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Research Article

Bipartite Graphs Associated with 3 Uniform Semigraphs of Trees and its Topological Indices

V. Kala Devi[1] and K. Marimuthu[2,*]

[1] *Department of Mathematics, Bishop Heber College, Trichy, Tamilnadu 620 017, India*
[2] *Department of Mathematics, Ramco Institute of Technology, Rajapalayam, Tamilnadu 626117, India*
***Corresponding author:** marismphil@gmail.com

Abstract. In this paper, we have studied special class of bipartite graphs associated with 3 uniform semigraphs of path graph $P_{m,1}$ and star graph $S_{m,1}$ and estimated some topological indices such as Wiener index, Detour index, Circular index, Cut Circular index, vertex PI index and vertex Co-PI index of these graphs.

Keywords. Semi graph; Bipartite graphs associated with semigraphs; Wiener index; Detour index; Vertex PI index; Vertex Co-PI index

MSC. 54A99

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1. Introduction

Let $G = (V(G), E(G))$ be a simple, connected and undirected graph, where $V(G)$ is the vertex set of G and $E(G)$ is the edge set of G. For any two vertices $u, v \in V(G)$, the shortest distance between u and v is denoted by $d(u,v)$, the longest distance between u and v is denoted by $D(u,v)$, the sum of the longest distance and shortest distance between u and v, called as circular distance is denoted by $d^0(u,v)$.

The Wiener index of G is defined as $W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d(u,v)$ with the summation taken over all pairs of distinct vertices of G. In the same manner the Detour index of G is defined as $D(G) = \frac{1}{2} \sum_{u,v \in V(G)} D(u,v)$, the Circular index of G is defined as $C(G) = \frac{1}{2} \sum_{u,v \in V(G)} (D(u,v) + d(u,v))$ and the Cut Circular index of G is defined as $CC(G) = \frac{1}{2} \sum_{u,v \in V(G)} (D(u,v) - d(u,v))$. Also, $C(G) = D(G) + W(G)$ and $CC(G) = D(G) - W(G)$. For an edge $e = (u,v) \in E(G)$, the number of vertices of G whose distance to the vertex u is smaller than the distance to the vertex v in G is denoted by $n_u^G(e)$ and the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u in G is denoted by $n_v^G(e)$, the vertices with equidistance from the ends of the edge $uv = e$ are not counted. The vertex PI index of G, denoted by PI(G), is defined as $\text{PI}(G) = \sum_{e=uv \in E(G)} [n_u^G(e) + n_v^G(e)]$. If G is a bipartite graph, then $\text{PI}(G) = |V(G)| \cdot |E(G)|$ [1]. The vertex Co-PI index of G, denoted by Co-PI(G), is defined as $\text{Co-PI}(G) = \sum_{e=uv \in E(G)} |n_u^G(e) - n_v^G(e)|$.

2. Semigraph and Bipartite Graphs Associated with Semi Graph

2.1 Semigraph

Semigraph is a natural generalization of graph where in an edge may have more than two vertices by containing middle vertices apart from the usual end vertices. Semigraphs, introduced by Sampathkumar [8], is an interesting type of generalization of the concept of graph. Kamath and Bhat [2] introduced adjacency domination in semigraphs. Also, Kamath and Hbber [3] introduced strong and weak domination in semigraphs. Semi graph have elegant pictorial representation [9] and several results have been extended from graph theory to semigraphs. Venkatakrishnan and Swaminathan [11] introduced bipartite theory of semigraphs. Given a semigraph they constructed bipartite graphs which represents the arbitrary graphs.

A semigraph S is a pair (V,X), where V is a non empty set whose elements are called vertices of S, and X is a set of $n-tuples$ of distinct vertices called edges of S for various $n \geq 2$ satisfying the following conditions:

(a) Any two edges have at most one vertex in common.

(b) Two edges $(u_1,u_2,\ldots,u_m)$ and $(v_1,v_2,\ldots,v_n)$ are considered to be equal if and only if (i) $m = n$ and (ii) either $u_i = v_i$ for $1 \leq i \leq n$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Thus, the edges $(u_1,u_2,\ldots,u_m)$ is same as $(u_m,u_{m-1},\ldots,u_1)$.

