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Research Article

3-Total Sum Cordial Labeling on Some New Graphs

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Abstract. Let $G = (V, E)$ be a graph with vertex set V and edge set E. Consider a vertex labeling $f : V(G) \to \{0,1,2\}$ such that each edge uv assign the label $(f(u)+f(v)) \pmod 3$. The map f is called a 3-total sum cordial labeling if $|f(i)-f(j)| \le 1$, for $i, j \in \{0,1,2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0,1,2\}$. Any graph which satisfied 3-total sum cordial labeling is called a 3-total sum cordial graph. Here we prove some graphs like wheel, globe and a graph obtained by switching and duplication of arbitrary vertex of a cycle are 3-total sum cordial graphs.

Keywords. 3-total sum cordial labeling; 3-total sum cordial graph; Globe; Vertex switching; Vertex duplication

MSC. 05C78

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1. Introduction

The graphs consider here are simple, finite, connected and undirected graphs for all other terminology and notation follow Harrary [3]. Let $G(V, E)$ be a graph where the symbols $V(G)$

and $E(G)$ denotes the vertex set and edge set. If the vertices or edges or both of the graphare assigned values subject to certain conditions it is known as graph labeling. Many of the results about graph labelings, including cordial labelings, are collected and updated in a survey by Gallian [2]. Cordial graphs was first introduced by Cahit [1] as a weaker version of both graceful graphs and harmonious graphs. The concept of sum cordial labeling of graph was introduced by Shiama [5] and that of k-sum cordial labeling by Pethanachi Selvam [4]. The concept of 3-total super sum cordial labeling of graphs was introduced by Tenguria and Verma [7]. Ghosh and Pal [6] discussed Fibonacci divisor cordial labeling on a graph obtained by switching and duplication of arbitrary vertex of the graph. Here brief summary of definitions are given which are useful for the present investigations.

Definition 1.1. Let $G = (V, E)$ be a graph. Let $f : V \to \{0, 1\}$, and for each edge uv, assign the label $|f(u) - f(v)|$. Then the binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f((1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i) =$ number of vertices having label I under f and $e_f(i)$ =number of edges having label i under f.

A graph G is Cordial if it admits Cordial Labeling.

Definition 1.2. Let G be a graph. Let f be a map from $V(G)$ to $\{0, 1, 2\}$. For each edge uv assign the label $[f(u) + f(v)] \pmod 3$. Then the map f is called 3-total sum cordial labeling of G, if $|f(i) - f(j)| \leq 1$; $i, j \in \{0, 1, 2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0, 1, 2\}$.

Definition 1.3. A globe is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by $Gl(n)$.

Definition 1.4. A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5. Duplication of a vertex v_k of a graph G produces a new graph G_1 by a vertex $v_{k'}$ with $N(v_{k'}) = N(v_k)$.

2. Main Results

Theorem 2.1. *Wheel W_n is a 3-total sum cordial graph.*

Proof. Let v be the apex vertex and $v_1, v_{2,} \dots, v_n$ be the rim vertices of wheel W_n.

Define $f(v) = 0$

$$f(v_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 2 & \text{if } i \text{ is even.} \end{cases}$$

Hence f is 3-total sum cordial labeling. □

Example 2.1. Wheel W_{11} is a 3-total sum cordial graph.

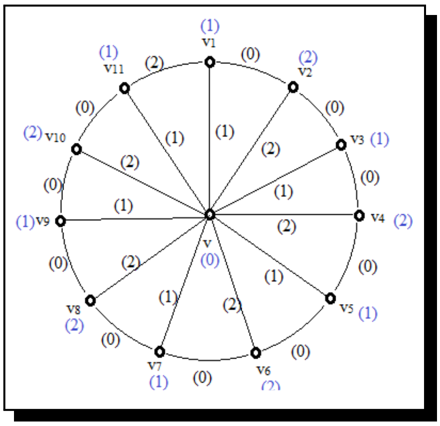

Figure 1. 3-total sum cordial labeling of W_{11}.

