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Research Article

On Soft Ig^* Closed Sets in Soft Ideal Topological Spaces

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Abstract. In this paper we introduce and study the notions of soft Ig^* closed sets in soft ideal topological spaces and investigate some of their properties.

Keywords. Soft Ig closed sets; Soft Ig^* closed sets; sIg^* closure; sIg^* interior

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [11] in 1999 as a general mathematical tool for dealing with uncertain objects. In [12], Molodtsov successfully applied the soft set theory in several directions, such as Smoothness, Gametheory, Operations research, Riemann integration and so on. After the introduction of soft sets [10], Shabir and Naz [14] initiated the study of soft topological spaces. Consequently the basic properties of soft sets in soft topological spaces were studied by several authors [1–3, 15, 16]. The notion of soft generalized closed sets was introduced by Kannan in [7]. The concept of soft g^* closed sets was introduced by Kalavathi *et al*. [8]. The notion of soft ideal in soft set theory was first given by Kandil *et al*. [6]. They also introduced soft local function in soft ideal topological spaces. Kale and Guler [9] studied the properties of soft ideal topological spaces. Then, Mustafa and Sleim in [13] introduced a different version of soft ideal. The concepts which are introduced in [4] were extended to soft ideal in [5].

2. Preliminaries

Definition 2.1 ([11]). Let X be an initial universal set and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universal set X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \phi$ i.e. $F_A = \{F(e) : e \in A \subseteq E, F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2 ([10]). Let $F_A, G_B \in SS(X)_E$. Then F_A is said to be a soft subset of G_B, denoted by $F_A \widetilde{\subseteq} G_B$, if

(1) $A \subseteq B$, and

(2) $F(e) \widetilde{\subseteq} G(e)$, for all $e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \widetilde{\supseteq} F_A$.

Definition 2.3 ([1]). The complement of a soft set (F,A), denoted by $(F,A)'$, is defined by $(F,A)' = (F',A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X \setminus F(e)$, for all $e \in A$ and F' is called the soft complement function of F. Clearly, $(F')'$ is the same as F and $((F,A)')' = (F,A)$.

Definition 2.4 ([14]). The difference of two soft sets (F,E) and (G,E) over the common universal set X, denoted by $(F,E) \setminus (G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) \setminus G(e)$.

Definition 2.5 ([14]). Let (F,E) be a soft over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.6 ([14]). Let $x \in X$. Then the soft set (x,E) over the common universal set X, where $x_E(e) = \{x\}$, for all $e \in E$, called the singleton soft point and denoted by x_E.

Definition 2.7 ([10]). A soft set (F,A) over the common universal set X is said to be a NULL soft set denoted by $\widetilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 2.8 ([10]). A soft set (F,A) over the common universal set X is said to be an absolute soft set denoted by $\widetilde{X}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly, we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 2.9 ([10]). The union of two soft sets (F,A) and (G,B) over the common universal set X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), \ e \in A \setminus B, \\ G(e), \ e \in B \setminus A, \\ F(e)\tilde{\cup}G(e), \ e \in A \cap B. \end{cases}$$

We write $(F,A)\tilde{\cup}(G,B) = (H,C)$.

Definition 2.10 ([10]). The intersection of two soft sets (F,A) and (G,B) over the common universal set X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e)\tilde{\cap}G(e)$. We write $(F,A)\tilde{\cap}(G,B) = (H,C)$.

Definition 2.11 ([14]). Let τ be a collection of soft sets over a universal set X with a fixed set of parameters E, then $\tau \in SS(X)_E$ is called a soft topology on X if

(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, for all $e \in E$,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. The members of τ are said to be soft open sets in X.

Definition 2.12 ([14]). Let (X,τ,E) be a soft topological space. A soft set (F,A) over the common universal set X is said to be soft closed set in X, if its complement $(F,A)'$ is a soft open set.

We denote the set of all soft open sets over X by $SO(X)$ and the set of all soft closed sets by SC(X).

Definition 2.13 ([14]). Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all soft closed super sets of (F,E). Clearly, $cl(F,E)$ is the smallest soft closed set over X which contains (F,E) i.e. $cl(F,E) = \tilde{\cap}\{(H,E) : (H,E)$ is soft closed set and $(F,E)\tilde{\subseteq}(H,E)\}$.

