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Research Article

Wiener, Hyper Wiener and Detour Index of Pseudoregular Graphs

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Abstract. The field of graph theory is rich in its theoretical and application area. Drugs and other chemical compounds are often modeled as polygonal shape, where each vertex represents an atom of the molecule and covalent bonds between atoms are represented by edges between the corresponding vertices. This polygonal shape derived from a chemical compound is often called its modecular graph and can be a path, a tree or in general graph. An indicator defined over this molecular graph, the Wiener index $W(G)$ is defined as $W(G)=\sum\limits_{u\neq v} d(u,v)$, where the sum is taken through all unordered pairs of vertices of G. Another indicator is the Hyper Wiener index $WW(G)$ is defined as $WW(G)=\frac{1}{2}\sum\limits_{u\neq v}\left(d(u,v)+d^2(u,v)\right)$, where $d^2(u,v)=d(u,v)^2$, and the Detour index $D(G)$ is defined as $D(G)=\sum\limits_{u\neq v} D(u,v)$, where $D(u,v)$ denotes the longest distance from u to v in G. In this paper, we computed the Wiener, Hyper Wiener, Detour index for a special graph namely, Pseudo-regular graph.

Keywords. Wiener index; Hyper wiener index; Pseudo-regular graphs.

MSC. 68R10; 94C15; 92E10

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1. Introduction

Chemical graph theory is used to model physical properties of molecules. Indices based on the graphical structure are used to model both the boiling point and melting point of the molecules. Molecular descriptors are terms that characterize a specific aspect of a molecule. Topological indices have been defined as those "Numerical values associated with Chemical contribution for correlation of chemical structure with various physical properties, chemical reactivity or biological activity. A topological representation of a molecule is called molecular graph. A molecular graph is a collection of points representing the atoms in the molecule and set of lines representing the covalent bonds. These points are named vertices and the lines are named edges in graph theory language. The topological distance between a pair of vertices u and v which is denoted by $d(u,v)$, is the number of edges of the shortest path joining u and v.

The Wiener index of G, $W(G)$ is defined as $W(G) = \sum\limits_{u \neq v} d(u,v)$, where the sum is taken through all unordered pairs of vertices of G. Wiener index was introduced by Harold Wiener, as an aid to determine the boiling point of Paraffin [1]. It is related to boiling point, heat of evaporation, heat of formation, Chromatographic relation times, surface tension, vapour pressure, partition coefficients, total electron energy of polymers, ultrasonic sound velocity, internal energy etc., [2]. For this reason Wiener index is widely studied by Chemists.

There is a lot of mathematical and chemical literature on the Wiener index especially on the Wiener index of trees. In the Mathematical literature, the Wiener index seems to be the first studied by Entringer *et al*. [4]. He showed that among all trees of order n the star $K_{1,n-1}$ and the path P_n have the minimum and Maximum Wiener index respectively.

The Hyper Wiener index is defined as $WW(G) = \frac{1}{2} \sum\limits_{u \neq v} \left(d(u,v) + d^2(u,v)\right)$. The Hyper Wiener index has been defined by Randic for any connected acyclic structures as a sum over all "pair contributions" C_{ij}, where subscripts i and j denote vertices i and j in the underlying graph. For the mathematical properties of Hyper Wiener index [5] and its applications in chemistry are refer to [5–7].

The Detour distance $D_G(u.v)$ is the length of the longest path between u and v. The Detour index $D(G)$, where G denotes the underlying graph, has been introduced by Amic and Trinajstic [8], and by John [9] independently $D(G) = \sum\limits_{u \neq v} D(u,v)$.

A graph G is called Pseudo-regular graph [20] if every vertex of G has equal average degree. The main goal of this paper is to find the Wiener, Hyper Wiener, Detour index for special graph, namely, Pseudo-regular graphs.

2. Pseudo-Regular Graphs

Let $G = (V,E)$ be a simple, connected undirected graph with n vertices and m edges. For any vertex $v_i \in V$, the degree of v_i is the number of edges incident on v_i. It is denoted by d_i or $d(v_i)$.

