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Research Article

Solving Multi Objective Linear Fractional Programming Problem Under Uncertainty via Robust Optimization Approach

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Abstract. In this article, a *Multi Objective Linear Fractional Programming* (MOLFP) problem with uncertain data in the objective function and the relationship between its *Robust Counterpart* (RC) formulations is studied. We use box uncertainty set for MOLFP problem and propose an approach to derive its corresponding RC formulation by reducing it into a single objective programming problem. It is shown that the corresponding RC formulation of MOLFP problem under box uncertainty set is a *Linear Programming* (LP) problem. A numerical example is worked out to illustrate the methodology and proposed approach.

Keywords. Box uncertainty; Multi objective programming; Linear fractional programming; Robust optimization

MSC. 90C29; 90C32; 90C05

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1. Introduction

Multi Objective Linear Fractional Programming (MOLFP) problem is one of the interesting subjects of nonlinear optimization that has been attracted the attention of many researches in the last few decades. It can be described in mathematical terms as follows

$$\begin{aligned} &\max z(x) = [z_1(x), z_2(x), \dots, z_k(x)] \\ &\text{s.t.} \;\; x \in X = \{x \in R^n : Ax \le b, x \ge 0\} \end{aligned} \tag{1}$$

In which X is a convex and bounded set, and

$$z_i(x) = \frac{p_i^T x + \alpha_i}{q_i^T x + \beta_i} = \frac{N_i(x)}{D_i(x)}, \quad i = 1, 2, \ldots, k, \tag{2}$$

$$p_i, q_i \in R^n,\ \alpha_i, \beta_i \in R$$

and also

$$x \in R^n,\ b \in R^m,\ A \in R^{m\times n} \text{ and } D_i(x) > 0 \quad \forall\ i. \tag{3}$$

MOLFP problem has been used in wide variety of application such as engineering, business, management, finance, production planning, economics and others. Generally MOLFP problem has been used for modeling real-world problems such as inventory/sales, profit/cost, debt/equity and others. There are many methodologies in the literature to solve MOLFP problems [4], [12], [13], [15] and [17]. Zimmermann [19] proposed a fuzzy approach to *Multi Objective Linear Programming* (MOLP) problems. Duran Toksarı [6] proposed an approach for fuzzy MOLFP problem using Taylor series. Sulaiman *et al*. [18] presented transformation technique for solving MOLFP problems by transforming into a single objective linear fractional program.

Due to uncertainty in the real physical world and phenomena's, we need to work with some techniques which deals with uncertainty. *Robust Optimization* (RO) is found very effective and efficient in this regard that has been shown as a specific and relatively new approach for handling optimization problems with uncertain data that has used in many applications. The very early work on robust optimization in 1970s is due to Soyster [16], who was one of the first researchers to investigate explicit to this approach. He considered a *Robust Counterpart* (RC) and proved that the model is feasible under perturbations. Ben-Tal and Nemirovski [1], and El-Ghaoui *et al*. [7] have introduced ellipsoidal uncertainties to the RO literature which caused in conic quadratic robust counterparts for linear formulations. In fact ellipsoidal uncertainties can be used to approximate more complicated uncertainty sets. Janak *et al*. [11] and Lin *et al*. [14] extended RO formulation of LP problems with uncertain data to *Mixed Integer Linear Programming* (MILP) problems. Bertsimas and Sim [2] developed the theory of the RO for discrete programming and LP problems. Hasanzadeh *et al*. [10] used an interactive method (weighted Tchebycheff) to solve the robust format of the multi objective R&D project portfolio selection programming with imprecise information. Goberna *et al*. [8] investigated the problem of robust solutions to MOLP problems with uncertain data in which the uncertainty happens both in the constraints and objective functions.

In the current study we have applied a box uncertainty in RO procedure that based on uncertainty approach to solve MOLFP problem with imprecise coefficients in the objective functions by reducing it into a LP problem. The developed approach in this study extends the RO concepts to MOLFP problem for solving such problem under uncertainty in the coefficients of the objective functions.

2. Preliminaries

In this section, some basic definitions and concepts of *Linear Fractional Programming* (LFP) along with box uncertainty with RO is introduced.

