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Research Article

Fuzzy Soft Min-Max Decision Making and its Applications

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Abstract. In this paper, we study fuzzy soft matrix and fuzzy soft max-min decision making method. Then we introduce a fuzzy soft min-max decision making method based on a respective decision making function and two more examples are given using the *AND* and *OR* products of fuzzy soft matrices.

Keywords. Bipolar fuzzy graph (BFG); Strong edge; Dominating set; Domination number; Global dominating set; Global domination number; Semi complete; Purely semi complete; Semi complementary; Semi global dominating set and semi global domination number

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1. Introduction

Fuzzy set theory was proposed by Zadeh [11] in 1965. The theory has been found extensive application in various field to handle uncertainty. In 1999, Molodstov [9] introduced soft set theory which is a new mathematical model that deals with uncertainties. In 2002, Roy and Maji [6] gave some applications of soft sets. In 2003, Maji *et al*. [7] studied the theory of soft sets and defined several properties of soft sets, binary operations. In 2001, Maji *et al*. [8] defined fuzzy soft sets and introduced some properties of fuzzy soft sets. Cagman *et al*. [1] defined soft set theory and uni-int decision making, soft matrices which were a representation of the soft sets and constructed a soft max-min decision making method.

In 2007 Maji *et al*. [10] presented a method to deal with decision making method on fuzzy soft sets. The algorithm defined in [10] is not applicable in general cases that is defined by Kong *et al*. [5]. Cagman *et al*. [4], they defined fuzzy soft set theory and its applications in decision making using aggregation operator and in 2012, Cagman *et al*. [3] defined fuzzy soft matrices and constructed a decision making problem.

In this paper, we define fuzzy soft min-max decision making method based on the respective decision making function can be successfully applicable to many decision making problems that yields the optimum solution. Also, we give an example for fuzzy soft matrix using *AND* and *OR*-products in decision making problems.

2. Preliminaries

Definition 2.1 ([9])**.** A soft set F_A over U is a set defined by a function f_A representing a mapping $f_A : E \to P(U)$ such that $f_A(x) = \phi$ if $x \notin A$.

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called $x-element$ of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary, empty, or have nonempty intersection. Thus a soft set over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.2 ([11])**.** Let U be a universe. A fuzzy set X over U is a set defined by a function μ_X representing a mapping

$$\mu_X : U \to [0,1]$$

μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows:

$$X = \{(\mu_X(u)/u) : u \in U, \mu_X(x) \in [0,1]\}.$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

We use $\Gamma_A, \Gamma_B, \Gamma_C, \dots,$ etc. for fs-sets and $\gamma_A, \gamma_B, \gamma_c, \dots,$ etc for their fuzzy approximate functions, respectively.

Definition 2.3 ([4]). Let U be an initial universe, E be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over U for all $x \in E$. Then an fs-set Γ_A over U is a set defined by a function γ_A representing a mapping,

$$\gamma_A : E \to F(U) \text{ such that } \gamma_A(x) = \phi, \text{ if } x \notin A.$$

Here, γ_A is called fuzzy approximate function of the fs-set Γ_A, and the value $\gamma_A(x)$ is a fuzzy set called x-element of the fs-set for all $x \in E$ and ϕ is the null fuzzy set. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}.$$

Note that the set of all fs-sets over U will be denoted by $FS(U)$.

Definition 2.4 ([3]). Let $\Gamma_A \in FS(U)$. Then a fuzzy relation form of Γ_A is defined by

$$R_A = \{(\mu_{R_A}(u,x)/(u,x)) : (u,x) \in U \times E\},$$

where the membership function of μ_{R_A} is written by

$$\mu_{R_A} : U \times E \to [0,1], \ \mu_{R_A}(u,x) = \mu_{\gamma_A(x)}(u).$$

If $U = \{u_1, u_2, \dots, u_m\}$, $E = \{e_1, e_2, \dots, e_n\}$ and $A \subseteq E$, then the set R_A can be presented by a table as in the following form,