If $E = (v_1,v_2,\ldots,v_n)$ is an edge of a semigraph, we say that v_1 and v_n are the end vertices of the edge E and v_i, $2 \leq i \leq n-1$, are the middle vertices or m-vertices of the edge e and also the vertices $v_1, v_2, \dots, v_n$, are said to belong to the edge e. A semigraph with p vertices and q edges is called a (p,q)-semigraph. Two vertices u and $v, u \neq v$, in a semigraph are adjacent if both off them belong to the same edge. The number of vertices in an edge e is called cardinality of e and it is denoted by $|e|$. A semigraph S is said to be r-uniform if the cardinality of each edge in S is r. By introducing n number of middle vertices to each edge of the graph C_m, where C_m is the cycle with m vertices, we get a semigraph with $(n+2)$ uniform which is denoted as $C_{m,n}$.

Example 2.1. Let $S = (V, X)$ be a semigraph, where $V = \{1, 2, \dots, 10\}$ and $X = \{(1,2),(3,6,8),(6,9,10),(2,10),(3,4,5),(1,5)\}$. The graph S is given in Figure 1.

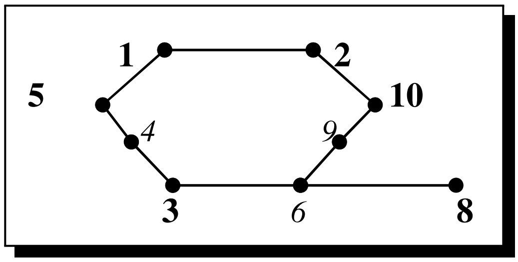

Figure 1

2.2 Bipartite Graphs Associated with Semigraph

Let V' be the another copy of the vertex set V of a semigraph S. Then the following graphs represents the bipartite graphs associated with the semigraph S.

Bipartite graph $A(S)$. The bipartite graph $A(S) = (V, V', X)$, where $X = \{(u, v')/u$ and v belong to the same edge of the semigraph $S\}$.

Bipartite graph $A^+(S)$. The bipartite graph $A^+(S) = (V, V', X)$, where $X = \{(u, v')/u$ and v belong to the same edge of the semigraph $S\} \cup \{(u, u')/u \in V, u' \in V'\}$.

Bipartite graph $CA(S)$. The bipartite graph $CA(S) = (V, V', X)$, where $X = \{(u, v')/u$ and v are consecutively adjacent in $S\}$.

Bipartite graph $CA^+(S)$. The bipartite graph $CA^+(S) = (V, V', X)$, where $X = \{(u, v')/u$ and v are consecutively adjacent in $S\} \cup \{(u, u')/u \in V, u' \in V'\}$.

Bipartite graph $VE(S)$. The bipartite graph $VE(S) = (V, X, Y)$, where V is vertex set and X is the set of edges of the semigraph S and $Y = \{(u, e)/u \in V$ and $e \in X\}$.

$P_{m,1}$ is a 3 uniform semigraph. The Bipartite graph $A(P_{5,1})$, the Bipartite graph $A^+(P_{5,1})$, the Bipartite graph $CA(P_{5,1})$, the Bipartite graph $CA^+(P_{5,1})$ and the Bipartite graph $VE(P_{5,1})$ are given in Figures 2–6, respectively.