A graph G is called regular if every vertex of G has equal degree. A bipartite graph is called semi regular if each vertex in the same part of a bipartition has the same degree. The 2-degree of v_i [20] is the sum of the degree of the vertices adjacent to v_i and denoted by t_i [11]. The average degree of v_i is defined as t_i/d_i. For any vertex $v_i \in V$, the average degree of v_i is also denoted by $m(v_i) = t_i/d_i$.

A graph G is called Pseudo-regular graph [20] if every vertex of G has equal average degree and $m(G) = \frac{1}{n} \sum_{v \in V(G)} m(u)$ is the average neighbor degree number of the graph G.

A graph is said to be r-regular if all its vertices are of equal degree r. Every regular graph is a Pseudo-regular graph [21]. But the Pseudo-regular graph need not be a regular graph. Pseudo-regular graph is shown in Figure 1 and Figure 2.

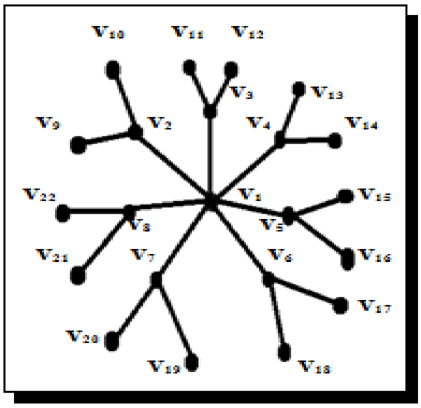

Figure 1

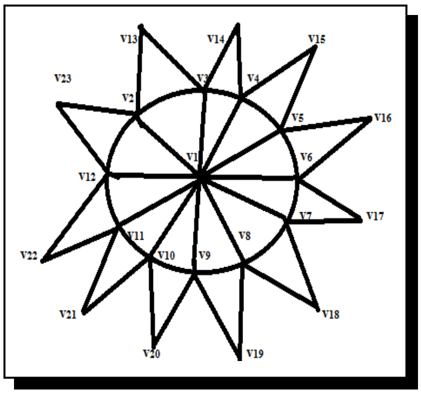

Figure 2

In Figure 1, there are 14 vertices are of degree 1, 7 vertices of degree 3 and 1 vertex is of degree 7. So totally there are 22 vertices in the graph. Average degree of vertices of degree 1 is equal to 3/1 = 3. Average degree of vertices of degree 2 is equal to 9/3 = 3. Average degree of vertices of degree 7 is equal to 21/7 = 3. Therefore, Average degree of each vertex is 3. Hence it is a Pseudo-regular graph. In Figure 2, average degree of each vertex is 5. Hence the graph in Figure 2 is also a Pseudo-regular graph.

The relevance of pseudo-regular graph for the theory of nanomolecules and nanostructure should become evident from the following. There exist polyhedral (planar, 3-connected) graphs and infinite periodic planar graphs belonging to the family of the Pseudo-regular graphs. Among polyhedral, the deltoidal hexecontahedron possesses Pseudo-regular property. The deltoidal hexecontahedron is a Catalan polyhedron with 60 deltoid faces, 120 edges and 62 vertices with degree 3, 4 and 5 and average degree of its vertices is 4.

A variety of their chemically relevant polyhedral and polyhedral-type structure, whose graphs are Pseudo-regular, can be found in the book [12, 13]. In this paper, the Pseudo-regular graphs can be drawn based on the value of average degree p (say) only.

3. Construction of Pseudo-Regular Graphs

In this section algorithms are proposed to construct Pseudo-regular graphs.

3.1 Algorithm to Construct the Pseudo-Regular Graph of type I denoted by G_I

Step 1. For any positive integer $p \geq 2$, draw a star graph $G = K_1,\ m$ where $m = p^2 - p + 1$.
Let v_0 be the central vertex of G and let $v_1,\ v_2, \dots, v_m$ be the n pendant vertices.

