Journal of Informatics and Mathematical Sciences
Vol. 9, No. 3, pp. 623–634, 2017
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

Proceedings of the Conference
Current Scenario in Pure and Applied Mathematics
December 22-23, 2016
Kongunadu Arts and Science College (Autonomous)
Coimbatore, Tamil Nadu, India

Research Article

On *fgspr*-Closed and *fgspr*-Open Mappings

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Abstract. The purpose of this paper is to introduce a new type of fuzzy generalized mappings namely *fgspr*-closed mappings, *fgspr*-open mappings, *fgspr**-closed mappings and *fgspr**-open mappings in fuzzy topological spaces and study some of their properties.

Keywords. *fgspr*-closed map; *fgspr*-open map; *fgspr**-closed map; *fgspr**-open map

MSC. 54A40

Received: January 4, 2017 **Accepted:** March 27, 2017

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [17]. Subsequently, several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. Fuzzy topology was introduced by Chang [6]. Azad [1] introduced fuzzy semi continuity in 1981. Balasubramanian and Sundaram

[2] introduced generalized fuzzy continuous functions in 1997 and Thakur and Singh [14] introduced fuzzy semi pre continuity in 1998. Gnanambal and Balachandran [9] introduced the concept of gpr-continuous functions in 1999. In 2010, Govindappa Navalagi *et al*. [8] defined the concept of generalized semi preregular closed sets and also introduced the notion of generalized semi preregular continuity and studied their properties. In 2011, Benchalli and Karnel [4] explained the concept of fuzzy gb-continuous maps and investigated their properties. In 2012, Balasubramanian and Lakshmi Sarada [3] defined the concept of gpr-closed and gpr-open mappings and studied their properties. In 2013, Vadivel *et al*. [15] explained the concept of fuzzy generalized preregular continuous mappings and investigated their properties. In 2016, Madabhavi and Patil [10] introduced fuzzyg-μ-closed maps, fuzzy g-μ-continuous maps and studied their properties.

In this paper, *fgspr*-closed mappings, *fgspr*-open mappings, *fgspr**-closed mappings and *fgspr**-open mappings are introduced and some of their properties are studied.

2. Preliminaries

Let X, Y and Z be fuzzy sets. Throughout this paper, (X,τ), (Y,σ) and (Z,η) (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let $f:(X,\tau)\to(Y,\sigma)$ be mapping from a fuzzy topological space X to fuzzy topological space Y. Let us recall the following definitions which we shall require later.

Definition 1. A fuzzy set λ in a fuzzy topological space (X,τ) is called

(i) a fuzzy preopen set [5] if $\lambda \le int(cl(\lambda))$ and a fuzzy preclosed set if $cl(int(\lambda)) \le \lambda$.

(ii) a fuzzy semi-preopen set [14] if $\lambda \le cl(int(cl(\lambda)))$ and a fuzzy semi-preclosed set if $int(cl(int(\lambda))) \le \lambda$.

(iii) a fuzzy regular open set [1] if $int(cl(\lambda)) = \lambda$ and a fuzzy regular closed set if $cl(int((\lambda)) = \lambda$.

Definition 2. A fuzzy set λ in a fuzzy topological space (X,τ) is called

(i) a fuzzy generalized closed set (briefly, *fg*-closed) [2] if $cl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy open set in X.

(ii) a fuzzy generalized pre closed set (briefly, *fgp*-closed) [7] if $pcl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy open set in X.

(iii) a fuzzy generalized semi-pre closed set (briefly, *fgsp*-closed) [11] if $spcl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy open set in X.

(iv) a fuzzy generalized preregular closed set (briefly, *fgpr*-closed) [15] if $pcl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy regular open set in X.

(v) a fuzzy generalized semi preregular closed set (briefly, *fgspr*-closed) [12] if $spcl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy regular open set in X.

Definition 3. Let X, Y be two fuzzy topological spaces. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called

(i) a fuzzy generalized continuous (briefly, *fg*-continuous) [2] if $f^{-1}(\lambda)$ is a fuzzy generalized open (fuzzy generalized closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.