R_A	e_1	e_2	$\dots$	e_n
u_1	$\mu_{R_A}(u_1,e_1)$	$\mu_{R_A}(u_1,e_2)$	$\dots$	$\mu_{R_A}(u_1,e_n)$
u_2	$\mu_{R_A}(u_2,e_1)$	$\mu_{R_A}(u_2,e_2)$	$\dots$	$\mu_{R_A}(u_2,e_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{R_A}(u_m,e_1)$	$\mu_{R_A}(u_m,e_2)$	$\dots$	$\mu_{R_A}(u_m,e_n)$

If $a_{ij} = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

which is called an $m \times n$ fs-matrix of the fs-set Γ_A over U. According to this definition, an fs-set Γ_A is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. It means that an fs-set Γ_A is formally equal to its soft matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any fs-set with its fs-matrix and use these two concepts as interchangeable.

The set of all $m \times n$ fs-matrices over U will be denoted by $FSM_{m\times n}$. We shall delete the subscript $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$, since $[a_{ij}] \in FSM_{m\times n}$ means that $[a_{ij}]$ is an $m \times n$ fs-matrix for $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$.

3. Product of fs-Matrices

Here, we studied the four products of fuzzy soft matrices are as follows:

Definition 3.1 ([3]). Let $[a_{ij}],[b_{ik}] \in FSM_{m\times n}$. Then And-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\wedge : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.2 ([3]). Let $[a_{ij}],[b_{ik}] \in FSM_{m\times n}$. Then Or-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\vee : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \vee [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.3 ([3]). Let $[a_{ij}],[b_{ik}] \in FSM_{m\times n}$. Then And-Not-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\overline{\wedge} : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \overline{\wedge} [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, 1-b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.4 ([3]). Let $[a_{ij}],[b_{ik}] \in FSM_{m\times n}$. Then Or-Not-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\underline{\vee} : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \underline{\vee} [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, 1-b_{ik}\}$ such that $p = n(j-1)+k$.

4. Fuzzy-Soft min-max Decision Making

Here we give a fuzzy soft min-max decision making ($FSmMDM$) method by using fuzzy soft min-max decision function. The method selects optimum alternatives from the set of the alternatives.

Definition 4.1. Let $[c_{ip}] \in FSM_{m\times n^2}$, $I_k = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leq kn\}$ for all $k \in I = \{1,2,\dots,n\}$. Then fuzzy-soft min-max decision function, denoted mM, is defined as follows

$$mM : FSM_{m\times n^2} \to FSM_{m\times 1}, \quad mM[c_{ip}] = [d_{i1}] = [\min_{k\in I}\{t_{ik}\}]$$

where

$$t_{ik} = \begin{cases} \max\limits_{p\in I_k}\{c_{ip}\}, & \text{if } I_k \neq \phi, \\ 0, & \text{if } I_k = \phi. \end{cases}$$

The one column fs-matrix $mM[c_{ip}]$ is called min-max decision fs-matrix.

Definition 4.2. Let $U = \{u_1, u_2, \dots, u_m\}$ be an initial universe and $mM[c_{ip}] = [d_{i1}]$. Then a subset of U can be obtained by using $[d_{i1}]$ as in the following way

$$opt_{[d_{i1}]}(U) = \{d_{i1}/u_i : u_i \in U, d_{i1} \neq 0\}$$

which is called an optimum fuzzy set on U.

We can make a fuzzy soft min-max decision by the following algorithm.

Step 1: Choose selected parameters as a subsets.

Step 2: Construct an fs-sets Γ_A and Γ_B over U.

Step 3: Construct the fuzzy soft matrix for each set of parameters.

Step 5: Find the convenient product of the fuzzy soft matrices.

Step 6: Find the min-max decision fuzzy soft matrix.

Step 7: Find an optimum fuzzy set on U.

Example 4.3. Let us consider a situation that people who met with an accident rushes to a multi-speciality hospital with multiple injuries. The following model helps to identify the patient who to be treated as an emergency case.