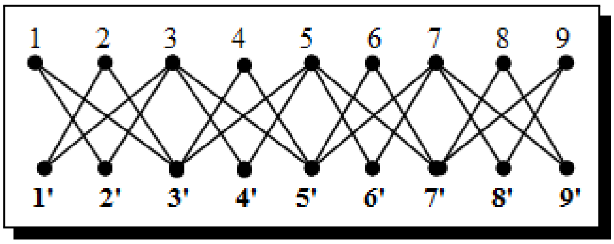

Figure 2

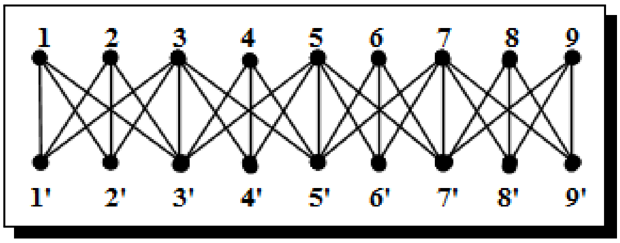

Figure 3

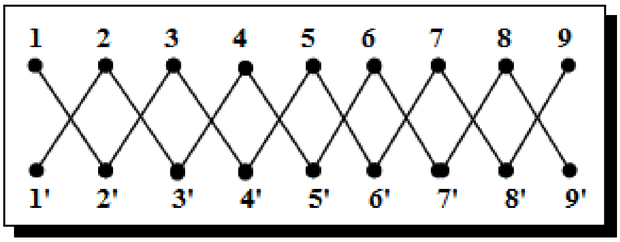

Figure 4

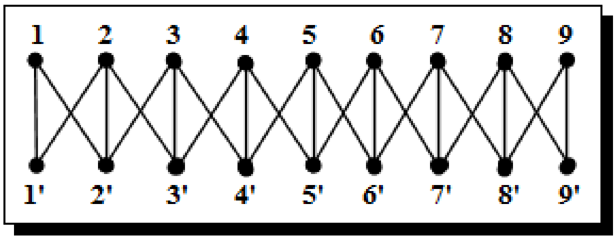

Figure 5

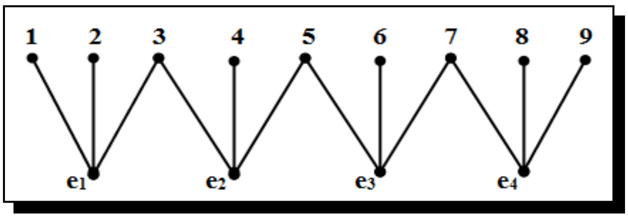

Figure 6

The Bipartite graph $CA(P_{5,1})$ is the disjoint union of two paths and which is a disconnected graph.

Theorem 2.2. *Let G be the Bipartite graph $A(P_{m,1})$. Then $W(G) = \frac{1}{3}[8m^3 + 12m^2 - 20m + 9]$, $PI(G) = 24m^2 - 36m + 12$ and Co-PI$(G) = \begin{cases} 12m^2 - 32m + 16, & \text{if } m \text{ is even} \\ 12m^2 - 32m + 20, & \text{if } m \text{ is odd} \end{cases}$ where $m = 3,4,5,\dots$.*

Proof. Let $U = V \cup V'$ where $V = \{1,2,\dots,2m-1\}$, $V' = \{1',2',\dots,(2m-1)'\}$ and $E = \{(u,v')/u$ and v belong to the same edge of the semigraph $P_{m,1}\}$ be the vertex set and edge set of the graph G, respectively.

Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1 \\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1' \\ i<j}}^{(2m-1)'} d(i',j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i,j')$. Then $W(G) = S_1 + S_2 + S_3$.

Case (i): m is even

$$S_1 + S_2 = 2(7m-11)P_3 + 8[(2m-7)P_5 + (2m-11)P_7 + \dots + 5P_{m-1} + P_{m+1}]$$
$$= 4(7m-11) + 8[4(2m-7) + 6(2m-11) + \dots + 5(m-2) + m],$$
$$S_3 = 6(m-1)P_2 + (18m-41)P_4 + 8[(2m-9)P_6 + (2m-13)P_8 + \dots + 7P_{m-2} + 3P_m]$$
$$= 6(m-1) + 3(18m-41) + 8[5(2m-9) + 7(2m-13) + \dots + 7(m-3) + 3(m-1)],$$
$$W(G) = S_1 + S_2 + S_3 = \frac{1}{3}[8m^3 + 12m^2 - 20m + 9].$$

Case (ii): m is odd

$$S_1 + S_2 = 2(7m-11)P_3 + 8[(2m-7)P_5 + (2m-11)P_7 + \dots + 7P_{m-2} + 3P_m]$$
$$= 4(7m-11) + 8[4(2m-7) + 6(2m-11) + \dots + 7(m-3) + 3(m-1)],$$
$$S_3 = 6(m-1)P_2 + (18m-41)P_4 + 8[(2m-9)P_6 + (2m-13)P_8 + \dots + 5P_{m-1} + P_{m+1}]$$
$$= 6(m-1) + 3(18m-41) + 8[5(2m-9) + 7(2m-13) + \dots + 5(m-2) + m],$$
$$W(G) = S_1 + S_2 + S_3 = \frac{1}{3}[8m^3 + 12m^2 - 20m + 9].$$

For any m, PI$(G) = |U(G)| \cdot |E(G)| = (4m-2) \times 6(m-1) = 12m^2 - 24m + 12$.