(ii) a fuzzy generalized semi preregular continuous (briefly, *fgspr*-continuous) [13] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.

(iii) a fuzzy generalized semi preregular irresolute (briefly, *fgspr*-irresolute) [13] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set λ in Y.

Definition 4. Let X, Y be two fuzzy topological spaces. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called

(i) a fuzzy closed mapping (briefly, f-closed) [16] if $f(\lambda)$ is a fuzzy closed set in Y, for every fuzzy closed set λ in X.

(ii) a fuzzy preclosed mapping (briefly, *fp*-closed) [5] if $f(\lambda)$ is a fuzzy preclosed set in Y, for every fuzzy closed set λ in X.

(iii) a fuzzy sp-closed mapping (briefly, *fsp*-closed) [14] if $f(\lambda)$ is a fuzzy sp-closed set in Y, for every fuzzy closed set λ in X.

(iv) a fuzzy gp-closed mapping (briefly, *fgp*-closed) [7] if $f(\lambda)$ is a fuzzy gp-closed set in Y, for every fuzzy closed set λ in X.

(v) a fuzzy gsp-closed mapping (briefly, *fgsp*-closed) [11] if $f(\lambda)$ is a fuzzy gsp-closed set in Y, for every fuzzy closed set λ in X.

(vi) a fuzzy gpr-closed mapping (briefly, *fgpr*-closed) [15] if $f(\lambda)$ is a fuzzy gpr-closed set in Y, for every fuzzy closed set λ in X.

The corresponding open mappings are defined in the similar manner.

Definition 5. A fuzzy topological space (X,τ) is said to be

(i) a fuzzy $T_{1/2}$ space [2] if every fg-closed is fuzzy closed.

(ii) a fuzzy semi preregular $T_{1/2}$ space [12] if every *fgspr*-closed is fuzzy semi preclosed.

(iii) a fuzzy semi preregular $T^*_{1/2}$ space [12] if every *fgspr*-closed is fuzzy closed.

3. *fgspr*-Closed Mappings

In this section, some properties of fuzzy generalized semi preregular closed mappings are studied.

Definition 6. Let X and Y be two fuzzy topological spaces. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy generalized semi preregular closed (briefly, *fgspr*-closed) if the image of every fuzzy closed set in X is a *fgspr*-closed set in Y.

Example 7. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.5),(b,0.4),(c,0.7)\}$, $\lambda_2 = \{(a,0.8),(b,1),(c,0.4)\}$ and $\lambda_3 = \{(a,0.5),(b,0.6),(c,0.3)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,1\}$. Define the mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only fuzzy closed set in X is λ_3 and $f(\lambda_3)$ is a *fgspr*-closed set in Y. Hence f is a *fgspr*-closed map.

Theorem 8. *Every f-closed map is a fgspr-closed map.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ is a f-closed map. Let λ be any fuzzy closed set in X. Then $f(\lambda)$ is a fuzzy closed set in Y, as f is a f-closed map. Therefore, $f(\lambda)$ is a *fgspr*-closed set in Y, since every fuzzy closed set is a *fgspr*-closed set. Hence f is a *fgspr*-closed map. □

The following example shows that the converse of the above theorem is not true.

Example 9. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.5),(b,0.2),(c,0.6)\}$, $\lambda_2 = \{(a,0.7),(b,1),(c,0.5)\}$ and $\lambda_3 = \{(a,0.5),(b,0.8),(c,0.4)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,1\}$. Define the mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only fuzzy closed set in X is λ_3 and $f(\lambda_3)$ is not a fuzzy closed set in Y but a *fgspr*-closed set in Y. Hence f is a *fgspr*-closed map but not a fuzzy closed map.

Theorem 10. *Every fuzzy pre-closed (fgp-closed, fsp-closed, fgsp-closed and fgpr-closed) map is fgspr-closed.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ is a fuzzy pre-closed (*fgp*-closed, *fsp*-closed, *fgsp*-closed and *fgpr*-closed) map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a fuzzy closed set in Y, as f is a fuzzy pre-closed (*fgp*-closed, *fsp*-closed, *fgsp*-closed and *fgpr*-closed) map. Therefore $f(\lambda)$ is a *fgspr*-closed set in Y, since every fuzzy pre-closed (*fgp*-closed, *fsp*-closed, *fgsp*-closed and *fgpr*-closed) set is a *fgspr*-closed set. Hence f is a *fgspr*-closed map. □

The following examples show that the converse of the above theorems are not true.

