Journal of Informatics and Mathematical Sciences
Vol. 11, No. 2, pp. 99–113, 2019
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications
http://www.rgnpublications.com
DOI: 10.26713/jims.v11i2.947

Research Article

Some Restrictions on R&D Networks

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Abstract. We use an R&D network model by Goyal and Moraga-Gonzalez [13] to maximize the equilibrium outcomes. We focus on the effectiveness of R&D expenditures and its effect on the outcomes. We find that characteristics of markets and networks influence values of the effectiveness. The change of this parameter has a negative impact on the individual and social outcomes.

Keywords. R&D network game; Market structure; Equilibria; Maximum outcomes

MSC. 91A13; 91A40; 91A80

Received: June 21, 2018 **Accepted:** March 27, 2019

1. Introduction

A network concept has introduced to cooperative agreements in R&D among firms (e.g. 1, 8, 9, 12). Among the theoretical R&D network models, a model due to Goyal and Moraga-Gonzalez [13] who addressed R&D cooperation between as a network. They used an idea that each collaboration between any two firms is a mutual benefit to build an undirected network. This whole structure is called a cooperative R&D network where nodes represent firms and links represents R&D partnerships between firms. The network model by Goyal and Moraga-Gonzalez consists of three stages: network structure, R&D expenditure and market competition. The formation of the R&D network does not involve costs of establishment of cooperative links. Also, firms are free to engage in R&D as individuals or with others in the network. If some firms decide to participate in R&D alone, then R&D spillovers will be enforced between them to ensure knowledge flow between them (i.e. firms can take advantages of partners' R&D investments).

The study of R&D cooperation under the network model involves different types of structures. Goyal and Moraga-Gonzalez considered two classes of the networks: symmetric networks (each firm has the same number of links) and asymmetric networks (the links distribution is heterogeneous). For symmetric networks, they provided results for an arbitrary number of firms without R&D spillovers. Whereas, for asymmetric networks, they provided results for three firms with spillover. The authors concluded that firms always seek to form as many as cooperative links in order to maximize their profits and this allows the complete network (each two nodes in the network are linked) to construct.

The work by Goyal and Moraga-Gonzalez has been extended by many authors (e.g., 2–6, 21, 22). The interests of these authors were various. Some focused on the effect of the cooperation structure on the outcomes. Others focused on optimal R&D structures and the effect of the network population and market structure on these structures.

The purpose of the current paper is to preserve the maximum outcomes of firms engaged in R&D. In particular, we focus on the values of the effectiveness of the R&D expenditures to have non-negative results. Firstly, we examine the impact of the characteristics of the markets and networks on the effectiveness. Secondly, we examine the impact of the effectiveness on the individual and social equilibria.

The results of the paper suggest that the market and network structures have effects on the effectiveness of the R&D expenditure. The first effect is the cost of the production. It is found that the R&D effectiveness and the production cost are inversely related. This indicates that the production cost determines the lowest sufficient value of the effectiveness. The second effect is the population of the R&D network. The growth of the network size affects the effectiveness where as the size grows, the effectiveness increases. This effect appears clearly when the competition intensity between firms is weak. The change of the values of the effectiveness has an impact on the equilibrium outcomes. For the equilibrium R&D expenditure and total welfare, the impact of increasing the effectiveness is negative. This indicates that the maximum outcomes for the two economic parameters are obtained if the effectiveness is sufficiently small. For the equilibrium profit, the previous statement is true if the competition intensity is not high in a low cooperation structure.

The paper is structured as follows. In Section 2, we provide foundations in the social network and microeconomics. Then, we introduce the Goyal and Moraga-Gonzalez model. In Section 3, we present our outcomes. In Section 4, we conclude our study.

2. Background

2.1 Network

Let $G(N,E)$ be a network with a set of nodes $N = \{i, j, k, \ldots\}$ and a set of links $E = \{ij, jk, \ldots\}$ connecting the nodes. The set of nodes that link to node i is defined as a neighbor set of that node: $N_i = \{j \in N : ij \in E\}$. The length of the neighbors' set of node i is a degree of that node $\deg(i)$. If all firms have the same degree, the network G is called regular (or symmetric). The density of the network G is a ratio of actual links in the network out of possible links that can be drawn with the same number of nodes [16]. Thus, if $|N| = n$ and $|E| = m$, then $D(G) = 2m/n(n-1)$.