Definition 2.14 ([16]). Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all soft open subsets of (G,E). Clearly, $int(G,E)$ is the largest soft open set over X which contained in (G,E) i.e. $int(G,E) = \tilde{\cup}\{(H,E) :$ (H,E) is an soft open set and $(H,E)\tilde{\subseteq}(G,E)\}$.

Definition 2.15 ([16]). The soft set $(F,E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = x$ and $F(e') = \phi$ for each $e' \in E \setminus e$, and the soft point (F,E) is denoted by x_e.

Definition 2.16 ([16]). The soft point x_e is an element of the soft set (G,A), denoted by $x_e \in (G,A)$, if for the element $e \in A$, $F(e)\tilde{\subseteq}G(e)$.

Definition 2.17 ([16]). A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood (briefly : nbd) of the soft point $x_e \in X_E$ if there exists a soft open set (H,E) such that $x_e \in (H,E) \widetilde{\subseteq} (G,E)$.

A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood of the soft set (F,E) if there exists a soft open set (H,E) such that $(F,E) \in (H,E) \widetilde{\subseteq} (G,E)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.

Definition 2.18 ([14]). Let (X,τ,E) be a soft topological space over the common universal set X and Y be a non null soft subset of X. Then $\widetilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e) = Y$ for all $e \in E$.

Definition 2.19 ([14]). Let (X,τ,E) be a soft topological space over the common universal set X, $(F,E) \in SS(X)_E$ and Y be a non null soft subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y,E), is defined as follows:

$$F_Y(e) = Y \widetilde{\cap} F(e) \quad \text{for all } e \in E.$$

In other words $(F_Y,E) = \widetilde{Y} \widetilde{\cap} (F,E)$.

Definition 2.20 ([14]). Let (X,τ,E) be a soft topological space over the common universal set X and Y be a non null soft subset of X. Then

$$\tau_Y = \{(F_Y,E) : (F,E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Definition 2.21. Let (X,τ,E) be a soft topological space over the common universal set X and $(F,E) \in SS_(X)$. Then (F,E) is said to be

(1) Soft generalized closed set (soft g closed) [7] in soft topological space (X,τ,E) if $cl(F,E) \widetilde{\subseteq} (U,E)$ whenever $(F,E) \widetilde{\subseteq} (U,E)$ and (U,E) is soft open set in X.

(2) Soft Regular closed set [2] in a soft topological space if $(F,E) = cl(int(F,E))$.

(3) Soft Q set [2] in a soft topological space (X,τ,E) if and only if $int(cl(F,E)) = cl(int(F,E))$.

(4) Soft g^* closed set [8] in a soft topological space (X,τ,E) if $cl(F,E) \widetilde{\subseteq} (U,E)$ whenever $(F,E) \widetilde{\subseteq} (U,E)$ and (U,E) is soft g open set in X.

Definition 2.22 ([6]). Let $\widetilde{I}$ be a non-null collection of soft sets over a universal set X with a fixed set of parameters E, then $\widetilde{I} \widetilde{\subseteq} SS(X)_E$ is called a soft ideal on X with a fixed set E if

(1) $(F,E) \in \widetilde{I}$ and $(G,E) \in \widetilde{I} \Rightarrow (F,E) \widetilde{\cup} (G,E) \in \widetilde{I}$,

(2) $(F,E) \in \widetilde{I}$ and $(G,E) \widetilde{\subseteq} (F,E) \Rightarrow (G,E) \in \widetilde{I}$,

i.e. $\widetilde{I}$ is closed under finite soft union and soft subsets.

Definition 2.23 ([6]). Let $(X,\tau,E,\widetilde{I})$ be a soft topological space with a soft ideal $\widetilde{I}$ over X with the set of parameters E. Then

$$(F,E)^*(\widetilde{I},\tau) \text{ (or } (F_E^*) = \widetilde{\cup}\{x_e \in \varepsilon : O_{x_e} \widetilde{\cap} (F,E) \notin \widetilde{I} \text{ for all } O_{x_e} \in \tau\}$$

is called the soft local function of (F,E) with respect to $\widetilde{I}$ and τ, where O_{x_e} is a τ-open soft set containing x_e.