Example 11. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.5),(b,0.2),(c,0.6)\}$, $\lambda_2 = \{(a,0.7),(b,0),(c,0.4)\}$ and $\lambda_3 = \{(a,0.5),(b,0.8),(c,0.4)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,1\}$. Define the mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only fuzzy closed set in X is λ_3 and $f(\lambda_3)$ is not a fuzzy preclosed and a fuzzy semi preclosed set in Y but a *fgspr*-closed set in Y. Hence f is a *fgspr*-closed map but not a fuzzy preclosed map and a fuzzy semi preclosed map.

Example 12. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.8),(b,0.7),(c,0.2)\}$, $\lambda_2 = \{(a,0.2),(b,0.3),(c,0.4)\}$, $\lambda_3 = \{(a,0.3),(b,0.5),(c,0.4)\}$ and $\lambda_4 = \{(a,0.1),(b,0.3),(c,0.2)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_3,\lambda_4,1\}$. Define the mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then the only fuzzy closed set in X is λ_2 and $f(\lambda_2)$ is not a fuzzy gp-closed set and a fuzzy gpr-closed set in Y but a *fgspr*-closed set in Y. Hence f is a *fgspr*-closed map but not a fuzzy gp-closed map and a fuzzy gpr-closed map.

Example 13. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.8),(b,0.6),(c,0.8)\}$, $\lambda_2 = \{(a,0.2),(b,0.4),(c,0.2)\}$ and $\lambda_3 = \{(a,0.3),(b,0.5),(c,0.4)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,\lambda_3,1\}$. Define the mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then the only fuzzy closed set in X is λ_2 and $f(\lambda_2)$ is not a fuzzy gsp-closed set in Y but a *fgspr*-closed set in Y. Hence f is a *fgspr*-closed map but not a fuzzy gsp-closed map.

Remark 14. From the above results we get the following diagram:

f-closed map		*fgp*-closed map		*fgpr*-closed map
	↘	↓	↙	
		fgspr-closed map		
	↗	↑	↖	
fp-closed map		*fsp*-closed map		*fgsp*-closed map

where $A \to B$ represents A implies B but not converse. The above diagram shows the relationships of *fgspr*-closed with some other existing fuzzy mappings.

The following theorem state under what conditions the reverse implications hold good.

Theorem 15. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T_{1/2}$ space then f is a fsp-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T_{1/2}$ space, $f(\lambda)$ is a *fsp*-closed set in Y. Hence f is a *fsp*-closed map. □

Theorem 16. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T_{1/2}$ space then f is a fgsp-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T_{1/2}$ space, $f(\lambda)$ is a *fsp*-closed set in Y. Every *fsp*-closed set is a *fgsp*-closed set. Therefore $f(\lambda)$ is a *fgsp*-closed set in Y. Hence f is a *fgsp*-closed map. □

Theorem 17. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T^*_{1/2}$ space then f is a f-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy closed set in Y. Hence f is a f-closed map. □

Theorem 18. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T^*_{1/2}$ space then f is a fp-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy closed set in Y. Every f-closed set is a *fp*-closed set. Therefore $f(\lambda)$ is a *fp*-closed set in Y. Hence f is a *fp*-closed map. □

Theorem 19. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T^*_{1/2}$ space then f is a fgp-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy closed set in Y. Every f-closed set is a *fgp*-closed set. Therefore $f(\lambda)$ is a *fgp*-closed set in Y. Hence f is a *fgp*-closed map. □

Theorem 20. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map and Y is fuzzy semi preregular $T^*_{1/2}$ space then f is a fgpr-closed map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y as f is a *fgspr*-closed map. Since Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy closed set in Y. Every f-closed set is a *fgpr*-closed set. Therefore $f(\lambda)$ is a *fgpr*-closed set in Y. Hence f is a *fgpr*-closed map. □