Step 2. Join $(p-1)$ new pendant vertices to every vertex of G except the central vertex.

Step 3. The new resulting graph is a Pseudo-regular graph of type I. It is denoted by G_I.

$$\begin{aligned}
|V(G_I)| &= 1 + p^2 - p + 1 + (p-1)(p^2 - p + 1) \\
&= 1 + p^2 - p + 1 + p^3 - p^2 + p - p^2 + p - 1 \\
&= p^3 - p^2 + p + 1, \\
|E(G_I)| &= (p^2 - p + 1) + (p-1)(p^2 - p + 1) = p^3 - p^2 + p\,.
\end{aligned}$$

G_I is a tree for any positive constant p and average degree of each vertex is p.

G_I type Pseudo-regular graph is given in Figure 1.

3.2 Algorithm to Construct the Pseudo-Regular Graph of type II denoted by G_{II}

Step 1. For any positive integer $p \geq 3$, draw a wheel graph $K_1 + C_m$ with m spokes, where $m = p^2 - 3p + 3$. Let v_o be the central vertex of $W_m + 1$ and hence $d(v_0) = p^2 - 3p + 3$.

Step 2. Let $\{v_1, v_2, \dots, v_m\}$ be the vertices of the cycle in C_m. Join $p-3$ pendant vertices to every vertex in the cycle except the central vertex. The resulting graph is a type II Pseudo-regular graph. It is denoted by G_{II}.

Step 3. $|V(G_{II})| = p^3 - 3p + 3 + 1 + (p-3)(p^2 - 3p + 3) = p^3 - 5p^2 + 9p - 5$.

For $p = 5$, we can get the following Pseudo-regular graph

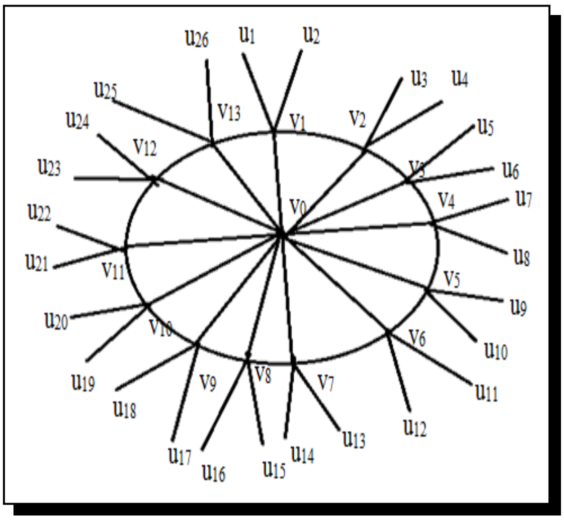

Figure 3

3.3 Algorithm to Construct the Pseudo-Regular Graph of type III denoted by G_{III}

Step 1. For any positive integer $p \geq 5$, draw a wheel graph $K_1 + C_m$ with m spokes, where $m = p^2 - 3p + 1$. Let v_o be the central vertex of wheel graph and hence $d(v_o) = p^2 - 3p + 1$.

Step 2. Let $\{v_1, v_2, \ldots, v_m\}$ be the vertices of the cycle in C_m. Introduce a new vertex for each edge of the cycle in a wheel graph and every new vertices should be joined to the end vertices of each edge of a cycle.

Step 3. Join $(p-5)$ pendant vertices to every vertex of the cycle in the wheel graph except the central vertex. $|V(G_{III})| = p^3 - 6p^2 + 10p - 2$. The resulting graph is pseudo-regular graph of type III. It is denoted by G_{III}.