Definition 2.24 ([6]). Let $(X,\tau,E,\widetilde{I})$ be a soft topological space with a soft ideal $\widetilde{I}$ over the common universal set X with the set of parameters E. Then the soft closure operator $cl^*: SS(X)_E \to SS(X)_E$ defined by:

$$cl^*(F,E) = (F,E)\widetilde{\cup}(F,E)^*$$

satisfies Kuratowski's axioms.

Definition 2.25 ([6]). Let $(X,\tau,E,\widetilde{I})$ be a soft topological space with a soft ideal $\widetilde{I}$ over X with the set of parameters E and $cl^*: SS(X)_E \to SS_(X)_E$ be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E, finer than τ, called the $*$-soft topology, denoted by $\tau^*(\widetilde{I})$ or τ^*, given by

$$\tau^*(\widetilde{I}) = \{(F,E) \in SS(X)_E : cl^*(F,E)' = (F,E)'\}.$$

Definition 2.26 ([5]). Let $(X,\tau,E,\widetilde{I})$ be a soft topological space with a soft ideal $\widetilde{I}$ over X. A soft set $(F,E) \in SS(X)_E$ is called

(1) Soft pre-$\widetilde{I}$-closed set if $cl(int^*(F,E))\widetilde{\subseteq}(F,E)$.

(2) Soft α-$\widetilde{I}$-closed set if $cl(int^*(cl((F,E)))\widetilde{\subseteq}(F,E)$.

(3) Soft semi-$\widetilde{I}$-closed set if $int^*(cl(F,E))\widetilde{\subseteq}(F,E)$.

(4) Soft β-$\widetilde{I}$-closed set if $int(cl(int^*(F,E))\widetilde{\subseteq}(F,E)$.

Definition 2.27 ([6]). A soft topological space (X,τ,E) together with a soft ideal $\widetilde{I}$ is defined as soft ideal topological space over the common universal set X and it is denoted by $(X,\tau,E,\widetilde{I})$.

3. Soft Ig^* closed set in soft ideal topological spaces

Definition 3.1. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. A soft set $(F,E) \in SS(X)_E$ is called a soft Ig closed set if $cl^*(F,E)\widetilde{\subseteq}(U,E)$ whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) is a soft open set.

Definition 3.2. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. A soft set $(F,E) \in SS(X)_E$ is called a soft Ig^* closed set if $cl^*(F,E)\widetilde{\subseteq}(U,E)$ whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) is a soft g open set.

Theorem 3.3. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X then every soft closed set is a soft Ig^* closed set.*

Proof. Proof of this theorem is obvious from the definition of soft closed set. □

Theorem 3.4. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. Every soft g^* closed set is a soft Ig^* closed set.*

Proof. Proof of this theorem is obvious from the definition of soft g^* closed set. □

Theorem 3.5. *Let* $(X,\tau,E,\widetilde{I})$ *be a soft ideal topological space over X, then every soft Ig* * *closed set is a soft Ig closed set.*

Proof. Proof of this theorem is obvious from the definition of soft Ig^* closed set. □

Remark 3.6. The converse of the above theorems need not be true in general as shown from the following example.

Example 3.7. Let $X = \{h_1,h_2,h_3\}, E = \{e_1,e_2\}$ and
$\tau = \{\phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)\}$ where
$(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_3\})\},(F_2,E) = \{(e_1,\{h_2\}),(e_2,\{h_1\})\}$,
$(F_3,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_3\})\},(F_4,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{\phi\})\}$,
$(F_5,E) = \{(e_1,\{h_1\}),(e_2,\{\phi\})\},(F_6,E) = \{(e_1,\{h_2\}),(e_2,\{\phi\})\}$,
$(F_7,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_3\})\},(F_8,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1\})\}$.
Let $\widetilde{I} = \{\phi,(A_1,E),(A_2,E),(A_3,E)\}$ where $(A_1,E) = \{(e_1,\{h_2\}),(e_2,\{h_1\})\}$,
$(A_2,E) = \{(e_1,\{h_2\}),(e_2,\{\phi\})\},(A_3,E) = \{(e_1,\{\phi\}),(e_2,\{h_1\})\}$.
Then $(X,\tau,E,\widetilde{I})$ is a soft ideal topological space over X. A soft set $(A,E) = \{(e_1,\{h_2\}),(e_2,\{\phi\})\}$ is a soft Ig^* closed set but not a soft g^* closed set and soft closed set.A soft set $(B,E) = \{(e_1,\{h_1\}),(e_2,\{h_2\})\}$ is soft *Ig* closed set but not a soft Ig^* closed set.