Consider the set of patients $U = \{u_1, u_2, u_3, u_4, u_5\}$ for $i = 1,2,3,4,5$ u_i's who met an accident requires immediate medical treatment which may be characterized by a set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$ for $j = 1,2,3,4,5,6$ the parameter e_j's stands for "Ortho", "Surgeon", "Cardiologist", "Neurologist", "Nephrologist", respectively. The situation is handled by two units, unit A and unit B of the Hospital. Unit A is parametrized with the set of Doctors, $A = \{e_1, e_3, e_4, e_5\}$ and unit B is parametrized with the set of Doctors, $B = \{e_2, e_4, e_5, e_6\}$.

We give a right decision for the patients on the basis of parameters using soft sets by $fsmMDM$ as follows.

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and

$$E = \{e_1, e_2, e_3, e_4, e_5\}$$

is a set of all parameters.

Step 1: Let us choose the set of parameters, $A = \{e_1, e_2, e_4, e_5\}$ and $B = \{e_2, e_3, e_4, e_5\}$, respectively.

Step 2: Now, constructs an fs-set Γ_A and Γ_A over U,

if $A = \{e_1, e_2, e_4, e_5\}$, then

$$\Gamma_A = \{(e_1, \{0.4/u_1, 0.7/u_2, 0.1/u_3, 0.5/u_4, 0.3/u_5\}), (e_2, \{0.5/u_1, 0.6/u_2, 0.2/u_4, 0.4/u_5\}),$$
$$(e_4, \{0.2/u_1, 0.4/u_2, 0.6/u_3, 0.3/u_4\}), (e_5, \{0.6/u_1, 0.5/u_2, 0.4/u_3, 0.4/u_4, 0.6/u_5\})\}$$

and

if $B = \{e_2, e_3, e_4, e_5\}$, then

$$\Gamma_B = \{(e_2, \{0.3/u_1, 0.6/u_2, 0.8/u_4,\}), (e_3, \{0.4/u_1, 0.5/u_2\}),$$
$$(e_4, \{0.1/u_1, 0.4/u_2\}), (e_5, \{0.4/u_1, 0.7/u_2, 0.2/u_4\})\}.$$

Step 3: Now, constructing the fuzzy soft matrices according to the parameters,

$$[a_{ij}] = \begin{bmatrix} 0.4 & 0.5 & 0.2 & 0.6 \\ 0.7 & 0.6 & 0.4 & 0.5 \\ 0.1 & 0 & 0.6 & 0.4 \\ 0.5 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0 & 0.6 \end{bmatrix}, \qquad [b_{ik}] = \begin{bmatrix} 0.3 & 0.4 & 0.1 & 0.4 \\ 0.6 & 0.5 & 0.4 & 0.7 \\ 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 4: We can find a product of fuzzy soft matrices using OR-product as follows:

$$[a_{ij}] \vee [b_{ik}] = \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.4 & 0.2 & 0.4 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.7 & 0.7 & 0.7 & 0.7 & 0.6 & 0.6 & 0.6 & 0.7 & 0.6 & 0.5 & 0.4 & 0.7 & 0.6 & 0.5 & 0.5 & 0.7 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0 & 0 & 0.6 & 0.6 & 0.6 & 0.6 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.8 & 0.5 & 0.5 & 0.5 & 0.8 & 0.2 & 0.2 & 0.2 & 0.8 & 0.3 & 0.3 & 0.3 & 0.8 & 0.4 & 0.4 & 0.4 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.4 & 0.4 & 0.4 & 0.4 & 0 & 0 & 0 & 0 & 0.6 & 0.6 & 0.6 & 0.6 \end{bmatrix}$$

Step 5: We can find a $min-max$ fuzzy soft matrix as

$$mM([a_{ij}] \vee [b_{ik}]) = \begin{bmatrix} 0.4 \\ 0.7 \\ 0 \\ 0.8 \\ 0 \end{bmatrix}$$

Step 6: Finally, we can find an optimum fuzzy set on U according to $mM([a_{ij}] \vee [b_{ik}])$

$$opt_{mM([a_{ij}] \vee [b_{ik}])}(U) = \{0.4, 0.7, 0.8\}$$

which means that u_4 is the optimum choice to the solution and so among the five patient u_4 to be admitted in emergency care unit.