2.2 Economic Model

Consider the linear-quadratic function of consumers given by [7] and [10]:

$$U = a\sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\lambda\sum_{j\neq i} q_i q_j\right) + I. \tag{2.1}$$

Here the demand parameters $a > 0$ denotes the willingness of consumers to pay and q_i is the quantity consumed of product i, and I measures the consumer's consumption of all other products. The parameter λ such that $-1 \le \lambda \le 1$ captures the marginal rate of substitution between different products. If $\lambda < 0$, products are complements; whereas if $\lambda > 0$, products are substitutes. In this paper, we consider the case when the products are independent $\lambda = 0$ or homogeneous $\lambda = 1$.

From the utility function (2.1), the inverse demand function for each product i

$$D_i^{-1} = p_i = a - q_i - \lambda\sum_{j\neq i} q_j, \quad i = 1,\dots,n. \tag{2.2}$$

The profit π_i for firm i is

$$\pi_i = (p_i - c_i)q_i = \left(a - q_i - \lambda\sum_{j\neq i}^{n} q_j - c_i\right)q_i, \tag{2.3}$$

where p_i is the price of product i produced by firm i and c_i is the production cost.

The total welfare is the total surplus of consumers plus the industry profit

$$TW = \frac{(1-\lambda)}{2}\sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2 + \sum_{i=1}^{n}\pi_i. \tag{2.4}$$

2.3 R&D Network Model

Goyal and Moraga-Gonzalez [13] readdressed the cooperation of firms in R&D as an undirected network. The network consists of nodes represent firms and links represent the R&D partnerships. Based on their R&D network model, if firms stay in the network without cooperating, R&D spillovers will be enforced between them to ensure a partial benefit from investments of firms in R&D.

Goyal and Moraga-Gonzalez studied the two general cases of the network structures: symmetric and asymmetric networks. In the case of symmetric networks, all firms have the same number of links where the spillover is set zero. In the case of asymmetric networks, distribution of the links is heterogeneous where the spillover term is involved. The focus of their study was to examine the impact of the cooperative links on R&D expenditures and on the incentives of firms to cooperate. Also, they examined the conflict between the individual and social optimal structures of the R&D cooperation.

For the purpose of this paper, we only consider the symmetric networks. This is because for asymmetric interactions, it is not possible to generalize the equilibria. In addition to that, the quantity of possible networks increase with the number of firms and this complicates the analysis of the outcomes.

Stages of the Model

Based on the Goyal and Moraga-Gonzalez paper, the R&D network model consists of three stages:

1. *The network structure stage*: Firms choose their research partners. In the symmetric networks, the cooperation of firms is called activity levels.
2. *The R&D expenditure stage*: Given the R&D network, firms choose the amounts of investment (effort) in R&D simultaneously and independently. The purpose of investment is to reduce the cost of production.
3. *The competition stage*: Given the R&D investments by each firm, firms compete in the product market by setting quantities (Cournot competition) in order to maximize their profits.

Cost Reduction

Goyal and Moraga-Gonzalez defined the effective R&D effort for each firm as follows:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n, \tag{2.5}$$

where x_i denotes R&D effort of firm i, N_i is the set of firms participating in R&D with firm i and $\beta \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in R&D with firm i. The effective R&D effort reduces firm i's marginal cost ($\overline{c}$) of production

$$c_i = \overline{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n. \tag{2.6}$$

The effort is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is γx_i^2, where $\gamma > 0$ indicates the effectiveness of R&D expenditure [11]. The profit π_i for firm i is the difference between revenue and production cost minus the cost of R&D

$$\pi_i = (p_i - c_i)q_i - \gamma x_i^2 = \left(a - q_i - \lambda \sum_{j \neq i}^{n} q_j - c_i\right) q_i - \gamma x_i^2, \tag{2.7}$$

where the marginal cost satisfy $a > \overline{c}$.

Stability of R&D Networks

The pairwise stability of a network is defined upon the profit of firms [15].

Definition 1. For any two firms i, j in a network G, then G is stable if the following two conditions are satisfied:

1. If $ij \in G$, $\pi_i(G) \geq \pi_i(G - ij)$ and $\pi_j(G) \geq \pi_j(G - ij)$,
2. If $ij \notin G$ and if $\pi_i(G) < \pi_i(G + ij)$, then $\pi_j(G) > \pi_j(G + ij)$,

where $G - ij$ is the network resulting from deleting a link ij from the network G and $G + ij$ is the network resulting from adding a link ij to the network G.

2.4 Nash Equilibria

Under Cournot competition, we identify the sub-game perfect Nash equilibrium by using backwards induction. First, we assume that the marginal cost $\overline{c}$ is constant and equal for all firms. Also, for symmetric networks, we assume that $deg(i) = k$ for each firm $i \in N$.