Theorem 3.8. *Let* $(X,\tau,E,\widetilde{I})$ *be a soft ideal topological space over X. If* (A,E)*and* (B,E) *are soft* Ig^* *closed sets, then* $(A,E)\widetilde{\cup}(B,E)$ *is also a soft* Ig^* *closed set.*

Proof. Let $(A,E)$and (B,E) are soft Ig^* closed sets in X. Let $(A,E)\widetilde{\cup}(B,E)\widetilde{\subseteq}(G,E)$ and (G,E) is a soft g open set. Then $(A,E)\widetilde{\subseteq}(G,E)$ and $(B,E)\widetilde{\subseteq}(G,E)$. Since both (A,E) and (B,E) are soft Ig^*closed sets, then $cl^*(A,E)\widetilde{\subseteq}(G,E)$ whenever $(A,E)\widetilde{\subseteq}(G,E)$ and $cl^*(B,E)\widetilde{\subseteq}(G,E)$ whenever $(B,E)\widetilde{\subseteq}(G,E)$. We know that $cl^*((A,E)\widetilde{\cup}(B,E)) = ((A,E)\widetilde{\cup}(B,E))\widetilde{\cup}((A,E\widetilde{)}\cup(B,E))^* = ((A,E)\widetilde{\cup}(B,E))\widetilde{\cup}((A,E)^*\widetilde{\cup}(B,E)^*) = ((A,E)\widetilde{\cup}(A,E)^*)\widetilde{\cap}((B,E)\widetilde{\cup}(B,E)^*) = cl^*(A,E)\widetilde{\cup}cl^*(B,E)$.
Hence $cl^*((A,E)\widetilde{\cup}(B,E)) = cl^*(A,E)\widetilde{\cup}cl^*(B,E)\widetilde{\subseteq}(U,E)$. Thus $(A,E)\widetilde{\cup}$ (B,E) is a soft Ig^* closed set. □

Remark 3.9. The following example shows that the intersection of two soft Ig^* closed sets need not be a soft Ig^* closed set in a soft ideal topological space $(X,\tau,E,\widetilde{I})$.

Example 3.10. Let $X = \{h_1,h_2,h_3\}), E = \{e_1,e_2\}$ and
$\tau = \{\widetilde{\phi},\widetilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E)\}$ where $(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_2\})\}$,
$(F_2,E) = \{(e_1,\{h_3\}),(e_2,\{h_3\})\},(F_3,E) = \{(e_1,\{h_1,h_3\}),(e_2,\{h_2,h_3\})\}$,
$(F_4,E) = \{(e_1,\{h_1,h_3\}),(e_2,\{h_2\})\},(F_5,E) = \{(e_1,\{h_3\}),(e_2,\{\phi\})\}$.
Let $\widetilde{I} = \{\widetilde{\phi},(A,E),(B,E),(C,E)\}$ where $(A,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{\phi\}\}$,
$(B,E) = \{(e_1,\{h_1\}),(e_2,\{\phi\})\},(C,E) = \{(e_1,\{h_2\}),(e_2,\{\phi\})\}$.

Then $(X,\tau,E,\widetilde{I})$ is a soft ideal topological space over X. Now the soft set$(A_1,E) = \{(e_1,\{h_1\}),(e_2,\{X\})\}$,$(A_2,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1\})\}$are soft Ig^* closed sets but their intersection $(A_1,E)\widetilde{\cap}(A_2,E)$is not a soft Ig^* closed set.