For $p = 6$, we can get the following Pseudo-regular graph

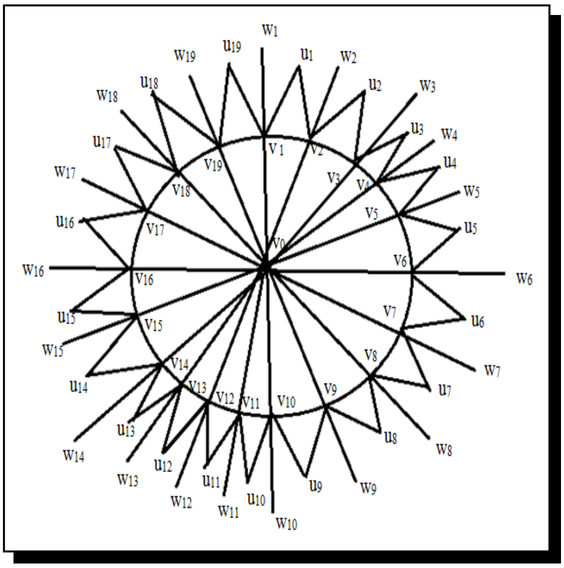

Figure 4

4. Main Results

Theorem 4.1. *For $p \ge 2$, the Wiener, Hyper Wiener and Detour index of type I Pseudo-regular graph G_I is*

$$W(G_I) = \frac{36p^5 - 235p^4 + 980p^3 - 2917p^2 + 4934p - 3336}{2},$$
$$WW(G_I) = \frac{180p^5 - 1185p^4 + 4606p^3 - 12667p^2 + 20470p - 13560}{2},$$
$$D(G_I) = \frac{36p^5 - 235p^4 + 980p^3 - 2917p^2 + 4934p - 3336}{2}.$$

Proof. Let $G = G_I$ be a type I Pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \dots, v_m, u_1, u_2, \dots, u_{m(p-1)}\}$ be the vertex set of G and v_0 as the central vertex of $K_{1,m}$ and $\{v_1, v_2, \dots, v_m\}$ are pendant vertices of $K_{1,m}$ where $m = p^2 - p + 1$ and the $(p-1)$ pendant vertices $\{u_1, u_2, \dots, u_{m(p-1)}\}$ are attached with m pendant vertices $v_1, v_2, \dots, v_m$. Let $E(G) = (v_0v_i; 1 \le i \le m\} \cup \{v_1u_j; 1 \le j \le p-1\} \cup \{v_2u_j; p \le j \le 2p-2\} \cup \{v_3u_j; 2p-1 \le j \le 3p-3\} \cup \dots \cup \{v_mu_j; (m-1)p - m + 2 \le j \le mp - m\}$.

Derivation of Wiener Index of G_I

The Wiener index of G is given by $W(G) = \sum_{u \ne v} d(u,v)$. Now

$$\begin{aligned}
W(G_I) &= \sum_{i=1}^{m} d(v_0, v_i) + \sum_{i=1}^{m} \sum_{j=i+1}^{m} d(v_i, v_j) + \sum_{i=1}^{m(p-1)} d(v_0, u_i) + \sum_{i=1}^{m} \sum_{j=1}^{m(p-1)} d(v_i, u_j) \\
&\quad + \sum_{1 \le i < j \le n} d(u_i, u_j) \\
&= W(K_{1,m}) + \sum_{i=1}^{m(p-1)} d(v_0, u_i) + \sum_{i=1}^{m} \sum_{j=1}^{m(p-1)} d(v_i, u_j) + \sum_{1 \le i < j \le n} d(u_i, u_j) \\
&= [p^4 - 2p^3 + 3p^2 - 2p + 1] + [2p^3 - 4p^2 + 4p - 2] + [163p^3 - 1001p^2 + 2153p - 1585] \\
&\quad + [36p^5 - 237p^4 + 654p^3 - 913p^2 + 624p - 164]/2 \\
&= \frac{36p^5 - 235p^4 + 980p^3 - 2917p^2 + 4934p - 3336}{2}.
\end{aligned}$$