Theorem 21. *If a function $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed map, then for each fuzzy set λ in X, fgspr-$cl(f(\lambda))\le f(cl(\lambda))$.*

Proof. Suppose f is a *fgspr*-closed map. If λ is a fuzzy set in X, then $cl(\lambda)$ is a fuzzy closed set in X. $f(cl(\lambda))$ is a *fgspr*-closed set in Y. Since $f(\lambda)\le f(cl(\lambda))$. This implies that *fgspr*-$cl(f(\lambda))\le fgspr$-$cl(f(cl(\lambda)))=f(cl(\lambda))$, as $f(cl(\lambda))$ is a *fgspr*-closed set in Y. That is *fgspr*-$cl(f(\lambda))\le f(cl(\lambda))$. □

Theorem 22. *A map $f:(X,\tau)\to(Y,\sigma)$ is fgspr-closed iff for each fuzzy set λ of Y and for each fuzzy open set μ such that $f^{-1}(\lambda)\le\mu$, there is a fgspr-open set γ of Y such that $\lambda\le\gamma$ and $f^{-1}(\gamma)\le\mu$.*

Proof. Suppose f is a *fgspr*-closed map. Let λ be a fuzzy set in Y and μ be a fuzzy open set in X such that $f^{-1}(\lambda)\le\mu$. Now, $1-\mu$ is a fuzzy closed set in X. Then $f(1-\mu)$ is a *fgspr*-closed set in Y since f is a *fgspr*-closed map. So, $1-f(1-\mu)$ is a *fgspr*-open set in Y. Thus, choose $\gamma=1-f(1-\mu)$ is a *fgspr*-open set in Y such that $\lambda\le\gamma$ and $f^{-1}(\gamma)\le\mu$.

Conversely, suppose that α is a fuzzy closed set in X. Then $1-\alpha$ is a fuzzy open set in X and $f^{-1}(1-f(\alpha))\le 1-\alpha$. Then there exists a *fgspr*-open set γ of Y such that $1-f(\alpha)\le\gamma$ and $f^{-1}(\gamma)\le 1-\alpha$ and so $\alpha\le 1-f^{-1}(\gamma)$. Hence $1-\gamma\le f(\alpha)\le f(1-f^{-1}(\gamma))\le 1-\gamma$. This implies that $f(\alpha)=1-\gamma$ since $1-\gamma$ is a *fgspr*-closed set. $f(\alpha)$ is a *fgspr*-closed set and thus f is a *fgspr*-closed map. □

Theorem 23. *If $f:(X,\tau)\to(Y,\sigma)$ be onto, fgspr-irresolute and fuzzy closed map. If (X,τ) is fuzzy semi preregular $T^*_{1/2}$ space, then (Y,σ) is also fuzzy semi preregular $T^*_{1/2}$ space.*

Proof. Let λ be a *fgspr*-closed set in Y. Since f is *fgspr*-irresolute, then $f^{-1}(\lambda)$ is a *fgspr*-closed set in X. As (X,τ) is fuzzy semi preregular $T^*_{1/2}$ space, $f^{-1}(\lambda)$ is a fuzzy closed set in X. Again f is a fuzzy closed map, $f(f^{-1}(\lambda))$ is a fuzzy closed set in Y. Since f is onto, $f(f^{-1}(\lambda))=\lambda$. Thus λ is a fuzzy closed set in Y. Hence (Y,σ) is fuzzy semi preregular $T^*_{1/2}$ space. □

Theorem 24. *If $f:(X,\tau)\to(Y,\sigma)$ is a f-closed map and $g:(Y,\sigma)\to(Z,\eta)$ is a fgspr-closed map then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-closed map.*

Proof. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a f-closed set in Y, since f is a f-closed map in Y. $g(f(\lambda))$ is a *fgspr*-closed set in Z as g is a *fgspr*-closed map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-closed set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr*-closed map. □

Theorem 25. *If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are fgspr-closed maps and Y is fuzzy semi preregular $T^*_{1/2}$ space then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-closed map.*