Example 4.4. Consider a set of patients $U = \{u_1, u_2, u_3, u_4, u_5\}$, $i = 1,2,3,4,5$, u_i's stands for different habits “smoking”, “taking healthy balanced diet”, “practicing yoga”, “chewing tobacco”, “using organic food”, respectively, which may be characterized by a set of parameters $E = \{e_1, e_2, e_3, e_4\}$ for $j = 1,2,3,4$ the parameter e_j's stands for “lung cancer patients”, “mouth cancer patients”, “set of people above 50 years”, “set of people below 50 years”, respectively.

Suppose that two Doctors X and Y analyse the causes of the cancer with their own set of parameters, then *fsmMDm* gives a right decision for the patients on the basis of their set of parameters using the *AND* operator.

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is the universal set and $E = \{e_1, e_2, e_3, e_4\}$ is the set of all parameters.

Step 1: Let us choose the set of parameters, $A = \{e_1, e_2, e_3\}$ and $B = \{e_1, e_2, e_4\}$, respectively.

Step 2: Now, constructs an fs-set Γ_A and Γ_A over U,

if $A = \{e_1, e_2, e_3\}$, then

$$\Gamma_A = \{(e_1, \{0.5/u_1, 0.6/u_2, 0.7/u_4, 0.8/u_5\}), (e_2, \{0.6/u_1, 0.2/u_3, 0.5/u_4, 0.5/u_5\}),$$
$$(e_3, \{0.9/u_1, 0.5/u_2, 0.6/u_3, 0.2/u_4, 0.7/u_5\})$$

and

if $B = \{e_1, e_2, e_4\}$, then

$$\Gamma_B = \{(e_1, \{0.6/u_1, 0.3/u_2, 0.8/u_4\}), e_2, \{0.8/u_1, 0.1/u_3, 0.6/u_4, 0.3/u_5\}),$$
$$(e_4, \{0.7/u_1, 0.4/u_2, 0.5/u_3, 0.9/u_4\})$$

Step 3: Now, constructing the fuzzy soft matrices according to the parameters,

$$[a_{ij}] = \begin{bmatrix} 0.5 & 0.6 & 0.9 \\ 0.6 & 0 & 0.5 \\ 0 & 0.2 & 0.6 \\ 0.7 & 0.5 & 0.2 \\ 0.8 & 0.5 & 0.7 \end{bmatrix}, \qquad [b_{ik}] = \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.3 & 0 & 0.4 \\ 0 & 0.1 & 0.5 \\ 0.8 & 0.6 & 0.9 \\ 0 & 0.3 & 0 \end{bmatrix}$$

Step 4: we can find a product of fuzzy soft matrices using *AND*-product as follows:

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.6 & 0.6 & 0.6 & 0.6 & 0.8 & 0.7 \\ 0.3 & 0 & 0.4 & 0 & 0 & 0 & 0.3 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0.1 & 0.2 & 0 & 0.1 & 0.5 \\ 0.7 & 0.6 & 0.7 & 0.5 & 0.5 & 0.5 & 0.2 & 0.2 & 0.2 \\ 0 & 0.3 & 0 & 0 & 0.3 & 0 & 0 & 0.3 & 0 \end{bmatrix}$$

Step 5: We can find a min-max fuzzy soft matrix as

$$Mm([a_{ij}] \wedge [b_{ik}]) = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0.2 \\ 0.3 \end{bmatrix}$$

Step 6: Finally, we can find an optimum fuzzy set on U according to $mM([a_{ij}] \wedge [b_{ik}])$

$$opt_{mM([a_{ij}] \wedge [b_{ik}])}(U) = \{0.5, 0.2, 0.3\}$$

which means that u_1 is the optimum choice to the solution and so *smoking* causes the cancer according to Doctor's X and Y decision.

5. Conclusion

This paper is a continuation of Naim Cagman and Serdar Enginoglu work. The authors in [3] provided a fuzzy soft decision making model on the fuzzy soft set theory and applied it to real life problem. Based on their method, we introduce an another decision making model by using respective decision making function and using the model we observe the problems with *AND* and *OR* operators on fs-sets that yields an optimum solution.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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