Derivation of Hyper Wiener Index of G_I

The Hyper Wiener index of G_I is given by $WW(G) = \frac{1}{2} \sum_{u \ne v} d(u,v) + d(u,v)^2$. Now

$$\begin{aligned}
WW(G_I) &= \frac{1}{2} \sum_{i=1}^{m} d(v_0, v_i) + d(v_0, v_i)^2 + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=i+1}^{m} d(v_i, v_j) + d(v_i, v_j)^2 \\
&\quad + \frac{1}{2} \sum_{i=1}^{m(p-1)} d(v_0, u_i) + d(v_0, u_i)^2 + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m(p-1)} d(v_i, u_j) + d(v_i, u_j)^2 \\
&\quad + \frac{1}{2} \sum_{1 \le i < j \le n} d(u_i, u_j) + d(u_i, u_j)^2
\end{aligned}$$

$$
\begin{aligned}
&= WW(K_{1,m}) + \frac{1}{2}\sum_{i=1}^{m(p-1)} d(v_0,u_i) + d(v_0,u_i)^2 + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-1)} d(v_i,u_j) + d(v_i,u_j)^2 \\
&\quad + \frac{1}{2}\sum_{1\le i<j\le n} d(u_i,u_j) + d(u_i,u_j)^2 \\
&= [3p^4 - 6p^3 + 8p^2 - 5p + 2] + [6p^3 - 12p^2 + 12p - 6] \\
&\quad + [650p^3 - 4000p^2 + 8608p - 6338] \\
&\quad + \left[\frac{180p^5 - 1191p^4 + 3306p^3 - 4659p^2 + 3240p - 876}{2}\right] \\
&= \frac{180p^5 - 1185p^4 + 4606p^3 - 12667p^2 + 20470p - 13560}{2}.
\end{aligned}
$$

Detour index of G_I

For G_I type Pseudo-regulargraph, the shortest distance between any two pair of vertices and the longest distance between any pair of vertices would be same. Hence Wiener index and Detour index should be equal.

Theorem 4.2. *For $p \ge 3$, the Wiener, Hyper Wiener and Detour index of type II Pseudo-regular graph G_{II} is*

$$
\begin{aligned}
W(G_I) = W(G_{II}) &= 4p^6 - 53p^5 + 288p^4 - 794p^3 + 1168p^2 - 1026p + 645, \\
WW(G_{II}) &= 20p^6 - 268p^5 + 1464p^4 - 4047p^3 - 6122p^2 - 4555p + 2184, \\
D(G_{II}) &= \frac{\begin{pmatrix} -10p^8 + 242p^7 - 2390p^6 + 12718p^5 - 39404p^4 \\ +70120p^3 - 63602p^2 + 17986p + 5208 \end{pmatrix}}{4}.
\end{aligned}
$$

Proof. Let $G = G_{II}$ be a type II Pseudo-regular graph.

Let $V(G) = \{v_0, v_1, v_2, \dots, v_m, u_1, u_2, \dots, u_{m(p-3)}\}$ be the vertex set of G and v_0 as the central vertex of W_m and $\{v_1, v_2, \dots, v_m\}$ are vertices of c_m in the clockwise direction and $\{u_1, u_2, \dots, u_{m(p-3)})\}$ are the pendant vertices joined to every vertex in the cycle except the central vertex, where $m = p^2 - 3p + 3$.

Let $E(G) = (v_0v_i; 1 \le i \le m\} \cup \{v_iv_{i+1}; 1 \le i \le m-1\} \cup \{v_mv_1\} \cup \{v_1u_j; 1 \le j \le p-3\} \cup \{v_2u_j; p-2 \le j \le 2(p-3)\} \cup \cdots \cup \{v_mu_j; m \le j \le m(p-3)\}$.

Derivation of Wiener index of G_{II}

The Wiener index of G is given by $W(G) = \sum_{u \ne v} d(u,v)$. Now

$$
\begin{aligned}
W(G_{II}) &= \sum_{i=1}^{m} d(v_0,v_i) + \sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i,v_j) + \sum_{i=1}^{m(p-3)} d(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i,u_j) \\
&\quad + \sum_{1\le i<j\le m(p-3)} d(u_i,u_j)
\end{aligned}
$$

$$
\begin{aligned}
&= W(K_1+C_m) + \sum_{i=1}^{m(p-3)} d(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i,u_j) + \sum_{1\le i<j\le n} d(u_i,u_j) \\
&= [5p^3-22p^2+6p+51]+[12p^2-70p+102] \\
&\quad +[3p^5-27p^4+90p^3-105p^2-94p+255] \\
&\quad +[4p^6-56p^5+315p^4-88p^3+1283p^2-868p+237] \\
&= 4p^6-53p^5+288p^4-794p^3+1168p^2-1026p+645.
\end{aligned}
$$