The general format of *Linear Fractional Programming* (LFP) [4] may be written as

$$\max \frac{p^T x + \alpha}{q^T x + \beta} \tag{4}$$
$$\text{s.t. } x \in X = \{x \in R^n : Ax \le b, x \ge 0\}$$

where $p, q \in R^n$, $A \in R^{m \times n}$, $\alpha, \beta \in R$.
For some values of $x \in X$ may be $q^T x + \beta = 0$. For convenience, assume that LFP satisfies $q^T x + \beta > 0$.
The general RC formulation of a LP problem [3] may be written as

$$\begin{aligned} &\max c^T x \\ &\text{s.t. } \tilde{a}_i^T x \le b, \ \ \forall\, a_i \in u_i \\ &\quad x \ge 0 \end{aligned} \tag{5}$$

where a_i represents the ith Constraint's coefficient of the uncertain data where $\tilde{a}_i \in u_i \in R^n$.
Then, $\tilde{a}_i^T x \le b_i$, $\forall\ \tilde{a}_i \in u_i$ if and only if $\max_{\{\tilde{a}_i \in u_i\}} \tilde{a}_i^T x \le b_i$, $\forall\ i$.

Definition 2.1. Consider $U = \{\xi \mid \|\xi\|_\rho \le \Psi\}$ contains of the uncertain data vectors, if $\rho = \infty$, then $U = \{\xi \mid |\xi_j| \le \Psi\}$ is the box uncertainty set. In which Ψ is a parameter that control the size of the U. Note that, if $\Psi = 1$, then the box uncertainty set represent an interval uncertainty set [20].

Theorem 2.1. *Let $U = \{\xi \mid |\xi_j| \le \Psi\}$ is a box uncertainty set, and let $\tilde{a}x \le b$ be a constraint that the left hand side (LHS) coefficients are subject to uncertainty. Then, the corresponding RC constraint of $\tilde{a}x \le b$ is reduced to $ax + \max_{\xi \in U}(\xi \hat{a} x) \le b$ that can be written in the following equivalent constraint:*

$$ax + \Psi(\hat{a}|x|) \le b, \tag{6}$$

where $\hat{a}$ denotes the constant perturbation around a.

Proof. See [20]. □

Theorem 2.2. *Let $U = \{\xi \mid |\xi_j| \le \Psi\}$ is a box uncertainty set, consider the LFP problem as defined in (4), where the coefficients of the objective function are all under uncertainties. Then, the corresponding RC of the uncertain LFP is equivalent as the following problem:*

$$\begin{aligned} &\min z \\ &s.t.\ p^T y + \alpha t + \Psi(\hat{p}^T y + \hat{\alpha} t) \le z \\ &\quad q^T y + \beta t + \Psi(\hat{q}^T y + \hat{\beta} t) = 1 \\ &\quad Ay - bt \le 0 \\ &\quad y \ge 0,\ t \ge 0 \end{aligned} \tag{7}$$

Proof. For a given LFP.

$$\min \frac{\tilde{p}^T x + \tilde{\alpha}}{\tilde{q}^T x + \tilde{\beta}}$$

$$\text{s.t. } Ax \le b \qquad (8)$$
$$x \ge 0$$

in which

$$\begin{aligned} \widetilde{p} &= p + \widehat{p}\xi_p, \quad \xi_p \in U \\ \widetilde{q} &= q + \widehat{q}\xi_q, \quad \xi_q \in U \\ \widetilde{\alpha} &= \alpha + \widehat{\alpha}\xi_\alpha, \quad \xi_\alpha \in U \\ \widetilde{\beta} &= \beta + \widehat{\beta}\xi_\beta, \quad \xi_\beta \in U \end{aligned} \qquad (9)$$

where p, q, α and β denotes the nominal value parameters, $\widehat{p}$, q, $\widehat{\alpha}$ and $\widehat{\beta}$ denotes the true value parameters and constant perturbation around their nominal value parameters and also ξ_p, ξ_q, ξ_α and ξ_β are independent random variables. Now by letting

$$t = \frac{1}{\widetilde{q}^T x + \widetilde{\beta}} \quad \text{and} \quad y = xt \qquad (10)$$

and using the Charnes-Cooper transformation [5], we get a LP in the following form:

$$\begin{aligned} &\min \widetilde{p}^T y + \widetilde{\alpha} t \\ &\text{s.t. } \widetilde{q}^T y + \widetilde{\beta} t = 1 \\ &\quad Ay - bt \le 0 \\ &\quad y \ge 0,\ t \ge 0 \end{aligned} \qquad (11)$$

