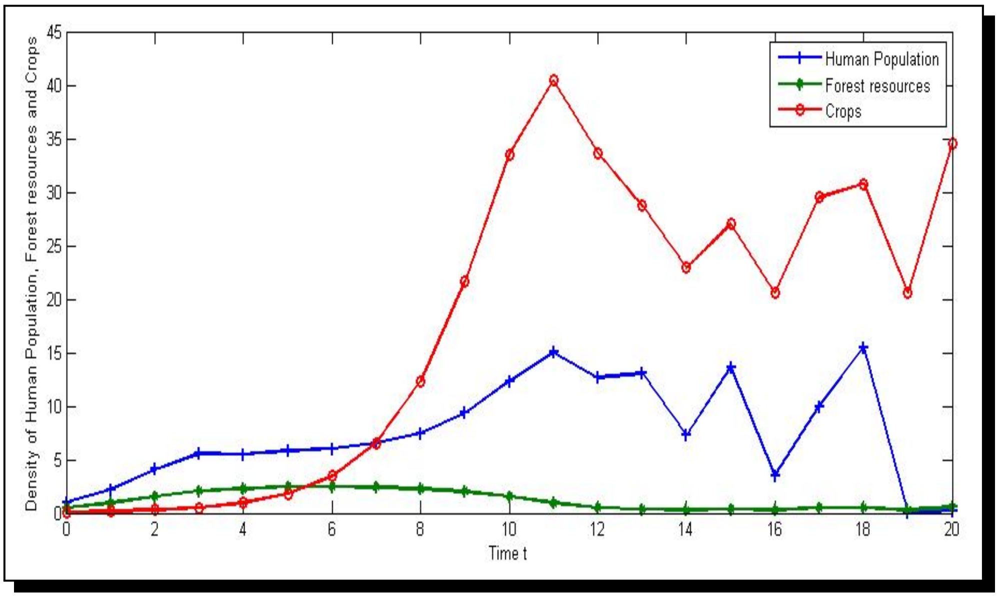

Figure 4. Dynamics of the fixed point $(\bar{H},\bar{B},0)$ with $r_0 = 0.3$

6. Conclusion

In this paper, we have considered a discrete-time model on deforestation due to human population and its effect on farm fields. We have considered the growth rate of human population and the forest resources to be logistic, and also the growth rate of human population depend on density of forest resources and crops in farm fields. We list the equilibrium points of the model and analyze the stability around each equilibrium point. We have proved our theoretical results using numerical simulations through MATLAB. And we have discussed different values for effects of farming in the numerical simulations.

Competing Interests

The authors declare they he have no competing interests.

Authors' Contributions

The authors wrote, read and approved the final manuscript.

References

[1] C. Celik and O. Duman, Chaos, Allee effect in a discrete-time predator?prey system, *Solitons and Fractals* **40** (2009), 1956 – 1962.

[2] U. Ufuktepe, S. Kapcak and O. Akman, Stability analysis of the Beddington model with Allee effect *Appl. Math. Inf. Sci.* **9** (2) (2015), 603 – 608.

[3] J.D. Murray, *Mathematical Biology*, Springer-Verlag, New York (1993).

[4] V. Ramdhani, Jaharuddin and E.H. Nugrahani, Dynamical system of modelling the depletion of forestry resources due to crowding by industrialization, *Applied Mathematical Sciences* **9** (82) (2015), 4067 – 4079.

[5] M. Agarwal and R. Pathak, Conservation of forestry biomass and wildlife population: a mathematical model, *Asian Journal of Mathematics and Computer Research* **4**(1) (2015), 1 – 15.

[6] B. Dubey, S. Sharma, P. Sinha and J.B Shukla, Modelling the depletion of forestry resources by population and population pressure augmented industrialization, *Applied Mathematical Sciences* **33** (2009), 3002 – 3014.

[7] M. Agarwala, T. Fatimaa and H.I. Freedman, Depletion of forestry resource biomass due to industrialization pressure: A ratio-dependent mathematical model, *Journal of Biological Dynamics* **4** (4) (2010), 381 – 396.

[8] M. Chaudhary, J. Dhar and G.P. Sahu, Mathematical model of depletion of forestry resource: effect of synthetic based industries, *International Journal of Biological, Biomolecular, Agricultural, Food and Biotechnological Engineering* **7** (4) (2013), 639 – 643.