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Inference on Reliability for Cascade Model

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Abstract. In this paper we consider two cases to obtain the system reliability for cascade model. For the first case we consider one-parameter exponential strength and two-parameter gamma stress. Under this assumption the reliability of the system in general terms is obtained. Secondly we consider two-parameter exponential strength and one-parameter gamma stress to obtain an expression for the reliability of a 3-cascade system. For both the cases all stress-strengths are random variables with given density. In all these cases numerical integration is used to evaluate the reliability for cascade system. Some numerical values of $R(1)$, $R(2)$, $R(3)$ and R_3 for particular values of the parameters involved are tabulated at the end.

1. Introduction

The strength of a component ([3], [4], [5]) can obviously be defined as the minimum stress required causing the component (or system) failure by considering the situation where a component works under the impact of stresses. If the stress equals or exceeds the strength of the component, it fails; otherwise it works. In practical situations, the magnitude of the stress is random, with considerable variations. Imperfections in the manufacture and non-uniformity in the materials give a random character to the component's strength, which is also a random variable. The reliability R of the component is given by

$$R = P[X \geq Y].$$

By cascade redundancy [2] we mean a standby redundancy where a standby component taking the place of a failed component is subjected to a modified value of the preceding stress. We assume that this modified value of stress is equal to 'k' times the stress on the preceding (failed) component. k is called attenuation factor. Here we shall assume that k is a constant through it may be changing from component to component or even it may be a random variable.

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Studies of a 3-cascade reliability for cascade redundancy using different distributions have been considered by many persons, e.g. Sriwastav and Kakati [6], etc. But there have been no studies where exponential and gamma distributions are for two-parameters.

The aim of this paper is to estimate the system reliability for the following two cases.

1. Strength follows one-parameter exponential distribution and stress follows two-parameter gamma distribution.
2. Strength follows two-parameter exponential distribution and stress follows one-parameter gamma distribution.

For both the cases we obtain the system reliability for 3-cascade system.

2. **The Model**

An n-cascade system is a special type of n-standby system [2]. Let $X_1, X_2, X_3, \ldots, X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2, Y_3, \ldots, Y_n$ are the stresses working on them. In cascade system after every failure the stress is modified by a factor k which is called attenuation factor such that

$$Y_2 = kY_1, \quad Y_3 = kY_2 = k^2 Y_1, \ldots, Y_i = k^{i-1} Y_1 \text{ etc.}$$

In stress-strength model the reliability R, of a component (or system) is defined as the probability that the strength of the component, say X (a random variable) is not less than the stress, say Y (another random variable), on it. Symbolically,

$$R = \Pr(X \geq Y) = \Pr(X - Y \geq 0).$$

Then the reliability R_n of the system is defined as

$$R_n = R(1) + R(2) + \ldots + R(n), \tag{2.1}$$

where $R(r) = P[X_1 < Y_1, X_2 < kY_1, \ldots, X_{r-1} < k^{r-2} Y_1, X_r \geq k^{r-1} Y_1]$,
$R(r)$ is the marginal reliability due to the rth component. Or we can write for cascade system,

$$R(r) = \int_{-\infty}^{\infty} [F_1(y_1) F_2(ky_1) F_3(k^2 y_1) \ldots \overline{F}_r(k^{r-1} y_1)] g(y_1) dy_1 .$$

2.1. *One-parameter exponential strength and two-parameter gamma stress*

Let $X_1, X_2, X_3, \ldots, X_n$ be one-parameter exponential strength [1], i.e., $f_i(x)$ with mean $\frac{1}{\lambda_i}$ and Y_1 be a two-parameter gamma stress then we have the following probability density functions

$$f_i(x,) = \begin{cases} \lambda_i e^{-\lambda_i x_i}, & x_i \geq 0,\ \lambda_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$g_i(y;\mu,\theta) = \begin{cases} \frac{1}{\theta^\mu \Gamma\mu} y^{\mu-1} e^{-\frac{y}{\theta}}, & y,\mu,\theta > 0 \\ 0, & \text{otherwise}. \end{cases}$$