Derivation of Hyper Wiener index of G_{II}

The Hyper Wiener index of G is given by $WW(G)=\frac{1}{2}\sum_{u\neq v} d(u,v)+d(u,v)^2$. Now

$$
\begin{aligned}
WW(G_I) &= \frac{1}{2}\sum_{i=1}^{m} d(v_0,v_i)+d(v_0,v_i)^2 + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i,v_j)+d(v_i,v_j)^2 \\
&\quad + \frac{1}{2}\sum_{i=1}^{m(p-3)} d(v_0,u_i)+d(v_0,u_i)^2 + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i,u_j)+d(v_i,u_j)^2 \\
&\quad + \frac{1}{2}\sum_{1\le i<j\le m(p-3)} d(u_i,u_j)+d(u_i,u_j)^2 \\
&= WW(K_1+C_m) + \frac{1}{2}\sum_{i=1}^{m(p-3)} d(v_0,u_i)+d(v_0,u_i)^2 + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} d(v_i,u_j)+d(v_i,u_j)^2 \\
&\quad + \frac{1}{2}\sum_{1\le i<j\le m(p-3)} d(u_i,u_j)+d(u_i,u_j)^2 \\
&= [15p^3-9p^2+32p+132]+[36p^2-210p+306] \\
&\quad +[12p^5-108p^4+360p^3-456p^2-166p+714] \\
&\quad +[20p^6-280p^5+1572p^4-4422p^3-5693p^2-4211p+1032] \\
&= 20p^6-268p^5+1464p^4-4047p^3-6122p^2-4555p+2184.
\end{aligned}
$$

Derivation of Detour index of G_{II}

The Detour index of G_{II} is given by $D(G_{II})=\sum\limits_{u\neq v} D(u,v)$. Now

$$
\begin{aligned}
D(G_{II}) &= \sum_{i=1}^{m} D(v_0,v_i) + \sum_{i=1}^{m}\sum_{j=i+1}^{m} D(v_i,v_j) + \sum_{i=1}^{m(p-3)} D(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} D(v_i,u_j) \\
&\quad + \sum_{1\le i<j\le m(p-3)} D(u_i,u_j) \\
&= D(K_1+C_m) + \sum_{i=1}^{m(p-3)} D(v_0,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{m(p-3)} D(v_i,u_j) + \sum_{1\le i<j\le n} D(u_i,u_j) \\
&= \left[\frac{5p^5-34p^4+64p^3+39p^2-240p+216}{2}\right] \\
&\quad +[-6p^5+101p^4-628p^3+1825p^2-2472p+1244]
\end{aligned}
$$

$$+[-p^8-20p^7-161p^6+675p^5-1496p^4+1201p^3+1712p^2-4331p+2499]$$
$$+\left[\frac{\begin{pmatrix}-6p^8+162p^7-1746p^6+10032p^5-33756p^4\\+67700p^3-77828p^2+45678p-10116\end{pmatrix}}{4}\right]$$
$$=\frac{\begin{pmatrix}-10p^8+242p^7-2390p^6+12718p^5-39404p^4\\+70120p^3-63602p^2+17986p+5208\end{pmatrix}}{4}.$$

Theorem 4.3. *For $p\geq 5$, the Wiener, Hyper Wiener and Detour index of type III Pseudo-regular graph G_{III} is*

$$W(G_{III})=18p^5-294p^4+2010p^3-6946p^2+11452p-6744,$$
$$WW(G_{III})=86p^5-1408p^4+9534p^3-32308p^2+51752p-29228,$$
$$D(G_{III})=p^9-27p^8+375p^7-3558p^6+24279p^5-115703p^4+364415p^3$$
$$-698354p^2+698394p-246645.$$

