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Research Article

Semi $c(s)$-Generalized Closed Sets in Topological Spaces

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Abstract. In this paper, we have introduced a new class of closed set, as a weaker form of closed set namely semi $c(s)$-generalized closed set in topological space.

Keywords. scg-closed, $sc*g$-closed, $sc(s)$ g-closed sets in topological space

MSC. 54C05

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1. Introduction

1.1 Strong and Weak Forms of Open Sets and Closed Sets in Topological Spaces

Stone [35] and Tong [36] were investigated regular open sets and strong regular open sets, which are strong forms of open sets in topological spaces. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets respectively. Semi open set, a weak form of open set was introduced by Levine [16]. Semi closed set was introduced by Biswas [8]. Njastad [24], Levine [19], Mashhour [20], Abd El-Monsef *et al*. [20], Andrijevic [3], Battacharyya and Lahiri [10], Arya and Nour [5], Maki *et al*. [21], Pallaniappan *et al*. [32], Maki *et al*. [21], Sundaram and Nagaveni [29] and Pushpalatha [27] have formulated α-closed sets, generalized closed sets, pre-closed sets, β-closed sets, semi generalized closed sets, generalized semi closed sets, αg-closed sets, regular generalized closed sets, generalized α-closed sets, weakly generalized closed sets, and strongly generalized closed sets, which are some weak forms of closed sets. Tong [36] and Hatir *et al*. [15] introduced B-sets and t-sets and α^*-sets as weaker forms of closed sets. B-sets are weak forms of open sets. Sundaram [33] introduced c-set and $c(s)$-set and Rajamani [28] introduced c^*-set. We recall the following definitions, which are used in this paper.

Definition 1.1. A subset S of X is called a

(i) regular closed [35] if $S = cl(int(S))$ and regular open [35] if $S = int(cl(S))$.

(ii) semi open [18] if there exist an open set G such that $G \subseteq S \subseteq cl(G)$ and semi closed [9] if there exist a closed set F such that $int(F) \subseteq S \subseteq F$. Equivalently, a subset S of X is called semi-open if $S \subseteq cl(int(S))$ and semi-closed if $S \subseteq int(cl(S))$ [3].

(iii) α-closed if $cl(int(cl(S))) \subseteq S$ and α-open if $S \subseteq int(cl(int(S)))$ [24].

(iv) pre-closed if $cl(int(S)) \subseteq S$ and pre-open if $S \subseteq int(cl(S))$ [20].

(v) β-closed [1] (semi pre-closed [5]) if $int(cl(S))) \subseteq S$ and a β-open [1] (semi pre-open [3]) if $S \subseteq cl(int(cl(S)))$.

Definition 1.2. For a subset S of X, the semi closure of S, denoted by $scl(S)$, is defined as the intersection of all semi closed sets containing S in X and the semi interior of S, denoted by $sint(S)$, is the union of all semi open sets contained in S in X [11]. Pre closure [3] of S, denoted by $pcl(S)$, pre interior of S, denoted by $pint(S)$, α-closure [24] of S, denoted by $\alpha cl(S)$, α-interior of S, denoted by $\alpha int(S)$, semi pre closure [3] of S, denoted by $spcl(S)$ and semi-pre interior of S, denoted by $spint(S)$.

Result 1.3. For a subset S of X,

(i) the semi closure is denoted by $scl(S)$, defined as $scl(S) = S \cup int(cl(S))$ [3].

(ii) the semi interior is denoted by $sint(S)$, defined as $sint(S) = S \cap cl(int(S)$ [3].

(iii) the pre closure is denoted by pcl(S), defined as $pcl(S) = S \cup cl(int(S))$ [3].

(iv) the pre interior is denoted by $pint(S)$, defined as $pint(S) = S \cap int(cl(S)$ [3]

(v) the α-closure is denoted by $\alpha cl(S)$, defined as $\alpha cl(S) = S \cup cl(int(cl(S)))$ [24].

(vi) the α-interior is denoted by $\alpha int(S)$, defined as $\alpha int(S) = S \cap int(cl(int(S)))$ [24].

(vii) the semi pre closure is denoted by $spcl(S)$, defined as $spcl(S) = S \cup int(cl(int(S)))$ [3].

(viii) the semi pre interior is denoted by $spint(S)$, defined as $spint(S) = S \cap cl(int(cl(S)))$ [3].

Definition 1.4. A subset A of X is called

(i) generalized closed (briefly g-closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [19].