Then,

$$\begin{aligned}
R(1) &= \int_0^\infty \overline{F}_1(y_1) g_1(y_1) dy_1 = \frac{1}{(1+\lambda_1\theta)^\mu}, \\
R(2) &= \int_0^\infty F_1(y_1)\overline{F_2}(ky_1) g_1(y_1) dy_1 \\
&= \frac{1}{(1+\lambda_2 k\theta)^\mu} - \frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta)^\mu}, \\
R(3) &= \int_0^\infty F_1(y_1) F_2(ky_1) \overline{F}_3(k^2 y_1) g_1(y_1) dy_1 \\
&= \frac{1}{(1+\lambda_3 k^2\theta)^\mu} - \frac{1}{(1+\lambda_2 k\theta+\lambda_3 k^2\theta)^\mu} - \frac{1}{(1+\lambda_1\theta+\lambda_3 k^2\theta)^\mu} \\
&\quad + \frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta+\lambda_3 k^2\theta)^\mu}.
\end{aligned}$$

In general,

$$\begin{aligned}
R(n) &= \frac{1}{(1+\lambda_n k^{n-1}\theta)^\mu} - \frac{1}{(1+\lambda_1\theta+\lambda_n k^{n-1}\theta)^\mu} - \ldots \\
&\quad + (-1)^{n+1} \frac{1}{(1+\lambda_1\theta+\lambda_2 k\theta+\ldots+\lambda_{n-1}k^{n-2}\theta+\lambda_n k^{n-1}\theta)^\mu}.
\end{aligned}$$

Then the system reliability R_3 for a 3-cascade system from (1) is given by

$$R_3 = R(1) + R(2) + R(3).$$

2.1.A. *Special case*

When $X_1, X_2, X_3, \ldots, X_n$ are one-parameter i.i.d. exponential strength with parameter λ then we have,

$$\begin{aligned}
R(1) &= \frac{1}{(1+\lambda\theta)^\mu}, \\
R(2) &= \frac{1}{(1+\lambda k\theta)^\mu} - \frac{1}{(1+\lambda\theta+\lambda k\theta)^\mu}, \\
R(3) &= \frac{1}{(1+\lambda k^2\theta)^\mu} - \frac{1}{(1+\lambda k\theta+\lambda k^2\theta)^\mu} - \frac{1}{(1+\lambda\theta+\lambda k^2\theta)^\mu} \\
&\quad + \frac{1}{(1+\lambda\theta+\lambda k\theta+\lambda k^2\theta)^\mu}.
\end{aligned}$$

Then in general,

$$R(n) = \frac{1}{(1+\lambda k^{n-1}\theta)^{\mu}} - \frac{1}{(1+\lambda\theta+\lambda k^{n-1}\theta)^{\mu}} - \dots$$
$$+(-1)^{n+1}\frac{1}{(1+\lambda\theta+\lambda k\theta+\ldots+\lambda k^{n-2}\theta+\lambda k^{n-1}\theta)^{\mu}}.$$

Then the system reliability R_3 for a 3-cascade system from (1) is given by

$$R_3 = R(1)+R(2)+R(3).$$

2.2. *Two-parameter exponential strength and one-parameter gamma stress*

Let $X_1, X_2, X_3, \ldots, X_n$ be two-parameter exponential strength and Y_1 be one-parameter gamma stress then we have the following probability density functions

$$f_i(x;\mu,\theta) = \begin{cases} \dfrac{1}{\theta_i} e^{-\frac{(x_i-\mu_i)}{\theta_i}}, & x_i > \mu_i,\ \mu_i \geq 0,\ \theta_i > 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$g_i(y) = \begin{cases} \dfrac{1}{\Gamma_m} e^{-y} y^{m-1}, & y \geq 0,\ m \geq 1 \\ 0, & \text{otherwise}. \end{cases}$$