If m is even, then

$$\text{Co-PI}(G) = \sum_{e=uv \in E(G)} |n_u^G(e) - n_v^G(e)|$$
$$= 4[4 + 12 + 20 + \dots + (4m-12)] + 8[8 + 16 + 24 + \dots + (4m-8)]$$
$$= 12m^2 - 32m + 16.$$

If m is odd, then

$$\text{Co-PI}(G) = \sum_{e=uv\in E(G)} |n_u^G(e) - n_v^G(e)|$$
$$= 8[4+12+20+\ldots+(4m-8)] + 4[8+16+24+\ldots+(4m-12)]$$
$$= 12m^2 - 32m + 20.$$ □

Theorem 2.3. *Let G be the Bipartite graph $A^+(P_{m,1})$. Then $W(G) = \frac{1}{3}[8m^3 + 12m^2 - 32m + 15]$, $D(G) = 32m^3 - 68m^2 + 48m - 11$, $PI(G) = 32m^2 - 44m + 14$, where $m = 3,4,5,\ldots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\ldots,2m-1\}$, $V' = \{1',2',\ldots,(2m-1)'\}$ and $E = \{(u,v')/u$ and v belong to the same edge of the semigraph $P_{m,1}\} \cup \{(u,u')/u \in V, u' \in V'\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1\\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1'\\ i<j}}^{(2m-1)'} d(i',j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i,j')$. Then $W(G) = S_1 + S_2 + S_3$.

Case (i): m is even

$$S_1 + S_2 = 2(7m-11)P_3 + 8[(2m-7)P_5 + (2m-11)P_7 + \ldots + 5P_{m-1} + P_{m+1}]$$
$$= 4(7m-11) + 8[4(2m-7) + 6(2m-11) + \ldots + 5(m-2) + m],$$
$$S_3 = (8m-7)P_2 + (16m-40)P_4 + 8[(2m-9)P_6 + (2m-13)P_8 + \ldots + 7P_{m-2} + 3P_m]$$
$$= (8m-7) + 3(16m-40) + 8[5(2m-9) + 7(2m-13) + \ldots + 7(m-3) + 3(m-1)],$$
$$W(G) = S_1 + S_2 + S_3 = \frac{1}{3}[8m^3 + 12m^2 - 32m + 15].$$

Case (ii): m is odd

$$S_1 + S_2 = 2(7m-11)P_3 + 8[(2m-7)P_5 + (2m-11)P_7 + \ldots + 7P_{m-2} + 3P_m]$$
$$= 4(7m-11) + 8[4(2m-7) + 6(2m-11) + \ldots + 7(m-3) + 3(m-1)],$$
$$S_3 = (8m-7)P_2 + (16m-40)P_4 + 8[(2m-9)P_6 + (2m-13)P_8 + \ldots + 5P_{m-1} + P_{m+1}]$$
$$= (8m-7) + 3(16m-40) + 8[5(2m-9) + 7(2m-13) + \ldots + 5(m-2) + m],$$
$$W(G) = S_1 + S_2 + S_3 = \frac{1}{3}[8m^3 + 12m^2 - 32m + 15].$$

Now $D(G) = S_1 + S_2 + S_3$, where $S_1 = S_2 = \frac{(2m-1)(2m-2)}{2} P_{4m-3}$, $S_3 = (2m-1)^2 P_{4m-2}$, and $D(G) = 32m^3 - 68m^2 + 48m - 11$.

For any m, $PI(G) = |U(G)| \cdot |E(G)| = (4m-2) \times (8m-7) = 32m^2 - 44m + 14$. □

Note. Since the Bipartite graph $A^+(P_{m,1})$ have $2m-1$ more edges than the Bipartite graph $A(P_{m,1})$ and $n_u^G(e) = n_{u'}^G(e) = 2m-1$, Co-PI($A(P_{m,1})$) and Co-PI($A^+(P_{m,1})$) are the same.