By substituting the inverse demand function (2.2) into the profit function (2.7) and calculating $\frac{\partial \pi_i}{\partial q_i} = 0$, we have $q_i = \left(a - c_i - \lambda \sum_{j \neq i} q_j\right)/2$ which is the best response function of quantity for product i. By substituting the best response functions into each other, the symmetric equilibrium output for each product i is

$$q_i^* = \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)\big((n-1)\lambda + 2\big)}. \tag{2.8}$$

To find the equilibrium profit, we substitute the equilibrium output (2.8) into the profit function which gives

$$\pi_i^* = \left[\frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)\big((n-1)\lambda + 2\big)}\right]^2 - \gamma x_i^2. \tag{2.9}$$

Thus the profit function can be expressed in the following form

$$\pi_i^* = {q_i^*}^2 - \gamma x_i^2. \tag{2.10}$$

Since $\beta = 0$, the cost function for each firm i is

$$c_i = \overline{c} - x_i - \sum_{j \in N_i} x_j, \quad i = 1, \dots, n, \tag{2.11}$$

where x_i denotes R&D effort of firm i, N_i is the set of firms connected with i.

Based on the network structure, we find the cost function and calculate the best response function of the R&D effort for each firm i $\left(\frac{\partial \pi_i}{\partial x_i} = 0\right)$. By plugging the best response functions into each other, we have the symmetric equilibrium for the R&D effort. To save space, we list the equilibria under Cournot competition in the Appendix (A) as given in [13].

3. The Effectiveness of R&D Expenditures: Restrictions and Effects

Goyal and Moraga-Gonzalez studied R&D networks consisted of firms compete by their quantities (Cournot competition). For homogeneous and independent products, they set restrictions on the value of the effectiveness of the R&D expenditures.

Given a symmetric network structure G with n firms (denoted G_n) such that the R&D spillovers between non-cooperating firms are zero. Let γ_h and γ_d be the lowest sufficient values of the R&D effectiveness under homogeneous and independent products, respectively. Then, according to Goyal and Moraga-Gonzalez, we have

$$\gamma_h > \max\left\{\frac{n^2}{(n+1)^2}, \frac{a}{4\overline{c}}\right\},$$
$$\gamma_d > \max\left\{\frac{n}{4}, \frac{an}{4\overline{c}}\right\}. \tag{3.1}$$

The previous inequalities are obtained by applying the following conditions:

Condition C1. The effort function should be non-negative.

Condition C2. The cost function should give non-negative results.

Condition C3. The second order condition for maximizing profit should be satisfied.

More detail about the previous conditions are provided in the Appendix B.

3.1 Restrictions on the Effectiveness

This section provides details related to the conditions applied to the effectiveness of the R&D expenditure. The focus is on determining the priority between the conditions to have suitable values of the effectiveness.

Proposition 1. *Given a symmetric network structure G_n with zero spillover such that the effectiveness γ satisfies* (3.1) *under Cournot competition. Then,*

(1) *For independent and homogeneous products, Condition C1 is satisfied if Condition C2 is applied.*

(2) *For independent products, Condition C2 is sufficient to have appropriate values for the effectiveness γ.*

(3) *For homogeneous products, both Conditions C2 and C3 are necessary to determine appropriate values of the effectiveness γ.*

The proof is given in the Appendix C.

From the conditions on the effectiveness (3.1), the value of γ is subject to the characteristics of the market and the growth of the R&D network. In particular, the suitable value of the effectiveness depends on the following three fundamental elements:

- Size of the market n,
- Intercept demand a,
- Marginal cost $\overline{c}$.

The following proposition states that the R&D effectiveness and the marginal cost are inversely related. As the marginal cost increases, the effectiveness decreases. Also, the great shift in value of effectiveness is when the products are independent. Moreover, the proposition states that the relationship between the appropriate values of the effectiveness and the network size is a positive relationship. Thus, for each market size, there exists values of γ_h and γ_d in which the equilibria are non-negative.

Proposition 2. *Given a symmetric network structure G_n with zero spillover.*

(1) *When the marginal cost increases, the sufficient value of γ decreases.*

(2) *When the network size increases, the sufficient value of γ increases.*

The proof is given in the Appendix C.

Example 1. Assume ten firms participating in a symmetric R&D network with zero spillover. Figure 1 shows the appropriate values of the effectivenesses γ_h and γ_d for different values of the intercept demand a and the marginal cost $\bar{c}$.