The above problem can be further equivalently transformed as follows:

$$\begin{aligned} &\min z \\ &\text{s.t. } \widetilde{p}^T y + \widetilde{\alpha} t \le z \\ &\quad \widetilde{q}^T y + \widetilde{\beta} t = 1 \\ &\quad Ay - bt \le 0 \\ &\quad y \ge 0,\ t \ge 0 \end{aligned} \qquad (12)$$

By substituting (9) in (12), and for immunize the problem (12) against infeasibility with uncertainty set U, we have

$$\begin{aligned} &\min z \\ &\text{s.t. } p^T y + \alpha t + \max_{\xi_p, \xi_\alpha \in U} (\widehat{p}^T \xi_p y + \widehat{\alpha}\xi_\alpha t) \le z \\ &\quad q^T y + \beta t + \max_{\xi_q, \xi_\beta \in U} (\widehat{q}^T \xi_q y + \widehat{\beta}\xi_\beta t) = 1 \\ &\quad Ay - bt \le 0 \\ &\quad y \ge 0,\ t \ge 0 \end{aligned} \qquad (13)$$

Now by using Theorem 2.1, we have

$$\begin{aligned} &\min z \\ &\text{s.t. } p^T y + \alpha t + \Psi(\widehat{p}^T |y| + \widehat{\alpha}|t|) \le z \\ &\quad q^T y + \beta t + \Psi(\widehat{q}^T |y| + \widehat{\beta}|t|) = 1 \end{aligned} \qquad (14)$$

$$Ay - bt \leq 0$$
$$y \geq 0,\ t \geq 0$$

Since y and $t \geq 0$, the above problem is equivalent to

$$\begin{aligned}
&\min z \\
&\text{s.t. } p^T y + \alpha t + \Psi(\widehat{p}^T y + \widehat{\alpha} t) \leq z \\
&\quad q^T y + \beta t + \Psi(\widehat{q}^T y + \widehat{\beta} t) = 1 \\
&\quad Ay - bt \leq 0 \\
&\quad y \geq 0,\ t \geq 0.
\end{aligned} \tag{15}$$

□

3. Robust Optimization and its Application to MOLFP Problem

MOLFP problem play a very important role rather than primary in optimization literature. Since the objective functions in MOLFP are in conflicts with each other, therefor we use the concept of Pareto optimality which is also called efficient solution to the problem.

Definition 3.1. Consider the MOLFP problem as defined in (1), a point $x^* \in R^n$ is called an efficient solution if there exists no $x \in R^n$ such that $\frac{N_i(x)}{D_i(x)} \geq \frac{N_i(x^*)}{D_i(x^*)}$, $i = 1,2,\dots,m$ and $\frac{N_i(x)}{D_i(x)} > \frac{N_i(x^*)}{D_i(x^*)}$, for at least one i, otherwise x^* is inefficient. The set of all efficient points is called efficient set solution.

Theorem 3.1. *Consider the MOLFP problem as defined in* (1)*, if x^* is an optimum solution of*

$$\begin{aligned}
&\max \left\{ \sum_{i=1}^{k} w_i \left(N_i(x) - (z_i)^* (D_i(x)) \right) \right\} \\
&s.t.\ Ax \leq b \\
&\quad x \geq 0
\end{aligned} \tag{16}$$

where $w_i \geq 0$, $\sum\limits_{i=1}^{k} w_i = 1$, and

$$(z_i)^* = \frac{N_i(x^*)}{D_i(x^*)} = \max_{x \in X} \frac{N_i(x)}{D_i(x)}, \quad i = 1,2,\dots,k. \tag{17}$$

Then x^* is an efficient solution of the MOLFP problem (1).