Proof. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y, since f is a *fgspr*-closed map in Y. As Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy closed set in Y. $g(f(\lambda))$ is a *fgspr*-closed set in Z as g is a *fgspr*-closed map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-closed set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr*-closed map. □

Theorem 26. *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be two maps such that $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-closed map.*

(i) *If f is f-continuous and surjective, then g is a fgspr-closed map.*

(ii) *If g is fgspr-irresolute and injective, then f is a fgspr-closed map*

Proof. (i) Let λ be a fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is a f-closed set in X, since f is f-continuous. As $g\circ f$ is a *fgspr*-closed map, $g\circ f(f^{-1}(\lambda))=g(\lambda)$ is a *fgspr*-closed set in Z. Thus $g:(Y,\sigma)\to(Z,\eta)$ is a *fgspr*-closed map.

(ii) Let μ be a fuzzy closed set in X. Then $g\circ f(\mu)$ is a *fgspr*-closed set in Z, since $g\circ f$ is a *fgspr*-closed map. As g is *fgspr*-irresolute, $g^{-1}(g\circ f)(\mu)$ is a *fgspr*-closed set in Y. Since g is injective, $g^{-1}(g\circ f)(\mu)=f(\mu)$ is a *fgspr*-closed set in Y. Therefore $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-closed map. □

4. *fgspr*-Open Mappings

In this section, some properties of fuzzy generalized semi preregular open mappings are studied.

Definition 27. Let X and Y be two fuzzy topological spaces. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy generalized semi preregular open (briefly, *fgspr*-open) if the image of every fuzzy open set in X is a *fgspr*-open set in Y.

Example 28. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.3),(b,0.5),(c,0.6)\}$, $\lambda_2 = \{(a,0.8),(b,0.6),(c,0.5)\}$ and $\lambda_3 = \{(a,0.2),(b,0.4),(c,0.5)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,1\}$. Define the mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only fuzzy open set in X is λ_3 and $f(\lambda_3)$ is a *fgspr*-open set in Y. Hence f is a *fgspr*-open map.

Theorem 29. *Every f-open map is a fgspr-open map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a f-open map. Let λ be any fuzzy open set in X. Then $f(\lambda)$ is a fuzzy open set in Y, as f is a f-open map. Therefore $f(\lambda)$ is a *fgspr*-open set in Y, since every fuzzy open set is a *fgspr*-open set. Hence f is a *fgspr*-open map. □

The following example shows that the converse of the above theorem is not true.

Example 30. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0.4),(b,0.5),(c,0.2)\}$, $\lambda_2 = \{(a,0.9),(b,0.7),(c,0.8)\}$ and $\lambda_3 = \{(a,0.1),(b,0.3),(c,0.2)\}$. Let $\tau = \{0,\lambda_1,1\}$ and $\sigma = \{0,\lambda_2,1\}$. Define the mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only fuzzy open set in X is λ_3 and $f(\lambda_3)$ is not a fuzzy open set in Y but a *fgspr*-open set in Y. Hence f is a *fgspr*-open map but not a fuzzy open map.

Theorem 31. *Let $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-open map and Y is fuzzy semi preregular $T_{1/2}$ space then f is a fsp-open map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-open map. Let λ be a fuzzy open set in X. Then $f(\lambda)$ is a *fgspr*-open set in Y as f is a *fgspr*-open map. Since Y is fuzzy semi preregular $T_{1/2}$ space, $f(\lambda)$ is a *fsp*-open set in Y. Hence f is a *fsp*-open map. □

Theorem 32. *Let $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-open map and Y is fuzzy semi preregular $T^*_{1/2}$ space then f is a f-open map.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-open map. Let λ be a fuzzy open set in X. Then $f(\lambda)$ is a *fgspr*-open set in Y as f is a *fgspr*-open map. Since Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy open set in Y. Hence f is a f-open map. □

Theorem 33. *If a function $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-open map, then for each fuzzy set λ in X, fgspr-int$(f(\lambda)) \ge f(int(\lambda))$.*