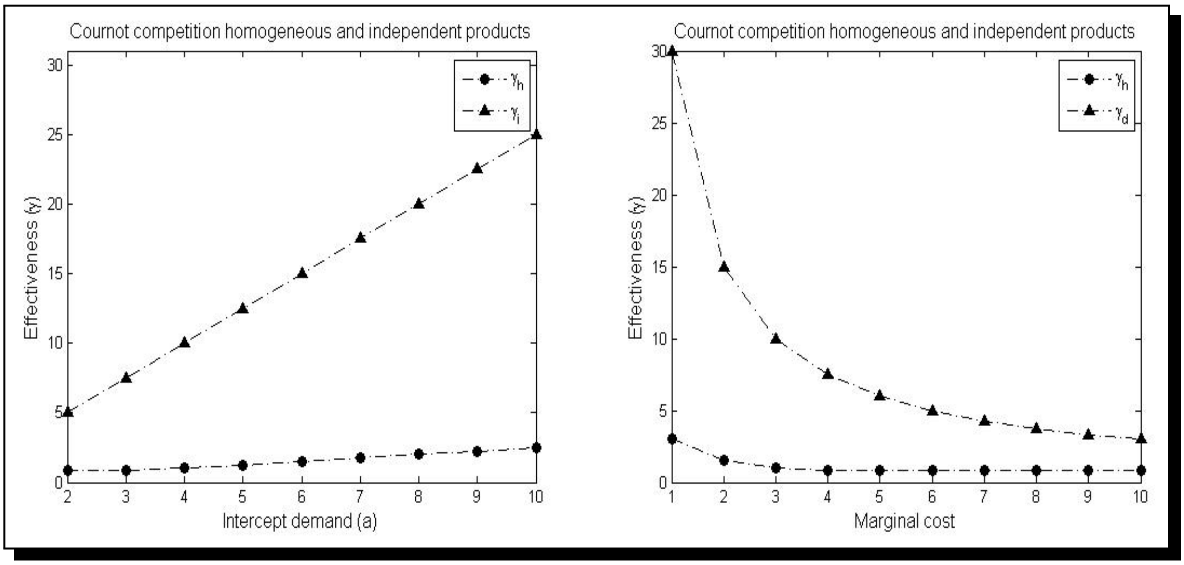

Figure 1. The appropriate values of the effectivenesses γ_h and γ_d for different values of the intercept demand a and the marginal cost $\bar{c}$. The parameters used to plot the figure on the right are $n = 10$ and $\bar{c} = 1$ and to the figure on left are $n = 10$ and $a = 12$.

Example 2. Assume a set of firms participating in a symmetric R&D network with zero spillover. Figure 2 shows the appropriate values of the effectivenesses γ_h and γ_d for $a = 2$, $\bar{c} = 1$, and different values of the market size n.

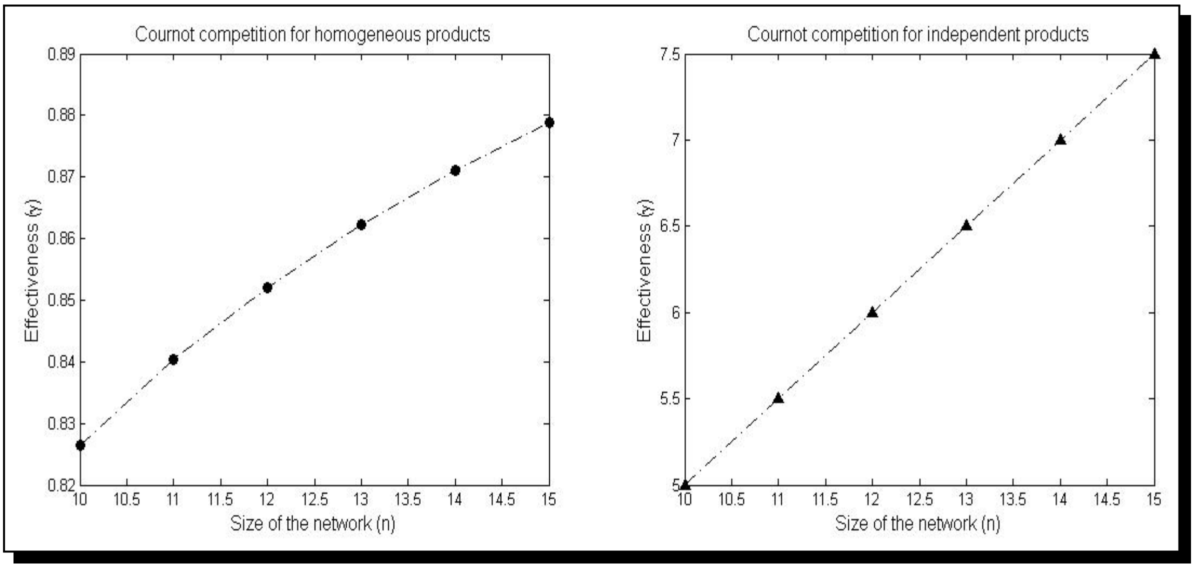

Figure 2. The appropriate values of the effectivenesses γ_h and γ_d for different activity levels. The parameters used to plot the figures are $a = 2$, $\bar{c} = 1$ for $n \in [10, 15]$.