Proof. See [9]. □

Theorem 3.2. *Let $U = \{\xi \mid |\xi_j| \leq \Psi\}$ is a box uncertainty set. Consider the MOLFP problem as defined in* (1)*, where the coefficients of the objective functions are all under uncertainties. Then, the corresponding RC of the uncertain MOLFP is equivalent to the following LP problem:*

$$\max z$$

$$s.t.\ \left\{ \sum_{i=1}^{k} w_i \left(\left(\sum_{j=1}^{n} p_{ij}^T x_j + \alpha_i + \Psi \left(\sum_{j=1}^{n} \widehat{p}_{ij}^T x_j + \widehat{\alpha}_i \right) \right) - (z_i)^* \left(\sum_{j=1}^{n} q_{ij}^T x_j + \beta_i + \Psi \left(\sum_{j=1}^{n} \widehat{q}_{ij}^T x_j + \widehat{\beta}_i \right) \right) \right) \right\} \geq z \tag{18}$$

$$Ax \leq b$$

$x \geq 0$

Proof. Let the coefficients of the objective functions of the MOLFP problem (1) are all under uncertainties. In other words, suppose that

$$\begin{aligned} &\max z(x) = [z_1(x), z_2(x), \dots, z_k(x)] \\ &\text{s.t. } x \in X = \{x \mid Ax \leq b,\ x \geq 0\} \end{aligned} \tag{19}$$

in which

$$z_i(x) = \frac{\tilde{p}_i^T x + \tilde{\alpha}_i}{\tilde{q}_i^T x + \tilde{\beta}_i} = \frac{N_i(x)}{D_i(x)},$$

where $\tilde{p}_i, \tilde{q}_i, \tilde{\alpha}_i, \tilde{\beta}_i \in U, A \in R^{m\times n}, b \in R^m, x, \tilde{p}_i, \tilde{q}_i \in R^n, \tilde{\alpha}_i, \tilde{\beta}_i \in R$ and $D_i(x) > 0$,

$$i = 1, 2, \dots, k. \tag{20}$$

Now using Theorem 3.1, we have

$$\begin{aligned} &\max \left\{ \sum_{i=1}^{k} w_i \left(N_i(x) - (z_i)^* \left(D_i(x) \right) \right) \right\} \\ &\text{s.t. } Ax \leq b \\ &\quad x \geq 0 \end{aligned} \tag{21}$$

is equivalent to the following problem:

$$\begin{aligned} &\max \left\{ \sum_{i=1}^{k} w_i \left(\left(\sum_{j=1}^{n} \tilde{p}_{ij}^T x_j + \tilde{\alpha}_i \right) - (z_i)^* \left(\sum_{j=1}^{n} \tilde{q}_{ij}^T x_j + \tilde{\beta}_i \right) \right) \right\} \\ &\text{s.t. } Ax \leq b \\ &\quad x \geq 0 \end{aligned} \tag{22}$$

The problem (22) can be equivalently transformed as follows:

$$\begin{aligned} &\max z \\ &\text{s.t. } \sum_{i=1}^{k} w_i \left(\left(\sum_{j=1}^{n} \tilde{p}_{ij}^T x_j + \tilde{\alpha}_i \right) - (z_i)^* \left(\sum_{j=1}^{n} \tilde{q}_{ij}^T x_j + \tilde{\beta}_i \right) \right) \geq z \\ &\quad Ax \leq b \\ &\quad x \geq 0 \end{aligned} \tag{23}$$

Now with considering

$$\begin{aligned} &\tilde{p}_{ij} = p_{ij} + \hat{p}_{ij}\xi_{ij},\ \xi_{ij} \in U \qquad \tilde{q}_{ij} = q_{ij} + \hat{q}_{ij}\xi_{ij},\ \xi_{ij} \in U \\ &\tilde{\alpha}_i = \alpha_i + \hat{\alpha}_i \xi_{i\alpha},\ \xi_{i\alpha} \in U \qquad \tilde{\beta}_i = \beta_i + \hat{\beta}_i \xi_{i\beta},\ \xi_{i\beta} \in U \end{aligned} \tag{24}$$

and by substituting (24) in (23), and also for immunize the problem (23) against infeasibility with predefined uncertainty set U, we have