Remark 3.11. In a soft topological space $(X,\tau,E,\widetilde{I})$, the concept of soft Ig^* closed sets and soft semi-$\widetilde{I}$- closed(soft α-$\widetilde{I}$- closed, soft pre-$\widetilde{I}$-closed, soft β-$\widetilde{I}$-closed) sets are independent. In the above Example 3.10,

The soft set $(D,E) = \{(e_1,\{h_3\}),(e_2,\{h_1,h_3\})\}$ is a soft semi-$\widetilde{I}$-closed set but not a soft Ig^* closed set.
The soft set $(G,E) = \{(e_1,\{h_2\}),(e_2,\{h_1,h_2\})\}$ is a soft Ig^* closed set but not a soft semi-$\widetilde{I}$-closed set.

The soft set $(H,E) = \{(e_1,\{h_2\}),(e_2,\{h_3\})\}$ is a soft pre-$\widetilde{I}$-closed set but not a soft Ig^* closed set.
The soft set $(I,E) = \{(e_1,\{h_2\}),(e_2,\{X\})\}$ is a soft Ig^* closed set but not a soft pre-$\widetilde{I}$-closed set.
The soft set $(J,E) = \{(e_1,\{h_2\}),(e_2,\{\phi\})\}$ is a soft α-$\widetilde{I}$-closed set but not a soft Ig^* closed set. The soft set$(K,E) = \{(e_1,\{h_1\}),(e_2,\{\phi\})\}$ is a soft Ig^* closed set but not a soft α-$\widetilde{I}$-closed set.
The soft set $(L,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_2\})\}$ is a soft β-$\widetilde{I}$-closed set but not a soft Ig^* closed set.
The soft set $(G,E) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_1\})\}$ is a soft Ig^* closed set but not a soft β-$\widetilde{I}$-closed set.

Theorem 3.12. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. If (A,E) is a soft Ig^* closed set in X and $(A,E)\widetilde{\subseteq}(B,E)\widetilde{\subseteq}cl^*(A,E)$, then (B,E) is a soft Ig^* closed set.*

Proof. Suppose that (A,E) is a soft Ig^* closed set in X and $(A,E)\widetilde{\subseteq}(B,E)\widetilde{\subseteq}cl^*(A,E)$. Let $(B,E)\widetilde{\subseteq}(U,E)$ and (U,E) is a soft g open set in X. Since (A,E) is a soft Ig^* closed, hence $cl^*(A,E)\widetilde{\subseteq}(U,E)$ whenever $(A,E)\widetilde{\subseteq}(U,E)$. Since $(B,E)\widetilde{\subseteq}cl^*(A,E)$, hence $cl^*(B,E)\widetilde{\subseteq}cl^*(A,E)\widetilde{\subseteq}(U,E)$. Thus (B,E) is a soft Ig^* closed set. □

Theorem 3.13. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. A soft set (A,E) is a soft Ig^* closed set if and only if $cl^*(A,E)\setminus(A,E)$ contains null soft g closed set.*

Proof. Suppose that (A,E) is soft Ig^* closed set in X. Let (F,E) be a non null soft g closed set such that $(F,E)\widetilde{\subseteq}cl^*(A,E)\setminus(A,E)$. Then $(F,E)'$ is a soft g open set. Now $(A,E)\widetilde{\subseteq}(F,E)'$ and $(A,E)\widetilde{\subseteq}cl^*(A,E)$. Since (A,E) is a soft g^* closed set, $cl^*(A,E)\widetilde{\subseteq}(F,E)'$. Hence $(F,E)\widetilde{\subseteq}(cl^*(A,E))'$. Thus (F,E) is null soft g closed set. Conversely, suppose $cl^*(A,E)\setminus(A,E)$ contains only null soft g closed set (F,E). We have $(F,E)'$ is a soft g open set in X and $(F,E)\widetilde{\subseteq}cl^*(A,E)$ and $(F,E)\widetilde{\subseteq}(A,E)'$. Hence $(A,E)\widetilde{\subseteq}(F,E)'$, since (F,E) is null soft g closed set. Hence $cl^*(A,E)\widetilde{\subseteq}(F,E)'$. □

Theorem 3.14. *Let$(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X then either e_F is soft g closed set or $\widetilde{X}\setminus e_F$ is soft Ig^* closed set.*

Proof. If suppose e_F is not a soft g closed set, then $\widetilde{X}\setminus e_F$ is not a soft g open set. Since $\widetilde{X}$ is the only soft g open set containing $\widetilde{X}\setminus e_F$. Hence $\widetilde{X}\setminus e_F$ is a soft Ig^* closed set. □