As can be observed from the figure, the increase of the value of γ_d with n is very high when compared to the value of γ_h. This indicates that the effectiveness is very sensitive to the network size if the firms sell independent products.

3.2 Effects of the Effectiveness on the Outcomes

In the context of the R&D interactions, different levels of the network perform different functions in the outcomes [2, 4, 13, 21]. In this section, we examine the impact of the effectiveness of the R&D expenditure on the maximum individual and social outcomes.

The upcoming proposition states that the outcomes of the R&D effort and total welfare and the effectiveness are inversely related. As the value of γ increases, the outcomes decline, regardless of the product type. This indicates that to have maximum R&D investment or total welfare, we have to consider the minimum appropriate value of the effectiveness. This rule is not always applied to the profit of firms. We found that when the density of the cooperation network is small, the increase of the effectiveness improves the profit.

Proposition 3. *Given a symmetric network structure G_n with zero spillover such that the effectiveness γ^* is sufficient to satisfy* (3.1) *under Cournot competition.*

(1) *The R&D effort and the total welfare decrease with increasing the R&D effectiveness, regardless of the production type.*
(2) *The profit decreases with increasing the R&D effectiveness for independent products. For homogeneous products, the statement is true if the network density is not small.*

The proof is given in the Appendix C.

As a result of the previous proposition, the effectiveness controls the behavior of the equilibrium outcomes by shifting the maximum values with respect to the network density to downwards (i.e., lower maximum values). However, the value of the effectiveness does not affect the network stability or the optimal level at which the effort, the profit or the total welfare are maximized. Meaning that, for each market size, let the effort, the profit and total welfare are maximized at the activity levels k_x^*, k_π^* and k_{TW}^*, respectively. Then those optimal levels are not affected by increasing the effectiveness γ.

Corollary 1. *Given a symmetric network structure G_n with zero spillover such that the effectiveness γ^* is sufficient to satisfy* (3.1) *under Cournot competition. Assume that the functions x^*, π^* and TW^* are maximized at k_x^*, k_π^* and k_{TW}^*, respectively. Then for any $\gamma > \gamma^*$, the activity levels k_x^*, k_π^* and k_{TW}^* are constant.*

There is no need to prove the previous result since it is straightforward from Proposition 3. Instead, we provide an example to explain the result.

Example 3. Assume ten firms participating in a symmetric R&D network with zero spillover. Under Cournot competition,

1. Figure 3 shows the impact of increasing the effectiveness γ on the equilibrium outcomes in the complete network (each two firms are linked) for independent and homogeneous products.