Proof. Let $G=G_{III}$ be a type III Pseudo-regular graph.
Let $V(G)=\{v_0,v_1,v_2,\dots,v_m,u_1,u_2,\dots,u_{m(p-5)},w_1,w_2,\dots,w_m\}$ be the vertex set of G and v_0 as the central vertex of w_m where $m=p^2-3p+1$ and $\{u_1,u_2,\dots,u_{m(p-5)}\}$ are the pendant vertices and $\{w_1,w_2,\dots,w_m\}$ are the vertices joined to the end vertices of each edge of a wheel graph except the central vertex.
Let $E(G)=(v_0v_i;1\leq i\leq m\}\cup\{v_iv_{i+1};1\leq i\leq m-1\}\cup\{v_mv_1;1\leq j\leq m(p-5)\}\cup\{u_1v_i;1\leq i\leq 2\}\cup\{u_2v_i;2\leq i\leq 32\}\dots\{u_mv_i;m-1\leq i\leq m\}\cup\{v_iw_j;1\leq j\leq p-5\}\cup\{v_2w_j;p-4\leq j\leq 2(p-5)\}\cup\{v_mw_j;m\leq j\leq m(p-5)\}$.

Derivation of Wiener index of G_{III}

The Wiener index of G is given by $W(G)=\sum\limits_{u\neq v}d(u,v)$. Now

$$\begin{aligned}W(G_{III})&=\sum_{i=1}^{m}d(v_0,v_i)+\sum_{i=1}^{m}\sum_{j=i+1}^{m}d(v_i,v_j)+\sum_{i=1}^{m(p-5)}d(v_0,w_i)+\sum_{i=1}^{m}d(v_0,u_i)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)}d(v_i,w_j)\\&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m}d(v_i,u_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)}d(w_i,w_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m}d(w_i,u_j)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)}d(u_i,w_j)\\&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m}d(u_i,u_j)\\&=W(K_1+C_m)+\sum_{i=1}^{m(p-5)}d(v_0,w_i)+\sum_{i=1}^{m}d(v_0,u_i)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)}d(v_i,w_j)+\sum_{i=1}^{m}\sum_{j=1}^{m}d(v_i,u_j)\\&\quad+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)}d(w_i,w_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m}d(w_i,u_j)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)}d(u_i,w_j)+\sum_{i=1}^{m}\sum_{j=1}^{m}d(u_i,u_j)\\&=[8p^3-54p^2+98p-30]+[2p^3-16p^2+32p-10]+[2p^2-6p+2]\end{aligned}$$

$$
\begin{aligned}
&+[2p^5-25p^4+173p^3-874p^2+2351p-2155]+[3p^4-18p^3+27p^2-3]\\
&+[35p^3-280p^2+560p-175]+[8p^5-136p^4+909p^3-2968p^2+4677p-2810]\\
&+[8p^5-136p^4+901p^3-2876p^2+4323p-2340]+[93p^2-583p+777]\\
&=18p^5-294p^4+2010p^3-6946p^2+11452p-6744.
\end{aligned}
$$

Derivation of Hyper wiener index of G_{III}

The Hyper-Wiener index of G is given by $WW(G)=\frac{1}{2}\sum_{u\neq v} d(u,v)+d(u,v)^2$. Now