(ii) semi generalized closed (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X [10].

(iii) generalized semi-closed (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [6].

(iv) generalized α-closed (briefly $g\alpha$-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X [21].

(v) α-generalized closed (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [21].

(vi) generalized semi-pre closed(briefly gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [14].

(vii) regular generalized closed (briefly rg-closed) $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [32].

(viii) weakly generalized closed (briefly wg-closed) $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X [23].

(ix) strongly generalized closed (briefly strongly g-closed) $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X [27].

(x) semi c generalized-closed (briefly scg-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is c-set in X [34].

(xi) semi c^* generalized-closed (briefly sc^*g-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is c^*-set in X [34].

The complements of the above mentioned closed sets are their respective open sets.

Definition 1.5. A subset S of X is called a

(i) regular closed if $S = cl(int(S))$ [35],

(ii) t-set if $int(S) = int(cl(S))$ [36],

(iii) α^*-set if $int(A) = int(cl(int(A)))$,

(iv) c-set if $S = G \subseteq F$ where G is open and F is α^*-set in X [33],

(v) c^*-set if $S = G \subseteq F$ where G is g-open and F is α^*-set in X [30],

(vi) $c(s)$-set if $S = G \subseteq F$ where G is g-open and F is t-set in X [33].

Remark 1.6. Every c-set in X is a c^*-set in X [28].

2. Semi $c(s)$-Generalized Closed Set in Topological Spaces

In 1970, Levine [19] introduced the concept of generalized closed (briefly g-closed) sets in topological spaces and investigated some of their properties. Semi closed sets was introduced by Biswas [8]. Nagaveni [23], Pushpalatha [27], Pallaniappan and Rao [32], and Arya and Nour [5] have introduced weakly generalized closed sets (wg-closed sets), strongly generalized closed sets (strongly g-closed sets), regular generalized closed sets (rg-closed sets) and generalized semi closed sets respectively. Tong [36] and Hatir *et al.* [15] introduced B-sets and t-sets and α^*-sets are weaker forms of closed sets, α^*-sets, t-sets and B-sets are weak forms of open sets. Sundaram [33] introduced c-set and $c(s)$ set and Rajamani [28] introduced c^*-set. We have introduced new class of set called $sc(s)g$-closed set in topological spaces and study some of their properties.

In this paper, we have introduced concept of semi $c(s)$-generalized closed set in topological spaces.

Definition 2.1. A subset A of X is called a semi $c(s)$-generalized closed (briefly $sc(s)$-g closed) set if $scl(A) \subseteq U$ whenever, $A \subseteq U$ and U is $c(s)$-set in X. The complement of $sc(s)$-g closed set is called a $sc(s)$-g open set in topological spaces.

Theorem 2.2. *Every closed set in X is $sc(s)$-g closed in X but not conversely.*

Proof. Assume that A is a closed set in X. Let U be a $c(s)$-set such that $A \subseteq U$. Since A is closed, $cl(A) = A$. Therefore, $cl(A) \subseteq U$. Since $scl(A) \subseteq cl(A)$, $scl(A) \subseteq U$. Hence A is (s)-g closed set in X. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.3. Consider the topological space $X = \{a,b,c\}$ with topology $\tau = \{\varphi, X, \{a\}, \{a,b\}\}$. The set $\{a,c\}$ is $sc(s)$-g closed set but not closed set in X.

Theorem 2.4. *Every semi closed set in X is (s)-g closed set in X but not conversely.*

Proof. Assume that A is a semi closed set. Let $A \subseteq U$, U is a $c(s)$-set. Since $scl(A) = A$, $scl(A) \subseteq U$. Therefore A is (s)-g closed set in X. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.5. Consider the topological space $X = \{a,b,c\}$ with topology $\tau = \{\varphi, X, \{a\}, \{a,b\}\}$. The set $\{a,c\}$ is scg-closed set but not semi closed set in X.

Theorem 2.6. *Every sc^*g-closed set in X is (s)-g closed set in X but not conversely.*

Proof. Assume that A is sc^*g-closed set in X. Let $A \subseteq G$, where G is $c(s)$-set. Since every $c(s)$-set in X is a c^*-set in X [28], G is a c^*-set and since A is sc^*g-closed, $scl(A) \subseteq G$. Therefore, A is a (s)-g closed set. □

The converse of the above theorem is need not be true as seen from the following example.