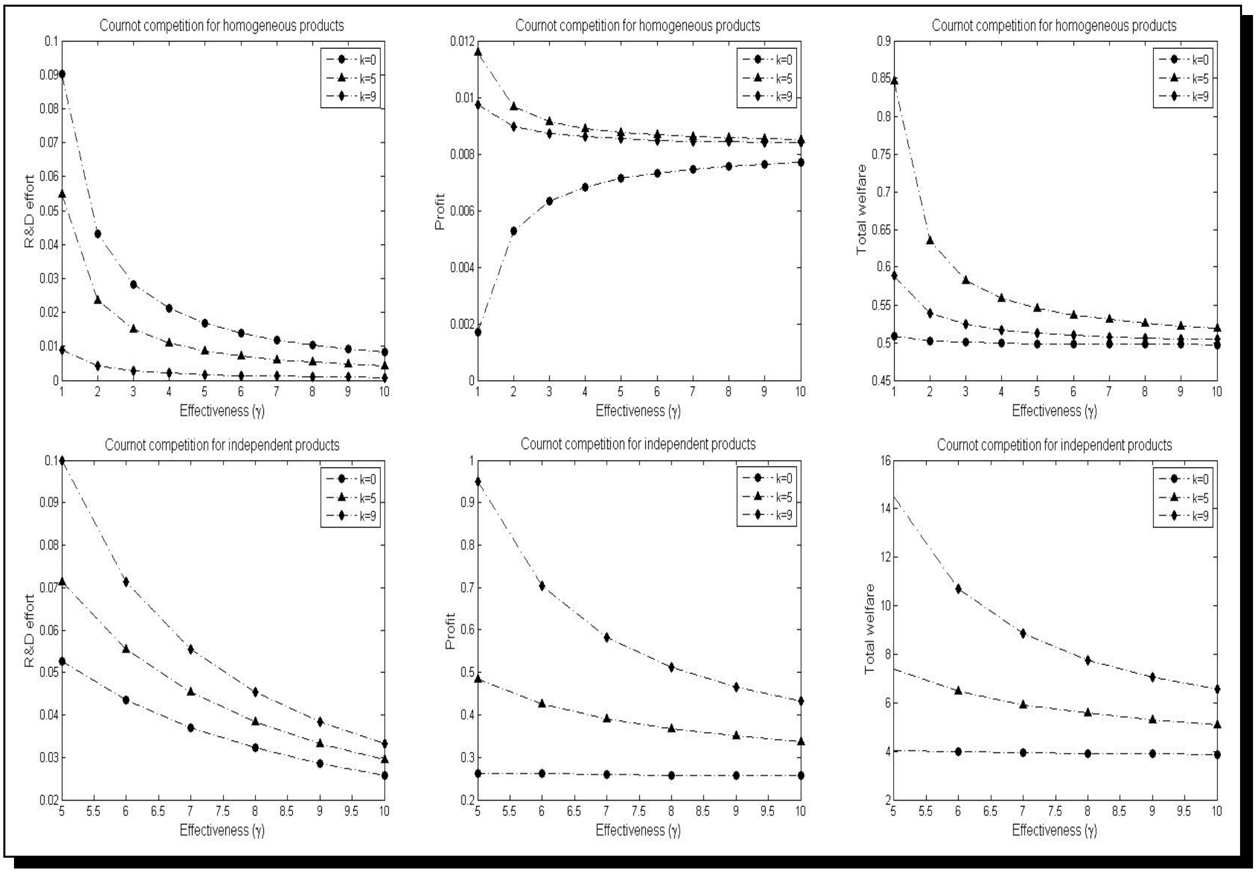

Figure 3. The impact of the effectiveness on the equilibria under Cournot competition. The graphs on the top are for homogeneous products $\gamma^* = 1$ and the graphs at the bottom are for independent products. The parameters used to plot the figures are $n = 10$, $a = 2$, $\overline{c} = 1$ and $\gamma^* = 1$ ($\gamma^* = 5$) for homogeneous (independent) products.

2. Figure 4 compares the activity levels k_x^*, k_π^* and k_{TW}^* for different values of the effectiveness $\gamma^* = 1$, $\gamma = 2$ and $\gamma = 3$.

4. Conclusion

In this paper, we studied possible conditions to maximize the outcomes of firms engaged in the R&D network. The results point out that the effectiveness of the R&D expenditures is affected by the market demand and the network formation. While increasing the production cost leads to minimize the value of the effectiveness, the growth of the network maximizes its value. This in turn has an effect on the equilibrium outcomes. As the effectiveness of the R&D expenditure raises, the individual and social benefits decline. According to this result, to obtain higher outcomes, the effectiveness should be reduced to lowest possible values.

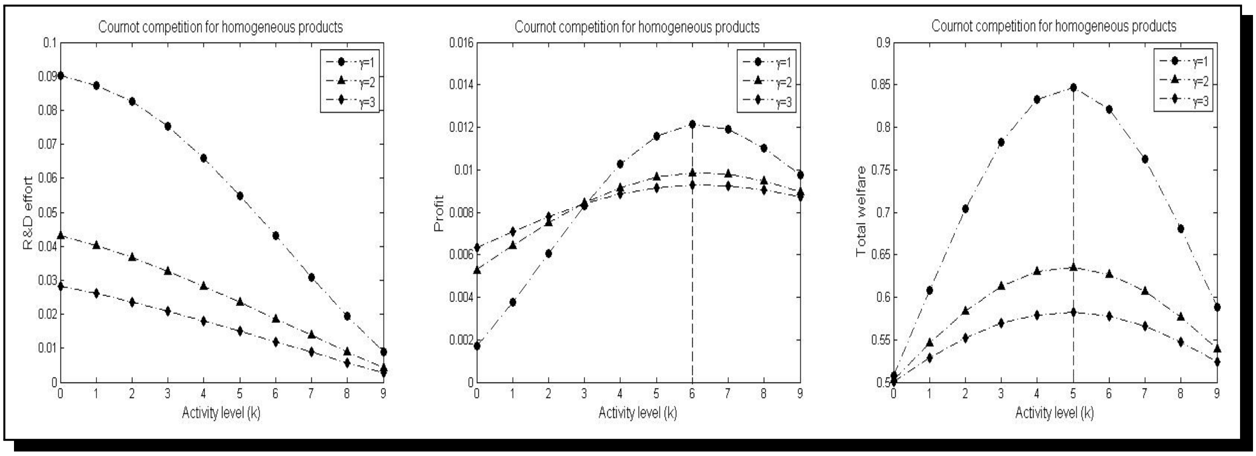

Figure 4. The maximum activity levels under Cournot competition for different values of the effectiveness. The figure shows the activity levels k^*_x, k^*_π and k^*_{TW} under homogeneous products for three values of the effectiveness $\gamma^* = 1$, and $\gamma = 2$ and $\gamma = 3$. The parameters used to plot the graphs are $n = 10$, $a = 2$, $\overline{c} = 1$.