Proof. Suppose f is a *fgspr*-open map. If λ is a fuzzy set in X, then $int(\lambda)$ is a fuzzy open set in X. $f(int(\lambda))$ is a *fgspr*-open set in Y. Since $f(\lambda) \ge f(int(\lambda))$. This implies that *fgspr*-*int*$(f(\lambda)) \ge fgspr$-*int*$(f(int(\lambda))) = f(int(\lambda))$, as $f(int(\lambda))$ is a *fgspr*-open set in Y. That is *fgspr*-*int*$(f(\lambda)) \ge f(int(\lambda))$. □

Theorem 34. *A map $f:(X,\tau)\to(Y,\sigma)$ is fgspr-open iff for each fuzzy set λ of Y and for each fuzzy closed set μ such that $f^{-1}(\lambda)\le\mu$, there is a fgspr-closed set γ of Y such that $\lambda\le\gamma$ and $f^{-1}(\gamma)\le\mu$.*

Proof. Suppose $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr*-open map. Let λ be a fuzzy set in Y and μ be a fuzzy closed set in X such that $f^{-1}(\lambda)\le\mu$. Now, $1-\mu$ is a fuzzy open set in X. Then $f(1-\mu)$ is a *fgspr*-open set in Y since f is a *fgspr*-open map. So, $1-f(1-\mu)$ is a *fgspr*-closed set in Y. Thus, choose $\gamma=1-f(1-\mu)$ is a *fgspr*-closed set in Y such that $\lambda\le\gamma$ and $f^{-1}(\gamma)\le\mu$.

Conversely, suppose that α is a fuzzy open set in X. Then $1-\alpha$ is a fuzzy closed set in X and $f^{-1}(1-f(\alpha))\le 1-\alpha$. Then there exists a *fgspr*-closed set γ of Y such that $1-f(\alpha)\le\gamma$ and $f^{-1}(\gamma)\le 1-\alpha$ and so $\alpha\le 1-f^{-1}(\gamma)$. Hence $1-\gamma\le f(\alpha)\le f(1-f^{-1}(\gamma))\le 1-\gamma$. This implies that $f(\alpha)=1-\gamma$ since $1-\gamma$ is a *fgspr*-open set. $f(\alpha)$ is a *fgspr*-open set and thus f is a *fgspr*-open map. □

Theorem 35. *If $f:(X,\tau)\to(Y,\sigma)$ be onto, fgspr-irresolute and fuzzy open map. If (X,τ) is fuzzy semi preregular $T^*_{1/2}$ space, then (Y,σ) is also fuzzy semi preregular $T^*_{1/2}$ space.*

Proof. Let λ be a *fgspr*-open set in Y. Since f is *fgspr*-irresolute, then $f^{-1}(\lambda)$ is a *fgspr*-open set in X. As (X,τ) is fuzzy semi preregular $T^*_{1/2}$ space, $f^{-1}(\lambda)$ is a fuzzy open set in X. Again f is a fuzzy open map, $f(f^{-1}(\lambda))$ is a fuzzy open set in Y. Since f is onto, $f(f^{-1}(\lambda))=\lambda$. Thus λ is a fuzzy open set in Y. Hence (Y,σ) is fuzzy semi preregular $T^*_{1/2}$ space. □

Theorem 36. *If $f:(X,\tau)\to(Y,\sigma)$ is a f-open map and $g:(Y,\sigma)\to(Z,\eta)$ is a fgspr-open map then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-open map.*

Proof. Let λ be a fuzzy open set in X. Then $f(\lambda)$ is a f-open set in Y, since f is a f-open map in Y. $g(f(\lambda))$ is a *fgspr*-open set in Z as g is a *fgspr*-open map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-open set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr*-open map. □

Theorem 37. *If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are fgspr-open maps and Y is fuzzy semi preregular $T^*_{1/2}$ space then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-open map.*

Proof. Let λ be a fuzzy open set in X. Then $f(\lambda)$ is a *fgspr*-open set in Y, since f is a *fgspr*-open map in Y. As Y is fuzzy semi preregular $T^*_{1/2}$ space, $f(\lambda)$ is a fuzzy open set in Y. $g(f(\lambda))$ is a *fgspr*-open set in Z as g is a *fgspr*-open map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-open set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr*-open map. □