$$\begin{aligned} &\max z \\ &\text{s.t. } \sum_{i=1}^{k} w_i \left(\begin{array}{l} \left(\sum_{j=1}^{n} p_{ij}^T x_j + \alpha_i + \max_{\xi_{ij}, \xi_{i\alpha} \in U} \left(\sum_{j=1}^{n} \hat{p}_{ij}^T \xi_{ij} x_j + \hat{\alpha}_i \xi_{i\alpha} \right) \right) \\ -(z_i)^* \left(\sum_{j=1}^{n} q_{ij}^T x_j + \beta_i + \max_{\xi_{ij}, \xi_{i\beta} \in U} \left(\sum_{j=1}^{n} \hat{q}_{ij}^T \xi_{ij} x_j + \hat{\beta}_i \xi_{i\beta} \right) \right) \end{array} \right) \geq z \end{aligned} \tag{25}$$

$$Ax \leq b$$
$$x \geq 0$$

Now by using Theorem 2.1, we can convert the problem (25) to the following form:

$$\max z$$
$$\text{s.t.} \sum_{i=1}^{k} w_i \left(\left(\sum_{j=1}^{n} p_{ij}^T x_j + \alpha_i + \Psi \left(\sum_{j=1}^{n} \widehat{p}_{ij}^T |x_j| + \widehat{\alpha}_i \right) \right) - (z_i)^* \left(\sum_{j=1}^{n} q_{ij}^T x_j + \beta_i + \Psi \left(\sum_{j=1}^{n} \widehat{q}_{ij}^T |x_j| + \widehat{\beta}_i \right) \right) \right) \geq z \tag{26}$$
$$Ax \leq b$$
$$x \geq 0$$

Since $x \geq 0$, the above problem is equivalent to following LP problem:

$$\max z$$
$$\text{s.t.} \left\{ \sum_{i=1}^{k} w_i \left(\left(\sum_{j=1}^{n} p_{ij}^T x_j + \alpha_i + \Psi \left(\sum_{j=1}^{n} \widehat{p}_{ij}^T x_j + \widehat{\alpha}_i \right) \right) - (z_i)^* \left(\sum_{j=1}^{n} q_{ij}^T x_j + \beta_i + \Psi \left(\sum_{j=1}^{n} \widehat{q}_{ij}^T x_j + \widehat{\beta}_i \right) \right) \right) \right\} \geq z \tag{27}$$
$$Ax \leq b$$
$$x \geq 0$$

□

4. Results and Discussion

In this section we describe the methodology process and solve a numerical example to illustrate the methodology and proposed approach.

4.1 Solution Procedure

The solution procedure of MOLFP problem under uncertainty in the coefficients of the objective functions is describes as follows:

Step 1: Solve each of the objective function on using Theorem 2.2 which yields to different z_i^*.

$$z_i^* = \max_{x \in X} \frac{N_i(x)}{D_i(x)}, \quad i = 1, 2, \ldots, k. \tag{28}$$

Step 2: After obtaining each z_i^* from *Step* 1 and choosing appropriate normalized weights w_i, on using Theorem 3.2, we convert the MOLFP problem (19) to get a LP problem as given in (27).

Step 3: Find the optimal solution of the LP problem (27) by any usual method.

4.2 Numerical Example

Consider a MOLFP problem with two objectives as follows:

$$\max z(x) = \begin{bmatrix} z_1(x) = \dfrac{\widetilde{p}_{11}x_1 + \widetilde{p}_{12}x_2 + \widetilde{\alpha}_1}{\widetilde{q}_{11}x_1 + \widetilde{q}_{12}x_2 + \widetilde{\beta}_1} \\ z_2(x) = \dfrac{\widetilde{p}_{21}x_1 + \widetilde{p}_{22}x_2 + \widetilde{\alpha}_2}{\widetilde{q}_{21}x_1 + \widetilde{q}_{22}x_2 + \widetilde{\beta}_2} \end{bmatrix}$$
$$\text{s.t. } x_1 - x_2 \geq 1 \tag{29}$$
$$2x_1 + 3x_2 \leq 15$$

$$x_1 \geq 3$$
$$x_1, x_2 \geq 0$$

Assume that the objective coefficients $\widetilde{p}_{ij}$, $\widetilde{q}_{ij}$, $\widetilde{\alpha}_i$, $\widetilde{\beta}_i$ are subject to uncertainty and they are defined as follows:

$$\widetilde{p}_{ij} = p_{ij} + \widehat{p}_{ij}\xi_{ij}$$
$$\left(p_{11} = -3, p_{12} = 2, p_{21} = 7, p_{22} = 1 \text{ and } \widehat{p}_{ij} = 0.1p_{ij},\ i,j = 1,2\right)$$
$$\widetilde{q}_{ij} = q_{ij} + \widehat{q}_{ij}\xi_{ij}$$
$$\left(q_{11}, q_{12} = 1, q_{21} = 5, q_{22} = 2 \text{ and } \widehat{q}_{ij} = 0.1q_{ij},\ \ i,j = 1,2\right)$$
$$\widetilde{\alpha}_i = \alpha_i + \widehat{\alpha}_i\xi_{i\alpha}$$
$$\left(\alpha_1, \alpha_2 = 0 \text{ and } \widehat{\alpha}_i = 0.1\alpha_{ij}, i = 1,2\right)$$
$$\widetilde{\beta}_i = \beta_i + \widehat{\beta}_i\xi_{i\beta}$$
$$\left(\beta_1 = 3, \beta_2 = 1 \text{ and } \widehat{\beta}_i = 0.1\beta_i, i = 1,2\right)$$

We first calculate $(z_1)^*$ as below

$$\begin{aligned} &\max z_1(x) = \frac{\widetilde{p}_{11}x_1 + \widetilde{p}_{12}x_2 + \widetilde{\alpha}_1}{\widetilde{q}_{11}x_1 + \widetilde{q}_{12}x_2 + \widetilde{\beta}_1} \\ &\text{s.t. } x_1 - x_2 \geq 1 \\ &\quad 2x_1 + 3x_2 \leq 15 \\ &\quad x_1 \geq 3 \\ &\quad x_1, x_2 \geq 0 \end{aligned} \tag{30}$$

Now on letting

$$t = \frac{1}{\widetilde{q}_{11}x_1 + \widetilde{q}_{12}x_2 + \widetilde{\beta}_1} \quad \text{and} \quad y = xt \tag{31}$$

and using the Charnes-Cooper transformation, we have

$$\begin{aligned} &\max z_1 = \widetilde{p}_{11}y_1 + \widetilde{p}_{12}y_2 + \widetilde{\alpha}_1 t \\ &\text{s.t. } \widetilde{q}_{11}y_1 + \widetilde{q}_{12}y_2 + \widetilde{\beta}_1 t = 1 \\ &\quad y_1 - y_2 - t \geq 0 \\ &\quad 2y_1 + 3y_2 - 15t \leq 0 \\ &\quad y_1 - 3t \geq 0 \\ &\quad y_1, y_2, t \geq 0 \end{aligned} \tag{32}$$

The above problem can be rewritten as the following:

$$\begin{aligned} &\max z_1 = -3y_1 + 2y_2 + \Psi(-0.3y_1 + 0.2y_2) \\ &\text{s.t. } y_1 + y_2 + 3t + \Psi(0.1y_1 + 0.1y_2 + 0.3t) = 1 \\ &\quad y_1 - y_2 - t \geq 0 \\ &\quad 2y_1 + 3y_2 - 15t \leq 0 \\ &\quad y_1 - 3t \geq 0 \\ &\quad y_1, y_2, t \geq 0 \end{aligned} \tag{33}$$

The optimal solution of the above LP with $\Psi = 1$ is obtained as

$$y_1 = 0.1957,\ y_2 = 0.1413,\ t = 0.0543 \tag{34}$$

Now by substituting (34) in (31) we obtain the optimum solution $x_1^* = 3.6$, $x_2^* = 2.6$ with the optimum value of the objective function as $(z_1)^* = -0.6087$.

Similarly, we can calculate $(z_2)^*$ as follows:

$$\begin{aligned}
&\max z_2(x) = \frac{\widetilde{p}_{21}x_1 + \widetilde{p}_{22}x_2 + \widetilde{\alpha}_2}{\widetilde{q}_{21}x_1 + \widetilde{q}_{22}x_2 + \widetilde{\beta}_2} \\
&\text{s.t. } x_1 - x_2 \geq 1 \\
&\quad 2x_1 + 3x_2 \leq 15 \\
&\quad x_1 \geq 3 \\
&\quad x_1, x_2 \geq 0
\end{aligned} \tag{35}$$