Acknowledgment

This research was supported by King Saud University, Deanship of Scientific Research, College of Science Research Center.

Appendix

Appendix A: The Equilibria Under Cournot Competition

A.1: The Equilibria for Homogeneous Products

For Independent Goods

R&D Effort:

$$x^* = \frac{(n-k)(a-\overline{c})}{\gamma(n+1)^2-(n-k)(k+1)}. \tag{A.1}$$

Cost Function:

$$c^* = \frac{\overline{c}\gamma(n+1)^2-(k+1)(n-k)a}{\gamma(n+1)^2-(n-k)(k+1)}. \tag{A.2}$$

Quantity Function:

$$q^* = \frac{\gamma(n+1)(a-\overline{c})}{\gamma(n+1)^2-(n-k)(k+1)}. \tag{A.3}$$

Profit Function:

$$\pi^* = \frac{(\gamma^2(n+1)^2-(n-k)^2)(a-\overline{c})^2}{(\gamma(n+1)^2-(n-k)(k+1))^2}. \tag{A.4}$$

Total Welfare Function:

$$TW^* = \frac{n(\gamma^2(n+2)(n+1)^2 - 2(n-k)^2)(a-\bar{c})^2}{2(\gamma(n+1)^2 - (n-k)(k+1))^2}. \tag{A.5}$$

A.2: The Equilibria for Independent Products

R&D Effort:

$$x^* = \frac{(a-\bar{c})}{4\gamma - k - 1}. \tag{A.6}$$

Cost Function:

$$c^* = \frac{2\big(4\bar{c}\gamma - (k+1)a\big)}{4\gamma - k - 1}. \tag{A.7}$$

Quantity Function:

$$q^* = \frac{2\gamma(a-\bar{c})}{4\gamma - k - 1}. \tag{A.8}$$

Profit Function:

$$\pi^* = \frac{\gamma(4\gamma-1)(a-\bar{c})^2}{(4\gamma - k - 1)^2}. \tag{A.9}$$

Total Welfare Function:

$$TW^* = \frac{n\gamma(6\gamma-1)(a-\bar{c})^2}{(4\gamma - k - 1)^2}. \tag{A.10}$$

Appendix B: The Conditions to Have Appropriate Values of the R&D Effectiveness

- **Under Cournot Competition:**

Condition C1. $x^* \geq 0$.

The effort function should be non-negative ($x^* \geq 0$). Then, from the effort functions ((A.1) and (A.6)), the effectiveness (γ) should satisfy

$$\gamma_h > \frac{(k+1)(n-k)}{(n+1)^2},$$

$$\gamma_d > \frac{(k+1)}{4}.$$

Condition C2. $c^* \geq 0$.

The cost functions ((A.2) and (A.7)) should give non-negative results. This is obtained if

$$\gamma_h \geq \frac{(k+1)(n-k)a}{(n+1)^2\bar{c}}, \quad \sqrt{\gamma_d} \geq \frac{(k+1)a}{4\bar{c}}.$$

Condition C3. $\frac{\partial^2 \pi}{\partial x^2} < 0$.

The second order condition for maximizing profit function ($\frac{\partial^2 \pi}{\partial x^2} < 0$) is satisfied if

$$\gamma_h > \left(\frac{n-k}{n+1}\right)^2, \quad \gamma_d > \frac{1}{4}.$$

Appendix C: Proofs of Propositions

Proof of Proposition 1. (1) Since $a > \overline{c}$, satisfying Condition C2 implies Condition C1.

For items (2) and (3), we have to calculate $\gamma_{C2} - \gamma_{C3}$ where γ_{C2} is the sufficient value of γ by the second condition and γ_{C3} is the sufficient value of γ by the third condition. For independent products, we have

$$\gamma_{C2} - \gamma_{C3} = \frac{(k+1)a - \overline{c}}{2\overline{c}}. \tag{A.11}$$

Since $a > \overline{c}$, then $\gamma_{C2} > \gamma_{C3}$ and this implies that Condition C2 is sufficient to have appropriate values for the effectiveness γ.