Theorem 3.15. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. If $(A,E)$is soft open and soft g^* closed set in$(X,\tau,E,\widetilde{I})$, then the following conditions are hold.*

(1) *(A,E) is soft closed set.*

(2) *$(A,E)$is soft Ig^* closed set.*

(3) *(A,E) is soft*

(4) *(A,E) is soft regular closed set.*

(5) *(A,E) is soft Q set.*

Proof. Proof is an immediate consequence of its definitions. □

Theorem 3.16. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X and $(A,E)\widetilde{\subseteq}\widetilde{Y}\widetilde{\subseteq}\widetilde{X}$, $\widetilde{Y}$ is a soft subspace of $\widetilde{X}$. Suppose that (A,E) is a soft Ig^* closed set in $(X,\tau,E,\widetilde{I})$, then (A,E) is a soft Ig^* closed set relative to $\widetilde{Y}$.*

Proof. Suppose (A,E) be a soft set in $(Y,\tau_Y,E,\widetilde{I_Y})$ such that (A,E) is a soft Ig^* closed set in $(X,\tau,E,\widetilde{I})$. Let $(A,E)\widetilde{\subseteq}\widetilde{Y}\widetilde{\cap}(G,E)$ where (G,E) is a soft g open set. Then $(A,E)\widetilde{\subseteq}(G,E)$ and $cl^*(A,E)\widetilde{\subseteq}(G,E)$. This implies that $\widetilde{Y}\widetilde{\cap}cl^*(A,E)\widetilde{\subseteq}\widetilde{Y}\bigcap(G,E)$. Thus (A,E) is a soft Ig^* closed set relative to $\widetilde{Y}$. □

Theorem 3.17. *If (A,E) is a soft Ig^* closed set and (F,E) is a soft closed set in a soft ideal topological space $(X,\tau,E,\widetilde{I})$ over X, then $(A,E)\widetilde{\cap}(F,E)$ is a soft Ig^* closed set.*

Proof. Suppose$(A,E)\widetilde{\cap}(F,E)\widetilde{\subseteq}(G,E)and(G,E)$is soft g open set. Then $(A,E)\widetilde{\subseteq}(G,E)\widetilde{\cup}(F,E)'$. Since (A,E) is a soft Ig^* closed set, we have $cl^*(A,E)\widetilde{\subseteq}(G,E)\widetilde{\cup}(F,E)'$ whenever $(A,E)\widetilde{\subseteq}(G,E)\widetilde{\cup}(F,E)'$. Now, $cl^*((A,E)\widetilde{\cap}(F,E))=((A,E)\widetilde{\cap}(F,E))\widetilde{\cup}((A,E)\widetilde{\cup}(F,E))^*\widetilde{\subseteq}((A,E)\widetilde{\cap}(F,E))\widetilde{\cup}((A,E)^*\widetilde{\cap}(F,E)^*)=((A,E)\widetilde{\cup}(A,E)^*)\widetilde{\cap}((F,E)\widetilde{\cup}(F,E)^*)=cl^*(A,E)\widetilde{\cap}cl^*(F,E)$. This implies that $cl^*((A,E)\widetilde{\cap}(F,E))\widetilde{\subseteq}cl^*(A,E)\widetilde{\cap}cl^*(F,E)\widetilde{\subseteq}(G,E)\widetilde{\cap}(F,E)\widetilde{\subseteq}(G,E)\widetilde{\subseteq}(G,E)\widetilde{\cup}(F,E)'$.
Thus $cl^*((A,E)\widetilde{\cap}(F,E))\widetilde{\subseteq}(G,E)\widetilde{\cup}(F,E)'$. Hence $(A,E)\widetilde{\cap}(F,E)$ is a soft Ig^* closed set. □

Definition 3.18. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. The soft set (A,E) is called a soft Ig^* open set if its complement $(A,E)'$ is a soft Ig^* closed set in $(X,\tau,E,\widetilde{I})$.
$sIg^*O(X)$ denotes the collection of all soft Ig^* open sets, and $sIg^*O(X,h)$ is the collection of all soft Ig^* open sets containing the point h of X in the soft ideal topological space $(X,\tau,E,\widetilde{I})$.