Then

$$R(1) = \int_0^\infty \overline{F}_1(y_1) g_1(y_1) dy_1 = 1 - e^{\frac{\mu_1}{\theta_1}}\left(1 - \frac{1}{\left(1+\frac{1}{\theta_1}\right)^m}\right),$$

$$R(2) = \int_0^\infty F_1(y_1)\overline{F}_2(ky_1) g_1(y_1) dy_1$$
$$= e^{\frac{\mu_1}{\theta_1}}\left[1 - \frac{1}{\left(1+\frac{1}{\theta_1}\right)^m}\right]$$
$$- e^{\frac{\mu_1}{\theta_1}+\frac{\mu_2}{\theta_2}}\left[1 - \frac{1}{\left(1+\frac{1}{\theta_1}\right)^m} - \frac{1}{\left(1+\frac{k}{\theta_2}\right)^m} + \frac{1}{\left(1+\frac{1}{\theta_1}+\frac{k}{\theta_2}\right)^m}\right],$$

$$R(3) = \int_0^\infty F_1\left(y_1\right) F_2\left(ky_1\right) \overline{F}_3\left(k^2 y_1\right) g_1\left(y_1\right) dy_1$$
$$= e^{\frac{\mu_1}{\theta_1}+\frac{\mu_2}{\theta_2}}\left[1 - \frac{1}{\left(1+\frac{k}{\theta_2}\right)^m} - \frac{1}{\left(1+\frac{1}{\theta_1}\right)^m} + \frac{1}{\left(1+\frac{1}{\theta_1}+\frac{k}{\theta_2}\right)^m}\right]$$

$$-e^{\frac{\mu_1}{\theta_1}+\frac{\mu_2}{\theta_2}+\frac{\mu_3}{\theta_3}}\left[1-\frac{1}{\left(1+\frac{k^2}{\theta_3}\right)^m}-\frac{1}{\left(1+\frac{k}{\theta_2}\right)^m}+\frac{1}{\left(1+\frac{k}{\theta_2}+\frac{k^2}{\theta_3}\right)^m}-\frac{1}{\left(1+\frac{1}{\theta_1}\right)^m}\right.$$
$$\left.+\frac{1}{\left(1+\frac{1}{\theta_1}+\frac{k^2}{\theta_3}\right)^m}+\frac{1}{\left(1+\frac{1}{\theta_1}+\frac{k}{\theta_2}\right)^m}-\frac{1}{\left(1+\frac{1}{\theta_1}+\frac{k}{\theta_2}+\frac{k^2}{\theta_3}\right)^m}\right].$$

Then the system reliability R_3 for a 3-cascade system from (1) is given by

$$R_3 = R(1)+R(2)+R(3)$$

Table 1. Reliability R_3 for one-parameter exponential (λ) strength and two-parameter gamma (μ,θ) stress, where $\lambda_1 = a$, $\lambda_2 = b$, $\lambda_3 = c$, $\theta = f$, $\mu = g$

b	c	k	g	$R(1)$	$R(2)$	$R(3)$	R_3
1	.5	.1	2	.2500	.5997	.1480	.9977
		.3		.2500	.4027	.3057	.9584
		.5		.2500	.2844	.3351	.8695
		.7		.2500	.2088	.2977	.7565
		.9		.2500	.1581	.2370	.6451
5	.5	.1	2	.2500	.2844	.4592	.9936
		.3		.2500	.0784	.6018	.9302
		.5		.2500	.0322	.5393	.8216
		.7		.2500	.0163	.4326	.6989
		.9		.2500	.0094	.3260	.5854
7	.5	.1	2	.2500	.2088	.5340	.9928
		.3		.2500	.0446	.6337	.9283
		.5		.2500	.0163	.5535	.8199
		.7		.2500	.0077	.4398	.6976
		.9		.2500	.0042	.3300	.5843
1	1.5	.1	1	.5000	.4329	.0649	.9978
		.3		.5000	.3344	.1265	.9609
		.5		.5000	.2667	.1207	.8874
		.7		.5000	.2179	.0912	.8090
		.9		.5000	.1815	.0624	.7439
5	1.5	.1	1	.5000	.2667	.2265	.9932
		.3		.5000	.1143	.3083	.9226
		.5		.5000	.0635	.2533	.8168
		.7		.5000	.0404	.1801	.7205
		.9		.5000	.0280	.1211	.6491
7	1.5	.1	1	.5000	.2179	.2742	.9920
		.3		.5000	.0787	.3397	.9184
		.5		.5000	.2667	.1207	.8874
		.7		.5000	.0246	.1910	.7156
		.9		.5000	.0165	.1281	.6446