$$
\begin{aligned}
WW(G_{III}) &= \frac{1}{2}\sum_{i=1}^{m} d(v_0,v_i)+d(v_0,v_i)^2+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=i+1}^{m} d(v_i,v_j)+d(v_i,v_j)^2\\
&+\frac{1}{2}\sum_{i=1}^{m(p-5)} d(v_0,w_i)+d(v_0,w_i)^2+\frac{1}{2}\sum_{i=1}^{m} d(v_0,u_i)+d(v_0,u_i)^2\\
&+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(v_i,w_j)+d(v_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d(v_i,u_j)+d(v_i,u_j)^2\\
&+\frac{1}{2}\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d(w_i,w_j)+d(w_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d(w_i,u_j)+d(w_i,u_j)^2\\
&+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(u_i,w_j)+d(u_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d(u_i,u_j)+d(u_i,u_j)^2\\
&= WW(K_1+C_m)+\frac{1}{2}\sum_{i=1}^{m(p-5)} d(v_0,w_i)+d(v_0,w_i)^2+\frac{1}{2}\sum_{i=1}^{m} d(v_0,u_i)+d(v_0,u_i)^2\\
&+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(v_i,w_j)+d(v_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d(v_i,u_j)+d(v_i,u_j)^2\\
&+\frac{1}{2}\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} d(w_i,w_j)+d(w_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} d(w_i,u_j)+d(w_i,u_j)^2\\
&+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} d(u_i,w_j)+d(u_i,w_j)^2+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} d(u_i,u_j)+d(u_i,u_j)^2\\
&=[24p^3-164p^2+300p-92]+[6p^3-48p^2+96p-30]+[6p^2-18p+6]\\
&\quad+[6p^5-60p^4+368p^3-2176p^2+6716p-6430]\\
&\quad+[12p^4-72p^3+100p^2+24p-20]+[172p^3-1376p^2+2752p-860]\\
&\quad+[40p^5-680p^4+4538p^3-14784p^2+23266p-13980]\\
&\quad+[40p^5-680p^4+4498p^3-14324p^2+21510p-11700]\\
&\quad+[458p^2-2984p+3878]\\
&=86p^5-1408p^4+9534p^3-32308p^2+51752p-29228.
\end{aligned}
$$

The Detour index of G is given by $D(G)=\sum_{u\neq v} D(u,v)$. Now

$$\begin{aligned}
D(G_{III}) &= \sum_{i=1}^{m} D(v_0,v_i)+\sum_{i=1}^{m}\sum_{j=i+1}^{m} D(v_i,v_j)+\sum_{i=1}^{m(p-5)} D(v_0,w_i)+\sum_{i=1}^{m} D(v_0,u_i)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} D(v_i,w_j)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} D(v_i,u_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} D(w_i,w_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} D(w_i,u_j)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} D(u_i,w_j)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} D(u_i,u_j)\\
&= W(K_1+C_m)+\sum_{i=1}^{m(p-5)} D(v_0,w_i)+\sum_{i=1}^{m} D(v_0,u_i)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} D(v_i,w_j)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} D(v_i,u_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m(p-5)} D(w_i,w_j)+\sum_{i=1}^{m(p-5)}\sum_{j=1}^{m} D(w_i,u_j)+\sum_{i=1}^{m}\sum_{j=1}^{m(p-5)} D(u_i,w_j)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m} D(u_i,u_j)\\
&= [16p^5-216p^4+1144p^3-2789p^2+2623p-59]\\
&\quad+[162p^4-2880p^3+19100p^2-56000p+61250]\\
&\quad+[p^5-11p^4+47p^3-96p^2+88p-22]\\
&\quad+[p^9-27p^8+339p^7-2628p^6\\
&\quad+13696p5-48165p4+109764p3-149968p2+106141p-27005]\\
&\quad+[2p6-18p5+59p4-84p^3+49p^2-12p+1]\\
&\quad+[162p^5-2736p^4+17262p^3-49050p^2+57600p-15750]\\
&\quad+[36p7-932p6+10244p^5-61844p^4+220756p^3-463700p^2+527300p-248500]\\
&\quad+[162p^5-2736p^4+17262p^3-49050p^2+57600p-15750]\\
&\quad+[16p^5-216p^4+1144p^3-2850p^2+3054p-810]\\
&= p^9-27p^8+375p^7-3558p^6+24279p^5-115703p^4+364415p^3\\
&\quad-698354p^2+698394p-246645.
\end{aligned}$$

5. Conclusion

For $p=3$, various types of Pseudo-regular graphs will be constructed. The topological indices of Pseudo-regular graphs for $p=3$ will be included in the future research study.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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