For homogeneous products, we have

$$\gamma_{C2} - \gamma_{C3} = \frac{(n-k)\big((k+1)a - \overline{c}(n-k)\big)}{(n+1)\overline{c}}. \tag{A.12}$$

The expression $n - k > 0$, but the expression $(k+1)a - \overline{c}(n-k)$ depends on the network size and the network structure. If the density is small (i.e. k is small), we have $\gamma_{C3} > \gamma_{C2}$ and this means Condition C3 is sufficient to have appropriate values for the effectiveness. Whereas, if the density is high, Condition C2 is sufficient to have appropriate values for the effectiveness. □

Proof of Proposition 2. (1) For independent products, from the inequalities (3.1), we have $\gamma_d > \max\{\frac{n}{4}, \frac{an}{4\overline{c}}\}$. Since $a > \overline{c}$, then $\frac{an}{4\overline{c}} > \frac{n}{4}$. Now as the marginal cost $\overline{c}$ increases, the fraction $\frac{an}{4\overline{c}}$ and γ_d decrease. For homogeneous products, we have $\gamma_h > \max\left\{\frac{n^2}{(n+1)^2}, \frac{a}{4\overline{c}}\right\}$. If n is not high, $\frac{a}{4\overline{c}} > \frac{n^2}{(n+1)^2}$, so as the marginal cost $\overline{c}$ increases, the effectiveness γ_h decreases.

(2) For independent products, we know that $\gamma_d > \frac{an}{4\overline{c}} > \frac{n}{4}$. Therefore, as the network size n increases, the effectiveness γ_d increases. For homogeneous products, as n increases, the fraction $\frac{n^2}{(n+1)^2}$ increases and this allows the effectiveness γ_h to take high values. □

Proof of Proposition 3. We prove the proposition by assuming $(a - \overline{c}) = 1$.

(1) R&D Eeffort: We want to show that for independent or homogeneous products, the R&D effort decreases with increasing the R&D effectiveness, i.e., $\frac{\partial x^*}{\partial \gamma} < 0$.

For independent products, we have

$$\frac{\partial x^*}{\partial \gamma} = \frac{-4}{(4\gamma - k - 1)^2}.$$

This implies $\frac{\partial x^*}{\partial \gamma} < 0$. For homogeneous products, we have

$$\frac{\partial x^*}{\partial \gamma} = \frac{(k-n)(n+1)^2}{[\gamma(n+1)^2 + (n-k)(k+1)]^2}.$$

Since $k < n$, then $\frac{\partial x^*}{\partial \gamma} < 0$. This means that for independent or homogeneous products, the R&D effort decreases with increasing γ.

(2) R&D Cost: Since the R&D effort (x^{*^2}) decreases with increasing the effectiveness of R&D expenditure, then the R&D cost (γx^{*^2}) should decrease for all types of the production.

(3) Profit: For independent products,

$$\frac{\partial \pi^*}{\partial \gamma} = \frac{-4\gamma(2k+1)+k+1}{(4\gamma-k-1)^3}.$$

This implies $\frac{\partial \pi^*}{\partial \gamma} < 0$ for any k.

For homogeneous products,

$$\frac{\partial \pi^*}{\partial \gamma} = \frac{2(n+1)^2(k-n)[\gamma(k+1)+(k-n)]}{(\gamma(n+1)^2+(n-k)(k+1))^3}.$$

By choosing an appropriate value of γ, the expression $\gamma(k+1)+(k-n)$ depends on the market size (n) and the network density (k). For homogeneous products, it is sufficient to assume that $\gamma = 1$, then the previous expression becomes $2k+1-n$. If k is small, the expression becomes negative and this implies $\frac{\partial \pi^*}{\partial \gamma} > 0$. The opposite occurs if k is not small. This indicates that the profit decreases with increasing γ if the products are homogeneous and k is not small.

(4) Total Welfare: For independent products,

$$\frac{\partial TW^*}{\partial \gamma} = \frac{n(-4\gamma(2+3k)+k+1)}{(4\gamma-k-1)^3}.$$

The expression $-4\gamma(2+3k)+k+1 < 0$ since for independent products, the effectiveness γ is large. This implies $\frac{\partial TW^*}{\partial \gamma} < 0$.

For homogeneous products,

$$\frac{\partial TW^*}{\partial \gamma} = \frac{2n(n-k)[-(n+1)^2\gamma((n+3)k+2)+(n-k)^2(k+1)]}{2(\gamma(n+1)^2+(n-k)(k+1))^3}.$$

This implies $\frac{\partial TW^*}{\partial \gamma} < 0$ for any k. This implies for independent or homogeneous products, the total welfare decreases with increasing γ. □

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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