Now on letting

$$t = \frac{1}{\widetilde{q}_{21}x_1 + \widetilde{q}_{22}x_2 + \widetilde{\beta}_2} \quad \text{and } y = xt \tag{36}$$

and using the Charnes-Cooper transformation, we have

$$\begin{aligned}
&\max z_2 = \widetilde{p}_{21}y_1 + \widetilde{p}_{22}y_2 + \widetilde{\alpha}_2 t \\
&\text{s.t. } \widetilde{q}_{21}y_1 + \widetilde{q}_{22}y_2 + \widetilde{\beta}_2 t = 1 \\
&\quad y_1 - y_2 - t \geq 0 \\
&\quad 2y_1 + 3y_2 - 15t \leq 0 \\
&\quad y_1 - 3t \geq 0 \\
&\quad y_1, y_2, t \geq 0
\end{aligned} \tag{37}$$

The above problem can be rewritten as the following:

$$\begin{aligned}
&\max z_2 = 7y_1 + y_2 + \Psi(0.7y_1 + 0.1y_2) \\
&\text{s.t. } 5y_1 + 2y_2 + t + \Psi(0.5y_1 + 0.2y_2 + 0.1t) = 1 \\
&\quad y_1 - y_2 - t \geq 0 \\
&\quad 2y_1 + 3y_2 - 15t \leq 0 \\
&\quad y_1 - 3t \geq 0 \\
&\quad y_1, y_2, t \geq 0
\end{aligned} \tag{38}$$

The optimal solution of the above LP with $\Psi = 1$ is obtained as

$$y_1 = 0.0974,\ y_2 = 0,\ t = 0.0130 \tag{39}$$

Now by substituting (39) in (36) we obtain the optimum solution $x_1^* = 7.5$, $x_2^* = 0$ with the optimum value of the objective function as $(z_2)^* = 1.3636$.

On using $(z_1)^* = -0.6087$ and $(z_2)^* = 1.3636$, the problem (31) is reformulated in a LP model as the following:

$$\begin{aligned}
&\max z \\
&\text{s.t. } w_1\big((-3x_1 + 2x_2 + \Psi(-0.3x_1 + 0.2x_2)) - (z_1)^*(x_1 + x_2 + 3 + \Psi(0.1x_1 + 0.1x_2 + 0.3)) \big) \\
&\qquad + w_2\big((7x_1 + x_2 + \Psi(0.7y_1 + 0.1y_2)) - (z_2)^*(5y_1 + 2y_2 + 1 + \Psi(0.5x_1 + 0.2x_2 + 0.1)) \big) \geq z \quad (40) \\
&\quad x_1 - x_2 \geq 1
\end{aligned}$$

$2x_1 + 3x_2 \le 15$

$x_1 \ge 3$

$x_1, x_2 \ge 0$

The optimal solution of the above LP problem with $w_1 = w_2 = 0.5$ and $\Psi = 1$ is obtained as $x_1^{**} = 3$, $x_2^{**} = 2$.

Finally, the efficient solution of the MOLFP problem (29) is given by $z_1^{**} = -0.6250$, $z_2^{**} = 1.1500$.

5. Conclusion

MOLFP problem under uncertainty in the coefficients of the objective functions and relationship between its robust counterparts is discussed in this article. We used box uncertainty set for MOLFP problem and proposed corresponding RC formulation by reducing it into a single objective LP problem. Furthermore, it is shown that the corresponding RC formulation of MOLFP problem under box uncertainty set is a linear programming problem. Finally in a numerical example we have shown the methodology and proposed approach.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] A. Ben-Tal and A. Nemirovski, Robust solutions of uncertain linear programs, *Operations Research Letters* **25**(1) (1999), 1 – 13, DOI: 10.1016/S0167-6377(99)00016-4.

[2] D. Bertsimas and M. Sim, Robust discrete optimization and network flows, *Mathematical Programming* **98**(1-3) (2003), 49 – 71, DOI: 10.1007/s10107-003-0396-4.

[3] D. Bertsimas, D. B. Brown and Caramanis, Theory and applications of robust optimization, *SIAM Review* **53**(3) (2011), 464 – 501, DOI: 10.1137/080734510.

[4] M. Chakraborty and S. Gupta, Fuzzy mathematical programming for multi objective linear fractional programming problem, *Fuzzy Sets and Systems* **125**(3) (2002), 335 – 342, DOI: 10.1016/S0165-0114(01)00060-4.