Example 2.7. Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{\varphi, X, \{a, b\}\}$. The set $\{a, c\}$ is (s)-g closed set but not sc^*g-closed set in X.

Theorem 2.8. *Every (s)-g closed set in X is gs-closed set in X but not conversely.*

Proof. Assume that A is $sc(s)$-g closed set in X. Let $A \subseteq U$, where U is a c(s)-set, then U can be written as $U = G \cap X$, where G is g-open and X is t-set. Since A is $sc(s)$-g closed set. Therefore, $scl(A) \subseteq G$ where G is open. Hence A is gs-closed set in X. □

The converse of the above theorem need not be true as seen from the following example.

Example 2.9. Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{\varphi, X, \{a\}\}$. The set $\{a, c\}$ is gs-closed set but not a (s)-g closed set in X.

Remark 2.10. From the above results, we obtain the following diagram:

closed → semi closed → sc^*g-closed → $sc(s)$-g closed → gs-closed

Figure 1

In the above diagram none of the implications can be reversed.

Remark 2.11. The concept of (s)-g closed set is independent with the following classes of sets namely pre-closed, β-closed, $g\alpha$-closed, wg-closed, g-closed, sg-closed, rg-closed, αg-closed and strongly g-closed sets in topological spaces.

Example 2.12. Consider the topological space $X = \{a, b, c\}$ with topologies $\tau_1 = \{\varphi, X, \{a\}\}$ and $\tau_2 = \{\varphi, X, \{a, b\}$. In (X, τ_1) the set $\{a, b\}$ is $sc(s)$-g closed set in X, but not pre-closed, β-closed, $g\alpha$-closed and sg-closed set in X. In (X, τ_2) the set $\{b, c\}$ is pre-closed, β-closed, $g\alpha$-closed, and sg-closed set but not (s)-g closed set in X.

Example 2.13. Consider the topological space $X = \{a, b, c\}$ with topologies $\tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\varphi, X, \{a\}, \{b, c\}\}$. In (X, τ_1) the set $\{a, c\}$ is sc*g-closed set in X, but not pre-closed, β-closed and sg-closed set in X. In (X, τ_2) the set $\{a, b\}$ is pre-closed, β-closed and sg-closed set but not $sc(s)$-g closed set in X.

Example 2.14. Consider the topological space $X = \{a, b, c\}$ with topologies $\tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\varphi, X, \{a, b\}\}$. In (X, τ_1) the set $\{b\}$ is both $sc(s)$-g closed and sc^*g-closed set in X, but not g-closed, αg-closed and strongly g-closed set in X. In (X, τ_2) the set $\{b, c\}$ is g-closed, αg-closed and strongly g-closed set but not (s)-g closed and sc^*g-closed set in X.

Example 2.15. Consider the topological space $X = \{a, b, c\}$ with topologies $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\varphi, X, \{a, b\}\}$. In (X, τ_1) the set $\{b\}$ is both scg-closed and sc^*g-closed set in X, but not rg-closed set and wg-closed set in X. In (X, τ_2) the set $\{b, c\}$ is rg-closed and wg-closed set but not scg-closed set in X.

Remark 2.16. From the above discussion and known results we have the following diagram:

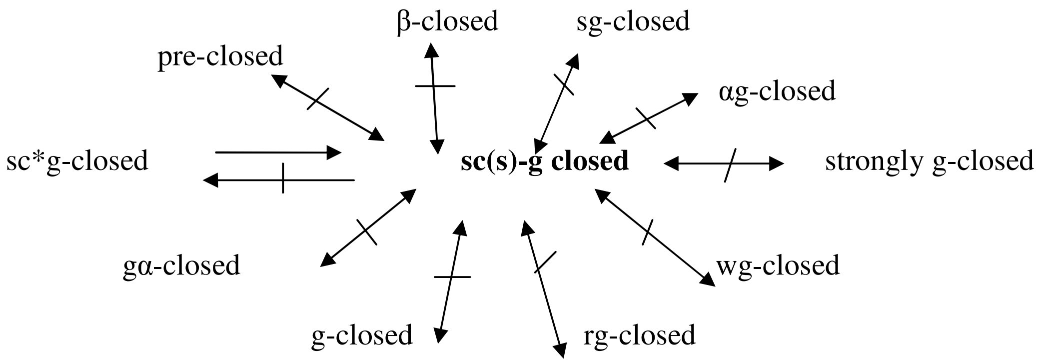

Figure 2

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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