Theorem 38. *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be two maps such that $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr-open map.*

(i) *If f is f-continuous and surjective, then g is a fgspr-open map.*

(ii) *If g is fgspr-irresolute and injective, then f is a fgspr-open map*

Proof. (i) Let λ be a fuzzy open set in Y. Then $f^{-1}(\lambda)$ is a f-open set in X, since f is f-continuous. As $g\circ f$ is a *fgspr*-open map, $g\circ f(f^{-1}(\lambda))=g(\lambda)$ is a *fgspr*-open set in Z. Thus $g:(Y,\sigma)\to(Z,\eta)$ is a *fgspr*-open map.

(ii) Let μ be a fuzzy open set in X. Then $g \circ f(\mu)$ is a *fgspr*-open set in Z, since $g \circ f$ is a *fgspr*-open map. As g is *fgspr*-irresolute, $g^{-1}(g \circ f)(\mu)$ is a *fgspr*-open set in Y. Since g is injective, $g^{-1}(g \circ f)(\mu) = f(\mu)$ is a *fgspr*-open set in Y. Therefore $f : (X, \tau) \to (Y, \sigma)$ is a *fgspr*-open map.

□

5. *fgspr**-Closed Mappings and *fgspr**-Open Mappings

In this section, some properties of *fgspr**-closed mappings and *fgspr**-open mappings are studied.

Definition 39. Let X and Y be two fuzzy topological spaces. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy generalized semi preregular* closed (briefly, *fgspr**-closed) if the image of every *fgspr*-closed set in X is a *fgspr*-closed set in Y.

Example 40. Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$ and consider the fuzzy sets $\lambda_1 = \{(a,0),(b,1),(c,0)\}$, $\lambda_2 = \{(a,0),(b,1),(c,1)\}$, $\lambda_3 = \{(a,1),(b,0),(c,0)\}$, $\lambda_4 = \{(a,1),(b,0),(c,1)\}$, $\lambda_5 = \{(a,1),(b,1),(c,0)\}$ and $\lambda_6 = \{(a,0),(b,0),(c,1)\}$. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$ and $\sigma = \{0, \lambda_5, 1\}$. Define the mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only *fgspr*-closed sets in X are λ_3, λ_4 and λ_6 and $f(\lambda_3)$, $f(\lambda_4)$ and $f(\lambda_6)$ are *fgspr*-closed sets in Y. Hence f is a *fgspr**-closed map.

Definition 41. Let X and Y be two fuzzy topological spaces. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy generalized semi preregular* open (briefly, *fgspr**-open) if the image of every *fgspr*-open set in X is a *fgspr*-open set in Y.

Example 42. Let $X = \{a,b,c\} = Y$ and consider the fuzzy sets $\lambda_1 = \{(a,1),(b,0),(c,0)\}$, $\lambda_2 = \{(a,1),(b,1),(c,0)\}$, $\lambda_3 = \{(a,1),(b,0),(c,1)\}$, $\lambda_4 = \{(a,0),(b,1),(c,1)\}$ and $\lambda_5 = \{(a,0),(b,0),(c,1)\}$. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$ and $\sigma = \{0, \lambda_3, 1\}$. Define the mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(c) = a$ and $f(b) = b$. Then the only *fgspr*-open sets in X are λ_1, λ_2, λ_3 and $f(\lambda_1)$, $f(\lambda_2)$ and $f(\lambda_3)$ are *fgspr*-open sets in Y. Hence f is a *fgspr**-open map.

Theorem 43. *Every fgspr*-closed (fgspr*-open) map is a fgspr-closed (fgspr-open) map.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is a *fgspr**-closed map. Let λ be a fuzzy closed set in X. Then λ is a *fgspr*-closed set in X, since every fuzzy closed set is a *fgspr*-closed set. Therefore $f(\lambda)$ is a *fgspr*-closed set in Y, as f is a *fgspr**-closed map. Hence f is a *fgspr*-closed map. □

The following example shows that the converse of the above theorem is not true.