Theorem 2.2. *Globe $Gl(n)$ is a 3-total sum cordial graph.*

Proof. Let $V(Gl(n)) = [u,v,w_i : 1 \le i \le n]$.

Define $f(u) = 1$

$f(v) = 2$

and $f(w_i) = 0$ for all i.

Then f is 3-total sum cordial labeling. □

Example 2.2. Globe $Gl(7)$ is a 3-total sum cordial graph.

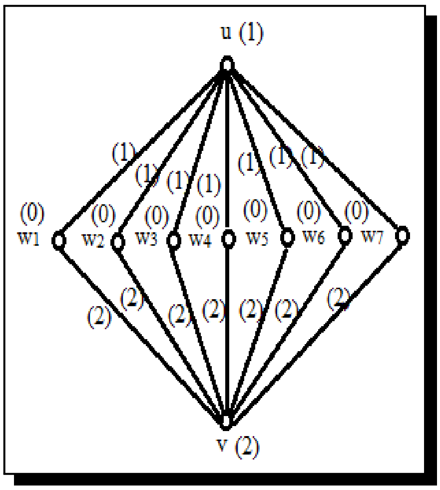

Figure 2. 3-total sum cordial labeling of $Gl(7)$.

Theorem 2.3. *The graph obtained by switching of an arbitrary vertex in cycle C_n is a 3-total sum cordial graph.*

Proof. Let $v_1, v_2, \dots, v_n$ be the successive vertices of C_n, and G_v denotes the graph obtained by switching of vertex v of G. Without loss of generality let the switched vertex be v_1. We note that $|V(G_{v1})| = n$ and $|E(G_{v1})| = 2n-5$. We define $f : V(G_{v1}) \to \{0,1,2\}$ as follows:

$$f(u_1) = 0$$

and $$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is even} \\ 2 & \text{if } i \text{ is odd.} \end{cases}$$

Hence f is 3-total sum cordial labeling. □

Example 2.3. The graph obtained by switching the vertex v_1 in cycle C_9 is a 3-total sum cordial graph.

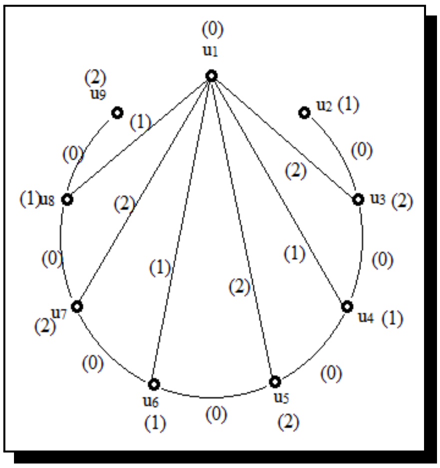

Figure 3. 3-total sum cordial labeling of the graph obtained by switching the vertex v_1 in cycle C_9.

Theorem 2.4. *The graph obtained by duplication of an arbitrary vertex in cycle C_n is a 3-total sum cordial graph.*

Proof. Let $u_1, u_2, \dots, u_n$ be the successive vertices of C_n, and G denotes the graph obtained by duplication of any vertex of C_n. Without loss of generality let the duplicated vertex be u_1. We note that $|V(G)| = n+1$ and $|E(G)| = n+2$. □

Case I: $n \equiv 0 \pmod 3$

Define f as $$f(u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod 3 \\ 1 & \text{if } i \equiv 2 \pmod 3 \\ 2 & \text{if } i \equiv 0 \pmod 3 \end{cases}$$

and $$f(u_1') = 0.$$

Then f is 3-total sum cordial labelling for Case I.

Example 2.4. The graph obtained by duplication of the vertex v_1 in cycle C_9 is a 3-total sum cordial graph.