[5] A. Charnes and W. W. Cooper, Programming with linear fractional functional, *Naval Research Logistics Quarterly* **9** (1962), 181 – 186, DOI: 10.1002/nav.3800090303.

[6] T. M. Duran, Taylor series approach to fuzzy multiobjective linear fractional programming, *Information Sciences* **178**(4) (2008), 1189 – 1204, DOI: 10.1016/j.ins.2007.06.010.

[7] L. El Ghaoui, F. Oustry and H. Lebret, Robust solutions to uncertain semi definite programs, *SIAM Journal on Optimization* **9**(1) (1998), 33 – 52, DOI: 10.1137/S1052623496305717.

[8] M. A. Goberna, V. Jeyakumar, G. Li and J. Vicente-Pérez, Robust solutions to multi-objective linear programs with uncertain data, *European Journal of Operational Research* **242**(3) (2015), 730 – 743, DOI: 10.1016/j.ejor.2014.10.027.

[9] N. Güzel and M. Sivri, Proposal of a solution to multiobjective linear fractional programming problem, *Sigma journal of Engineering and Natural Sciences* **2** (2005), 43 – 50, `http://www.ytusigmadergisi.com/scientific/2005-2-5-tam.pdf`.

[10] F. Hassanzadeh, H. Nemati and M. Sun, Robust optimization for interactive multiobjective programming with imprecise information applied to R&D project portfolio selection, *European Journal of Operational Research* **238**(1) (2014), 41 – 53, DOI: 10.1016/j.ejor.2014.03.023.

[11] S. L. Janak, X. Lin and C. A. Floudas, A new robust optimization approach for scheduling under uncertainty: II. Uncertainty with known probability distribution, *Computers & Chemical Engineering* **31**(3) (2007), 171 – 195, DOI: 10.1016/j.compchemeng.2006.05.035.

[12] J. S. H. Kornbluth and R. E. Steuer, Goal programming with linear fractional criteria, *European Journal of Operational Research* **8**(1) (1981), 58 – 65, DOI: 10.1016/0377-2217(81)90029-1.

[13] J. S. H. Kornbluth and R. E. Steuer, Multiple objective linear fractional programming, *Management Science* **27**(9) (1981), 1024 – 1039, DOI: 10.1287/mnsc.27.9.1024.

[14] X. Lin, S. L. Janak and C. A. Floudas, A new robust optimization approach for scheduling under uncertainty: I. Bounded uncertainty, *Computers & Chemical Engineering* **28**(6-7) (2004), 1069 – 1085, DOI: 10.1016/j.compchemeng.2003.09.020 .

[15] I. Nykowski and Z. Zolkiewski, A compromise procedure for the multiple objective fractional programming problem, *European Journal of Operational Research* **19**(1) (1985), 91 – 97, DOI: 10.1016/0377-2217(85)90312-1.

[16] A. L. Soyster, Convex programming with set-inclusive constraints and applications to inexact linear programming, *Operations Research* **21**(5) (1973), 1154 – 1157, DOI: 10.1287/opre.21.5.1154.

[17] N. A. Sulaiman and B. K. Abdulrahim, Using transformation technique to solve multi-objective linear fractional programming problem, *International Journal of Research and Reviews in Applied Sciences* **14**(3) (2013), 559 – 567, `https://www.arpapress.com/Volumes/Vol14Issue3/IJRRAS_14_3_09.pdf`.

[18] N. A. Sulaiman, G. W. Sadiq and B. K. Abdulrahim, New arithmetic average technique to solve multiobjective linear fractional programming problem and it is comparison with other techniques, *International Journal of Research and Reviews in Applied Sciences* **18**(2) (2014), 122 – 131, `https://www.arpapress.com/Volumes/Vol18Issue2/IJRRAS_18_2_03.pdf`.

[19] H. J. Zimmermann, Fuzzy mathematical programming, *Computer and Operations Research* **10**(4) (1983), 291 – 298, DOI: 10.1016/0305-0548(83)90004-7.

[20] L. Zukui, D. Ran and A. F. Christodoulos, A comparative theoretical and computational study on robust counterpart optimization: I. Robust linear optimization and robust mixed integer linear optimization, *Industrial Engineering Chemistry Research* **50** (2011), 10567 – 10603, DOI: 10.1021/ie200150p.