Remark 3.19. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X then every soft open set is a soft Ig^* open set and every soft Ig^* open set is a soft Ig open set. But the converse may not be true.

Example 3.20. Let $X=\{h_1,h_2,h_3\}, E=\{e_1,e_2\}$ and
$\tau=\{\phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E)\}$ where
$(F_1,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1\})\}, (F_2,E)=\{(e_1,\{h_1\}),(e_2,\{h_2,h_3\})\}$,
$(F_3,E)=\{(e_1,\{h_2\}),(e_2,\{h_1\})\}, (F_4,E)=\{(e_1,\{h_1\}),(e_2,\{h_3\})\}$,

$(F_5,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_3\})\}$,$(F_6,E) = \{(e_1,\{X\}),(e_2,\{h_1,h_3\})\}$, $(F_7,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{X\})\}$. Let $\widetilde{I} = \{\phi,(A_1,E),(A_2,E),(A_3,E)\}$ where $(A_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_3\})\}$,$(A_2,E) = \{(e_1,\{h_1\}),(e_2,\{\phi\})\}$, $(A_3,E) = \{(e_1,\{\phi\}),(e_2,\{h_3\})\}$. Here the soft set $(A,E) = \{(e_1,\{h_1\}),(e_2,\{\phi\})\}$ is a soft Ig^* open set, but it is not a soft open set and the soft set $(B,E) = \{(e_1,\{h_3\}),(e_2,\{h_1\})\}$ is a soft Ig open set, but it is not a soft Ig^* open set.

Theorem 3.21. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. A soft set $(F,E) \in SS(X)_E$ is called a soft Ig^* open set if and only if $(U,E)\widetilde{\subseteq}int^*(F,E)$ whenever $(U,E)\widetilde{\subseteq}(F,E)$ and (U,E) is a soft g closed set in X.*

Proof. Proof follows immediately from the definition of soft Ig^* closed set. □

Definition 3.22. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. Let $(A,E) \in SS(X)_E$. The union of all soft Ig^* open sets contained in (A,E) is called the soft Ig^* interior of (A,E) and is denoted by $sIg^*int(A,E)$.

Definition 3.23. Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. Let $(A,E) \in SS(X)_E$. The intersection of all soft Ig^* closed sets containing (A,E) is called the soft Ig^* closure of (A,E) and is denoted by $sIg^*cl(A,E)$. For a soft set $(A,E) \in SSX_E$, we have

$$int(A,E)\widetilde{\subseteq}sIg^*int(A,E)\widetilde{\subseteq}sIgint(A,E)\widetilde{\subseteq}(A,E)\widetilde{\subseteq}sIgcl(A,E)\widetilde{\subseteq}sIg^*cl(A,E)\widetilde{\subseteq}cl(A,E).$$

Theorem 3.24. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X then for any two soft subsets (A,E) and (B,E) we have*

(1) (A,E) *is soft Ig^* open if and only if* $(A,E) = sIg^*int(A,E)$

(2) (A,E) *is soft Ig^* closed if and only if* $(A,E) = sIg^*cl(A,E)$

(3) $(A,E)\widetilde{\subseteq}(B,E)$ *then* $sIg^*int(A,E)\widetilde{\subseteq}sIg^*int(B,E)$ *and* $sIg^*cl(A,E)\widetilde{\subseteq}sIg^*cl(B,E)$

(4) $sIg^*int(A,E)\widetilde{\cup}sIg^*int(B,E)\widetilde{\subseteq}sIg^*int((A,E)\widetilde{\cup}(B,E))$

(5) $sIg^*cl((A,E)\widetilde{\cap}(B,E))\widetilde{\subseteq}sIg^*cl((A,E)\widetilde{\cap}sIg^*cl(B,E)$

(6) $sIg^*cl(A,E)\widetilde{\cup}sIg^*cl(B,E) = sIg^*cl((A,E)\widetilde{\cup}(B,E))$

(7) $sIg^*int(A,E)\widetilde{\cap}sIg^*int(B,E) = sIg^*int((A,E)\widetilde{\cap}(B,E))$

(8) $sIg^*cl(\widetilde{X} \setminus (A,E)) = \widetilde{X} \setminus sIg^*int(A,E)$