Theorem 2.4. *Let G be the Bipartite graph $CA^+(P_{m,1})$. Then $W(G) = \frac{1}{3}[16m^3 - 12m^2 - 4m + 3]$, $D(G) = 32m^3 - 74m^2 + 64m - 21$, $PI(G) = 24m^2 - 32m + 10$, Co-PI$(G) = 8m^2 - 16m + 8$, where $m = 3,4,5,\ldots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\ldots,2m-1\}$, $V' = \{1',2',\ldots,(2m-1)'\}$ and $E = \{(u,v')/u$ and v are consecutively adjacent in the semigraph $P_{m,1}\} \cup \{(u,u')/u \in V,\ u' \in V'\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1\\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1'\\ i<j}}^{(2m-1)'} d(i',j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i,j')$.

Then $W(G) = S_1 + S_2 + S_3$, where

$$\begin{aligned}
S_1 + S_2 &= (8m-10)P_3 + (8m-18)P_5 + \ldots + 6P_{2m-1}\\
&= 2(8m-10) + 4(8m-18) + \ldots + 6(2m-2),\\
S_3 &= (6m-5)P_2 + [(8m-14)P_4 + (8m-22)P_6 + \ldots + 2P_{2m}]\\
&= (6m-5) + [3(8m-14) + 5(8m-22) + \ldots + 2P_{2m}],\\
W(G) &= S_1 + S_2 + S_3 = \frac{1}{3}[16m^3 - 12m^2 - 4m + 3].
\end{aligned}$$

Now, $D(G) = S_1 + S_2 + S_3$, where

$$\begin{aligned}
S_1 + S_2 &= 4[P_{2m+1} + P_{2m+3} + \ldots + P_{4m-5}] + (4m^2 - 10m + 10)P_{4m-3}\\
&= 4[(2m) + (2m+2) + \ldots + (4m-6)] + (4m^2 - 10m + 10)(4m-4)\\
&= 4[(\frac{m-2}{2})(6m-6)] + (4m^2 - 10m + 10)(4m-4),\\
S_3 &= (2m-1)P_2 + 2[P_{2m+2} + P_{2m+4} + \ldots + P_{4m-4}] + (4m^2 - 6m + 4)P_{4m-2}\\
&= (2m-1) + 2[(\frac{m-2}{2})(6m-4)] + (4m^2 - 6m + 4)(4m-3),\\
D(G) &= 32m^3 - 74m^2 + 64m - 21.
\end{aligned}$$

For any m, $PI(G) = |U(G)| \cdot |E(G)| = (4m-2) \times (6m-5) = 24m^2 - 32m + 10$ and Co-PI$(G) = \sum_{e=uv \in E(G)} |n_u^G(e) - n_v^G(e)| = 4[(4m-6) + (4m-10) + \ldots + 2] = 8m^2 - 16m + 8$. □

Theorem 2.5. *Let G be the Bipartite graph $VE(P_{m,1})$. Then $W(G) = D(G) = 3m^3 - 3m^2 - 3m + 3$, $PI(G) = 9m^2 - 15m + 6$ and Co-PI$(G) = 6m^2 - 13m + 8$, where $m = 3,4,5,\ldots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\dots,2m-1\}$, $V' = \{e_1, e_2, \dots, e_{m-1}\}$ and $E = \{(e_i, j)/1 \le i \le m-1,\ j = 2i-1,\ 2i,\ 2i+1\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1 \\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1}^{m-1} \sum_{\substack{j=1 \\ i<j}}^{m-1} d(e_i, e_j)$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1}^{m-1} d(i, e_j)$. Then $W(G) = S_1 + S_2 + S_3$, where

$$\begin{aligned}
S_1 + S_2 &= (4m-5)P_3 + [(5m-11)P_5 + (5m-16)P_7 + \dots + 14P_{2m-5} + 9P_{2m-3}] + 4P_{2m-1} \\
&= 2(4m-5) + [4(5m-11) + 6(5m-16) + \dots + 14(2m-4) + 9(2m-2)] + 4(2m-2), \\
S_3 &= 3(m-1)P_2 + [(4m-8)P_4 + (4m-12)P_6 + \dots + 8P_{2m-4}] + 4P_{2m-2} \\
&= 6(m-1) + [3(4m-8) + 5(4m-12) + \dots + 8(2m-5)] + 4(2m-3), \\
W(G) &= 3m^3 - 3m^2 - 3m + 3.
\end{aligned}$$

Since G is a tree and $d(u,v) = D(u,v)$, for all $u, v \in G$, Wiener index and Detour index are the same. For any m, $PI(G) = |U(G)| \cdot |E(G)| = (3m-2) \times (3m-3) = 9m^2 - 15m + 6$.