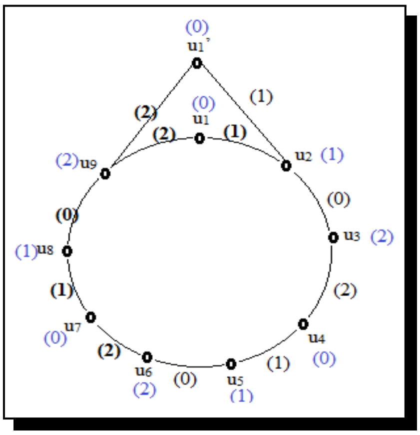

Figure 4. 3-total sum cordial labeling of the graph obtained by duplicating the vertex u_1 in cycle C_9.

Case II: $n \equiv 1 \pmod 3$

$$\text{Define } f \text{ as } f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 2 & \text{if } i \equiv 0 \pmod 3 \end{cases}$$

and $\quad f(u_1') = 0$.

Then f is 3-total sum cordial labelling for Case II.

Example 2.5. The graph obtained by duplication of the vertex u_1 in cycle C_{10} is a 3-total sum cordial graph.

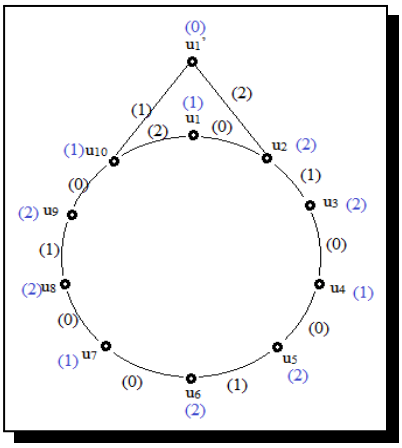

Figure 5. 3-total sum cordial labeling of the graph obtained by duplicating the vertex u_1 in cycle C_{10}.

Case III: $n \equiv 2 \pmod 3$

$$\text{Define } f \text{ as } f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 2 & \text{if } i \equiv 0 \pmod 3 \end{cases}$$

and $\quad f(u_1') = 2$.

Then f is 3-total sum cordial labelling for Case III.

Example 2.6. The graph obtained by duplication of the vertex u_1 in cycle C_{11} is a 3-total sum cordial graph.

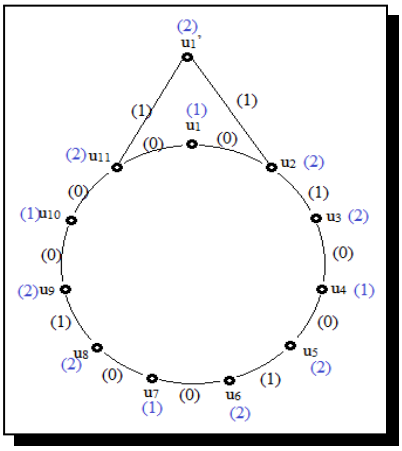

Figure 6. 3-total sum cordial labeling of the graph obtained by duplicating the vertex u_1 in cycle C_{11}.

Theorem 2.5. *Helm H_n is a 3-total sum cordial graph.*

Proof. Let u be the apex vertex and the other vertices are $u_1, u_2, u_3, \dots, u_n$ and $v_1, v_2, \dots, v_n$.

Case I: $n \equiv 0 \pmod 3$

Define f as $f(u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod 3 \\ 1 & \text{if } i \equiv 2 \pmod 3 \\ 2 & \text{if } i \equiv 0 \pmod 3 \end{cases}$

and $\quad f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 \pmod 3 \\ 1 & \text{if } i \equiv 2 \pmod 3 \\ 2 & \text{if } i \equiv 0 \pmod 3 \end{cases}$

$f(u) = 0$.

Then f is 3-total sum cordial labelling for Case I.

Example 2.7. Helm H_9 is a 3-total sum cordial graph.