Table 1 is also self explanatory. With some fixed values of $\lambda_1 = a = 1$, $\theta = f = 1$ if $\lambda_2 = b$, $\lambda_3 = c$ and $\mu = g$ increases with increasing k (i.e., $k = .1, .3, .5, .7, .9$) reliability decreases.

Table 2. Reliability R_3 for Special Case: where $\lambda_1 = \lambda_2 = \lambda_3 = a$ (identical), $\theta = f$, $\mu = g$

a	k	g	$R(1)$	$R(2)$	$R(3)$	R_3
1	.1	1	.5000	.4329	.0656	.9985
	.2		.5000	.3788	.1113	.9901
	.3		.5000	.3344	.1379	.9724
	.4		.5000	.2976	.1487	.9463
	.5		.5000	.2667	.1478	.9144
3	.1	3	.0156	.4426	.4871	.9453
	.2		.0156	.2339	.5105	.7600
	.3		.0156	.1373	.3847	.5376
	.4		.0156	.0868	.2509	.3533
	.5		.0156	.0580	.1522	.2258
5	.1	5	.0001	.1316	.6717	.8034
	.2		.0001	.0312	.3824	.4137
	.3		.0001	.0102	.1515	.1618
	.4		.0001	.0041	.0516	.0558
	.5		.0001	.0019	.0169	.0189

Here also the change in the values of reliability is as expected. If $\lambda = a$, $\mu = g$, and k increases then reliability deceases with fixed values of $\theta = f = 1$.

Table 3. Reliability R_3 for two-parameter exponential (μ, θ) strength and one-parameter gamma (m) stress

m	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	.2	.6496	.3121	.0374	.9990
	.4	.6496	.2794	.0641	.9931
	.6	.6496	.2513	.0794	.9803
	.8	.6496	.2269	.0846	.9611
	1	.6496	.2057	.0820	.9373
	1.2	.6496	.1871	.0744	.9111
2	.2	.4160	.4896	.0912	.9967
	.4	.4160	.4134	.1486	.9780
	.6	.4160	.3513	.1724	.9396
	.8	.4160	.3001	.1697	.8857
	1	.4160	.2575	.1502	.8236
	1.2	.4160	.2218	.1227	.7604
3	.2	.2602	.5827	.1501	.9931
	.4	.2602	.4631	.2323	.9556
	.6	.2602	.3706	.2522	.8830
	.8	.2602	.2981	.2290	.7874
	1	.2602	.2407	.1841	.6850
	1.2	.2602	.1947	.1336	.5885

Here we obtain the reliability for $\mu_i < \theta_i$, $i = 1, 2, 3$ and we take $\mu_1 = a = 0.1$, $\mu_2 = b = 0.2$, $\mu_3 = c = 0.3$, $\theta_1 = d = 2$, $\theta_2 = d = 2$, $\theta_3 = f = 4$.

It is noted from the above Table 3, that if m and k increases ($k = .2, .4, .6, .8, 1, 1.2$) reliability R_3 decreases for certain fixed values of μ_1, μ_2, θ_1, θ_2, θ_3.

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