Example 44. Let $X = \{a,b,c\} = Y$ and consider the fuzzy sets $\lambda_1 = \{(a,0),(b,1),(c,0)\}$, $\lambda_2 = \{(a,0),(b,1),(c,1)\}$, $\lambda_3 = \{(a,1),(b,0),(c,0)\}$, $\lambda_4 = \{(a,1),(b,0),(c,1)\}$ and $\lambda_5 = \{(a,1),(b,1),(c,0)\}$ and $\lambda_6 = \{(a,0),(b,0),(c,1)\}$. Let $\tau = \{0, \lambda_5, 1\}$ and $\sigma = \{0, \lambda_1, \lambda_2, 1\}$. Define the mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = f(b) = a$ and $f(c) = c$. Then the only f-closed set in X is λ_6 and $f(\lambda_6)$ is a *fgspr*-closed sets in Y. Hence f is a *fgspr*-closed map. But λ_1 and λ_2 are *fgspr*-closed sets in X and $f(\lambda_1)$ and $f(\lambda_2)$ are not *fgspr*-closed sets in Y. Hence f is not a *fgspr**-closed map.

Theorem 45. *If $f:(X,\tau)\to(Y,\sigma)$ is a fgspr-closed (fgspr-open) map and $g:(Y,\sigma)\to(Z,\eta)$ is a fgspr*-closed (fgspr*-open) map then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr*-closed (fgspr*-open) map.*

Proof. Let λ be a fuzzy closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y, since f is a *fgspr*-closed map in Y. $g(f(\lambda))$ is a *fgspr*-closed set in Z as g is a *fgspr**-closed map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-closed set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr**-closed map. □

Theorem 46. *Composition of two fgspr*-closed (fgspr*-open) mappings is fgspr*-closed (fgspr*-open).*

(i.e.) If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ is a fgspr*-closed (fgspr*-open) mappings then $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr*-closed (fgspr*-open) map.

Proof. Let λ be a *fgspr*-closed set in X. Then $f(\lambda)$ is a *fgspr*-closed set in Y, since f is a *fgspr**-closed map in Y. $g(f(\lambda))$ is a *fgspr*-closed set in Z as g is a *fgspr**-closed map. That is $g\circ f(\lambda)=g(f(\lambda))$ is a *fgspr*-closed set in Z. Hence $g\circ f:(X,\tau)\to(Z,\eta)$ is a *fgspr**-closed map. □

Theorem 47. *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be two maps such that $g\circ f:(X,\tau)\to(Z,\eta)$ is a fgspr*-closed (fgspr*-open) map.*

(i) *If f is fgspr-irresolute and surjective, then g is a fgspr*-closed (fgspr*-open) map.*

(ii) *If g is fgspr-irresolute and injective, then f is a fgspr*-closed (fgspr*-open) map.*

Proof. (i) Let λ be a *fgspr*-closed set in Y. Then $f^{-1}(\lambda)$ is a *fgspr*-closed set in X, since f is *fgspr*-irresolute. As $g\circ f$ is a *fgspr**-closed map, $g\circ f(f^{-1}(\lambda))=g(\lambda)$ is a *fgspr*-closed set in Z. Thus $g:(Y,\sigma)\to(Z,\eta)$ is a *fgspr**-closed map.

(ii) Let μ be a *fgspr*-closed set in X. Then $g\circ f(\mu)$ is a *fgspr*-closed set in Z, since $g\circ f$ is a *fgspr**-closed map. As g is *fgspr*-irresolute, $g^{-1}(g\circ f)(\mu)$ is a *fgspr*-closed set in Y. Since g is injective, $g^{-1}(g\circ f)(\mu)=f(\mu)$ is a *fgspr*-closed set in Y. Therefore $f:(X,\tau)\to(Y,\sigma)$ is a *fgspr**-closed map. □

6. Conclusion

It is an interesting exercise to work on *fgspr*-closed mappings and *fgspr*-open mappings with some other existing fuzzy mappings. Composition of these mappings have also been studied. Similarly,other forms of *fgspr*-closed sets and *fgspr*-open sets can be used to define *fgspr**-closed mappings and *fgspr**-open mappings. This new concept and its properties will be useful for future research in this field.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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