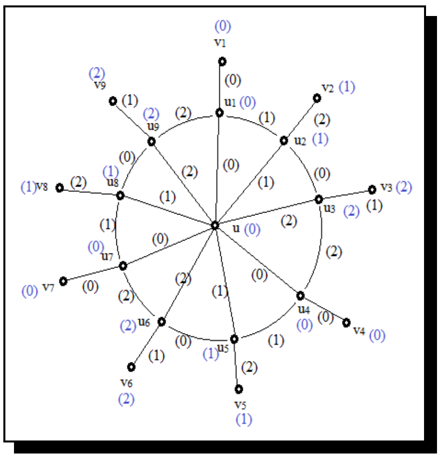

Figure 7. 3-total sum cordial labeling of H_9.

Case II: $n \equiv 1 \pmod 3$

Define f as $f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 0 & \text{if } i \equiv 0 \pmod 3 \text{ for } i = 1 \text{ to } (n-1) \end{cases}$

and $\quad f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 0 & \text{if } i \equiv 0 \pmod 3 \text{ for } i = 1 \text{ to } (n-1) \end{cases}$

$$f(u_n) = 1; \quad f(v_n) = 2.$$

$$f(u) = 0.$$

Then f is 3-total sum cordial labelling for Case II. □

Example 2.8. Helm H_{10} is a 3-total sum cordial graph.

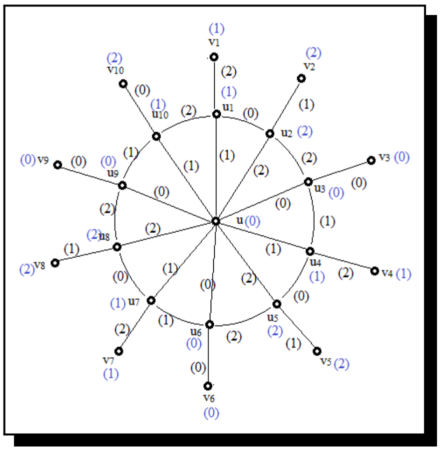

Figure 8. 3-total sum cordial labeling of H_{10}.

Case III: $n \equiv 2 \pmod 3$

Define f as

$$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 0 & \text{if } i \equiv 0 \pmod 3 \text{ for } i = 1 \text{ to } (n-2) \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod 3 \\ 2 & \text{if } i \equiv 2 \pmod 3 \\ 0 & \text{if } i \equiv 0 \pmod 3 \text{ for } i = 1 \text{ to } (n-2) \end{cases}$$

$$f(u_{n-1}) = 1; \quad f(v_{n-1}) = 1$$

$$f(u_n) = 2; \quad f(v_n) = 1.$$

$f(u) = 0.$

Then f is 3-total sum cordial labelling for Case III.

Example 2.9. Helm H_{11} is a 3-total sum cordial graph.

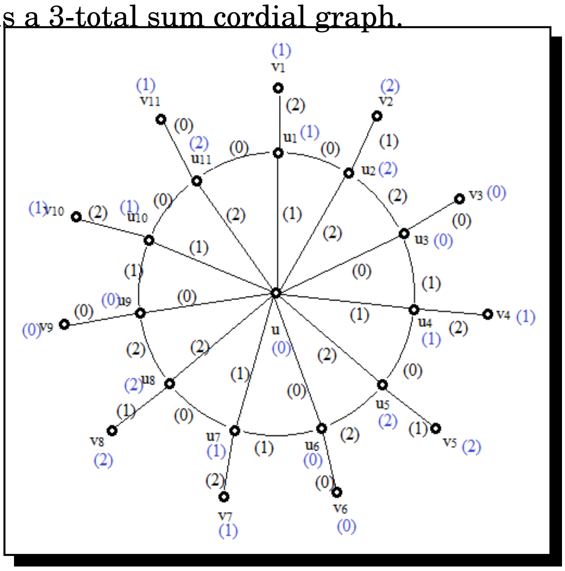

Figure 9. 3-total sum cordial labeling of H_{11}.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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