(9) $sIg^*int(\widetilde{X} \setminus (A,E)) = \widetilde{X} \setminus sIg^*cl(A,E)$

(10) $sIg^*cl(sIg^*cl(A,E)) = sIg^*cl(A,E)$

(11) $sIg^*int(sIg^*int(A,E)) = sIg^*int(A,E)$

(12) $sIg^*int(\phi,E) = (\phi,E)$ *and* $sIg^*int(X,E)) = (X,E)$

(13) $sIg^*cl(\phi,E) = (\phi,E)$ *and* $sIg^*cl(X,E) = (X,E)$

(14) $(sIg^*cl(A,E))'\widetilde{\supseteq}sIg^*cl(A,E)'$

(15) $sIg^*int(A,E)'\widetilde{\supseteq}(sIg^*int(A,E))'$

Remark 3.25. The following example shows that the converse of (3) to (5) and (14) to (15) of the above theorem are need not be true in a soft ideal topological space $(X,\tau,E,\widetilde{I})$.

Example 3.26. Let $X=\{h_1,h_2,h_3\})$ and $E=\{e_1,e_2\}$ and $\tau=\{\phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_3\})\},(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_3\})\}$, $(F_3,E)=\{(e_1,\{\phi\}),(e_2,\{h_3\})\}$. Let $\widetilde{I}=\{\widetilde{\phi},(A,E)\}$ where $(A,E)=\{(e_1,\{h_1\}),(e_2,\{\phi\})\}$. Then $(X,\tau,E,\widetilde{I})$ is a soft ideal topological space over X.

Now $(A,E)=\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\widetilde{\subseteq}(B,E)=\{(e_1,\{h_1\}),(e_2,\{h_2,h_3\})\}$ but $sIg^*int(B,E)\widetilde{\not\subset}sIg^*int(A,E)$ and $sIg^*cl(B,E)\widetilde{\not\subset}\ sIg^*cl(A,E)$.

For this soft sets $(A,E)=\{(e_1,\{h_3\}),(e_2,\{h_1\})\},(B,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_3\})\}$, $(C,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_3\})\}$ and $(D,E)=\{(e_1,\{h_2\}),(e_2,\{X\})\}$, we have

$sIg^*cl(A,E)\widetilde{\cap}sIg^*cl(B,E)\widetilde{\not\subset}sIg^*cl((A,E)\widetilde{\cap}(B,E))$,

$sIg^*int((C,E)\widetilde{\cup}(D,E))\widetilde{\not\subset}sIg^*int((C,E)\widetilde{\cup}sIg^*int(D,E)$,

$sIg^*cl(A,E)'\widetilde{\not\subset}(sIg^*cl(A,E))'$ and $(sIg^*int(A,E))'\widetilde{\not\subset}sIg^*int(A,E)'$.

Theorem 3.27. *Let $(X,\tau,E,\widetilde{I})$ be a soft ideal topological space over X. Let $(A,E)\in SSX_E$ then $e_F\in sIg^*cl(A,E)$ if and only if every soft Ig^* open set (U,E) of X containing e_F, $(A,E)\widetilde{\cap}(U,E)\neq\phi$.*

Proof. Suppose that $e_F\in sIg^*cl(A,E)$ and (U,E) is any soft Ig^* open set containing e_F such that $(A,E)\widetilde{\cap}(U,E)=\phi$. Then $\widetilde{X}\setminus(U,E)$ is soft Ig^* closed set containing (A,E). Thus $sIg^*cl(A,E)\widetilde{\subseteq}\widetilde{X}\setminus(U,E)$, which is a contradiction.

Conversely, assume that for every sIg^* open set of X containing e_F, whose intersection with (A,E) is non empty. Suppose that $e_F\notin sIg^*cl(A,E)$ and (V,E) is soft Ig^* closed set containing (A,E). Then $e_F\widetilde{\subseteq}\widetilde{X}\setminus(V,E)$ and $e_F\notin(V,E)$. Thus $\widetilde{X}\setminus(V,E)$ is soft Ig^* open set containing e_F and $\widetilde{X}\setminus(V,E)\widetilde{\cap}(A,E)=\phi$, which is a contradiction. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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