Finally, let $e = (u, v')$. If m is even, then $|n_u^G(e) - n_{v'}^G(e)|$ = either $2, 4, 8, 10, \dots, 3m-4$.

Co-PI(G) = $2[2 + 4 + 8 + \dots + (3m-8)] + (m-1)(3m-4) = 6m^2 - 13m + 8$. If m is odd, then $|n_u^G(e) - n_{v'}^G(e)|$ = either $1, 5, 7, 11, \dots$ or $3m-8$.

Co-PI(G) $= 2[1 + 5 + 7 + \dots + (3m-8)] + (m-1)(3m-4) = 6m^2 - 13m + 8$. □

Theorem 2.6. *Let G be the Bipartite graph $A(S_{m,1})$. Then $W(G) = 2m^2 - 36m + 17$, $D(G) = 44m^2 - 68m + 27$, $PI(G) = 24m^2 - 36m + 12$ and Co-PI$(G) = 16m^2 - 48m + 32$, where $m = 3, 4, 5, \dots$.*

Proof. Let $U = V \cup V'$, where $V = \{1, 2, \dots, 2m-1\}$, $V' = \{1', 2', \dots, (2m-1)'\}$ and $E = \{(u, v')/u$ and v belong to the same edge of the semigraph $S_{m,1}\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1 \\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1' \\ i<j}}^{(2m-1)'} d(i', j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i, j')$. Then $W(G) = S_1 + S_2 + S_3$, where $S_1 + S_2 = (4m^2 - 6m + 2)P_3 = 8m^2 - 12m + 4$, $S_3 = (6m-5)P_2 + (4m^2 - 10m + 6)P_4 = 12m^2 - 24m + 13$ and $W(G) = 20m^2 - 36m + 17$. $D(G) = S_1 + S_2 + S_3$, where $S_1 + S_2 = (4m-4)P_5 + (4m^2 - 10m + 6)P_7 = 24m^2 - 44m + 20$, $S_3 = (2m-1)P_4 + (4m^2 - 6m + 2)P_6 = 20m^2 - 24m + 7$ and $D(G) = 44m^2 - 68m + 27$.

Finally for any m, $PI(G) = |U(G)| \cdot |E(G)| = (4m-2) \times (6m-6) = 24m^2 - 36m + 12$.

Finally, let $e = (1, v')$, then $n_1^G(e) = 4m - 5$, $n_{v'}^G(e) = 3$ and let $e = (v, 1')$. Otherwise $n_u^G(e) = n_{v'}^G(e) = 2m - 1$. Co-PI$(G) = (4m-4)(4m-8) = 16m^2 - 48m + 32$. □

Theorem 2.7. *Let G be the Bipartite graph $A^+(S_{m,1})$. Then $W(G) = 20m^2 - 40m + 21$, $PI(G) = 32m^2 - 44m + 14$, where $m = 3, 4, 5, \dots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\dots,2m-1\}$, $V' = \{1',2',\dots,(2m-1)'\}$ and $E = \{(u,v')/u$ and v belong to the same edge of the semigraph $S_{m,1}\} \cup \{(u,u')/u \in V, u' \in V'\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1\\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1'\\ i<j}}^{(2m-1)'} d(i',j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i,j')$. Then $W(G) = S_1+S_2+S_3$, where $S_1+S_2 = (4m^2-6m+2)P_3 = 8m^2-12m+4$, $S_3 = (8m-7)P_2 + (4m^2-12m+8)P_4 = 12m^2-28m+17$ and $W(G) = 20m^2-40m+21$.

For any m, $PI(G) = |U(G)| \cdot |E(G)| = (4m-2) \times (8m-7) = 32m^2-44m+14$. □

Note. Since the Bipartite graph $A^+(S_{m,1})$ have $2m-1$ more edges than the Bipartite graph $A(S_{m,1})$ and $n_u^G(e) = n_{u'}^G(e) = 2m-1$, Co-PI($A(S_{m,1})$) and Co-PI($A^+(S_{m,1})$) are the same.

Theorem 2.8. *Let G be the Bipartite graph $CA^+(S_{m,1})$. Then $W(G) = 28m^2-60m+33$, $D(G) = 68m^2-88m+23$, $PI(G) = 24m^2-32m+10$ and Co-PI$(G) = 16m^2-48m+32$, where $m = 3,4,5,\dots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\dots,2m-1\}$, $V' = \{1',2',\dots,(2m-1)'\}$ and $E = \{(u,v')/u$ and v consecutively adjacent in the semigraph $S_{m,1}\} \cup \{(u,u')/u \in V,\ u' \in V'\}$ be the vertex set and edge set of the graph G respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1\\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1'}^{(2m-1)'} \sum_{\substack{j=1'\\ i<j}}^{(2m-1)'} d(i',j')$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1'}^{(2m-1)'} d(i,j')$. Then $W(G) = S_1+S_2+S_3$, where $S_1+S_2 = (m^2+3m-4)P_3 + (3m^2-9m+6)P_5 = 14m^2-30m+16$, $S_3 = (6m-5)P_2 + (3m^2-7m+4)P_4 + (m^2-3m+2)P_6 = 14m^2-30m+17$ and $W(G) = 28m^2-60m+33$. For any m, $PI(G) = |U(G)| \cdot |E(G)| = (4m-2) \times (6m-5) = 24m^2-32m+10$.

Finally, let $e = (u,u')$, $u = 1,2,\dots,2m-1$, then $n_u^G(e) = n_{u'}^G(e) = 2m-1$, let $e = (1,u')$, $u = 2,4,6,\dots,2m-2$, then $n_1^G(e) = 4m-6$, $n_{u'}^G(e) = 4$ and let $e = (u,1')$, $u = 2,4,6,\dots,2m-2$, then $n_u^G(e) = 4$, $n_{1'}^G(e) = 4m-6$. Otherwise, $n_u^G(e) = 4m-4$, $n_{v'}^G(e) = 2$. Then Co-PI$(G) = 2(m-1)(4m-10) + (2m-1)(4m-6) = 16m^2-48m+32$. □

Theorem 2.9. *Let G be the Bipartite graph $VE(S_{m,1})$. Then $W(G) = D(G) = 15m^2-36m+21$, $PI(G) = 9m^2-15m+6$ and Co-PI$(G) = 9m^2-25m+16$, where $m = 3,4,5,\dots$.*

Proof. Let $U = V \cup V'$, where $V = \{1,2,\dots,2m-1\}$, $V' = \{e_1,e_2,\dots,e_{m-1}\}$ and $E = \{(e_i,j)/1 \le i \le m-1, j = 1,2i,2i+1\}$ be the vertex set and edge set of the graph G, respectively. Let $S_1 = \sum_{i=1}^{2m-1} \sum_{\substack{j=1\\ i<j}}^{2m-1} d(i,j)$, $S_2 = \sum_{i=1}^{m-1} \sum_{\substack{j=1\\ i<j}}^{m-1} d(e_i,e_j)$ and $S_3 = \sum_{i=1}^{2m-1} \sum_{j=1}^{m-1} d(i,e_j)$. Then $W(G) = S_1+S_2+S_3$, where $S_1+S_2 = \frac{1}{2}(m^2+3m-4)P_3 + (2m^2-6m+4)P_5 = 9m^2-21m+12$, $S_3 = 3(m-1)P_2 + (2m^2-6m+4)P_4 = 6m^2-15m+9$ and $W(G) = 15m^2-36m+21$. Since G is a tree and $d(u,v) = D(u,v)$, for all $u,v \in G$, Wiener index and Detour index are the same. For

any m, $PI(G) = |U(G)| \cdot |E(G)| = (3m-2) \times (3m-3) = 9m^2 - 15m + 6$. Finally, let $e = (1, e_i)$, $i = 1, 2, \dots, m-1$, then $n_1^G(e) = 3m-5$, $n_{e_i}^G(e) = 3$. Otherwise, $n_u^G(e) = 1$, $n_{v'}^G(e) = 3m-3$. Then Co-PI$(G) = (m-1)(3m-8) + (2m-2)(3m-4) = 9m^2 - 25m + 16$. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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