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Research Article

Different Strategies to Obtain Higher Outcomes

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Abstract. In this paper, we examine the maximum outcomes under different strategies of firms. The outcomes suggest that the behavior of the equilibria is not affected by the type of competition. However, when comparing the outcomes, we find that interacting firms in a marketplace to maximize the individual and social outcomes is strongly affected by their positions in a marketplace.

Keywords. Network game; Equilibria; Outcomes maximization

MSC. 91A13; 91A20; 91A43

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1. Introduction

This paper combines three strategies of firms in order to conclude a better choice ensures high individual and social outcomes. The first strategy is a production type. It relates to a market structure or a competition toughness [15], where firms are assumed to have a choice to operate by setting their products in a differentiated product market or in a substituted product market. The second strategy is a market competition type. When firms operate in the market, they are assumed to compete by setting their production quantity (Cournot competition) or their production price (Bertrand competition). The third strategy is a market participation in research and development (R&D). In a market, firms are assumed to conduct R&D in order to reduce the cost of the production and obtain a maximum profit. In this case, firms have a choice to cooperate with other firms in a market or to participate in R&D alone.

The paper contributes to the theoretical literature of game theory through studying the interests acquired by comparing different firms' strategies in the market. In particular, this paper carries two main objectives. The first objective is to compare between the strategies of firms to conclude a case that guarantees maximum returns to the individual and social outcomes. The second objective is to examine the impact of the firms' positions in the market on their interactions (considering Cournot or Bertrand competition).

The study of the R&D cooperation in this paper is based on a network game [7]. The game can be briefly described as follows[1]. The structure that displays firms cooperate in R&D can be described as a network where the players (firms) are represented by nodes and the R&D partnerships (cooperation) are represented by links. The game consists of three stages: network formation, R&D investment (effort) and market competition. In the network, if any two firms cooperate, they are linked; otherwise there is an R&D spillover to ensure knowledge flow between non-cooperating firms. The structure of the network defines the marginal costs of the production of firms where these costs decrease with increasing the individual effort and effort of other firms in the network.

The outcomes of this paper can be summarized as follows. Firstly, the behavior of the equilibria under Cournot and Bertrand competition with respect to the structure of the market and the R&D network is consistent. When firms conduct R&D, the outcomes improve, in the sense that the individual outcomes (investment, production and profit) and social outcomes (total welfare) increase. This is because the R&D investment reduces the production cost. However, the benefit behind the R&D cooperation depends on the competition toughness (or production type). In a weak competitive market (firms produce complementary or independent goods), the collaboration always generates higher outcomes. In a competitive market (firms produce substituted goods), the R&D investment is lower and the profit is higher if the collaboration is made. Also, the cooperation is a negative factor reduces the social welfare, especially if the substitution between the products is high.

Secondly, the contrast between the equilibria under Cournot and Bertrand is influenced by the value of the differentiation degree between the products. Generally, the gap between the two competitions with respect to the differentiation degree is non-monotonic. Also, as the complementation degree or the substitution degree between products increases, the contrast between the two competitions increases. However, the contrast decreases as the products become independent where firms do not have a preference in terms of choosing the quantities or prices to compete in the market. This leads to an important insight that firms' position in the market is really matter to determine how the equilibrium outcomes under Cournot competition depart from those under Bertrand competition.

The paper proceeds as follows. In the next section, we review some economic and networks issues. In the third section, we compare among different strategies of firms. Then, we study the

[1]There are many papers based on Goyal and Moraga-Gonzalez model (e.g., 14, 17, 16, 1).

impact of the differentiation degree on the contrast between the equilibrium outcomes under Cournot and Bertrand competition. In the fifth section, we conclude the paper.

2. Background and Theoretical Development

2.1 Networks

A **network** is formed by a set of vertices (nodes) and a set of edges (links) connecting these vertices ([11, 10]). We define N as a set of all vertices labeled by letters $i,j,k,\dots$, where $|N| = n$ and $E = \{ij, jk, \dots\}$ is a set of all edges in the network where $|E| = m$ is the number of links. Then $G(N,E)$ denotes a network with nodes N and links E, and for simplicity the network is denoted by G. For the purpose of this article, we focus on undirected networks; meaning that each link between any two vertices runs in both directions (i.e., each two links ij and ji in G are the same). We also focus on simple networks that have neither parallel edges (edges that have the same end vertices) nor loops (edges where their start and end vertices are the same).

2.2 Economic Model

The emphasis in this paper is on the linear-quadratic function of consumers given by [6] and [3]:

$$U = a\sum_{i=1}^{n} q_i - \frac{1}{2}\left(\alpha \sum_{i=1}^{n} q_i^2 + 2\lambda \sum_{j\neq i} q_i q_j\right) + I. \tag{2.1}$$

The parameters $a > 0$ denotes the willingness of consumers to pay and $\alpha > 0$ is the diminishing marginal rate of consumption. Without loss of generality, we assume that $\alpha = 1$ to simplify the analysis. The parameter q_i is the quantity consumed of good i and I measures the consumers' consumption of all other products. The parameter $\lambda \in [-1,1]$ is the differentiation degree. If $\lambda < 0$, goods are complements and if $\lambda > 0$, goods are substitutes. For integer values, if $\lambda = -1$, $\lambda = 0$ or $\lambda = 1$, goods are perfect complements, independent, homogeneous, respectively.

The inverse demand function for each good i is

$$D_i^{-1} = p_i = a - q_i - \lambda \sum_{j\neq i} q_j, \quad i = 1,\dots,n. \tag{2.2}$$

The effort is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is γx_i^2, where $\gamma > 0$ indicates the effectiveness of R&D expenditure [5]. The profit π_i for firm i is the difference between revenue and production cost minus the cost of R&D

$$\pi_i = (p_i - c_i)q_i - \gamma x_i^2 = \left(a - q_i - \lambda \sum_{j\neq i}^{n} q_j - c_i\right) q_i - \gamma x_i^2, \tag{2.3}$$

where p_i is the price of good i produced by firm i and c_i is the production cost.

The Total Welfare (TW) is the total surplus of consumers plus the industry profit

$$TW = \frac{(1-\lambda)}{2}\sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2 + \sum_{i=1}^{n} \pi_i. \tag{2.4}$$

2.3 R&D Network Model

In the R&D network, nodes represent firms and links represent R&D partnerships. Since the R&D cooperation is a mutual benefit, each link between any two firms runs in both directions (i.e., undirected networks). The focus of this paper is on [7] model. In their model, if firms cooperate in R&D, they are linked in an undirected network and spillover is set at one where the cost of link formation is assumed to be negligible. If firms do not cooperate, they are not linked and there is a spillover ($\beta \in [0,1)$) between non-linked firms.

Goyal and Moraga-Gonzalez examined an oligopolistic market under Cournot competition with linear demand for symmetric and asymmetric networks[2]. They focused on the impact of the cooperative links on R&D investment and on the incentives of firms to cooperate. Moreover, their study investigated the situations in which the conflict between the stability and efficiency of R&D networks occurs[3]. For the stability of the R&D network with homogeneous goods, they found that the complete network is individually profitable.

• Some Special R&D Networks

A complete network is a graph such that each two firms are linked. A star network consists of a hub firm located at the center of the network and linked to other firms (periphery) such that the latter are not linked to each other. An empty network is a graph containing firms without links between them.

• Stages of the Model

In Goyal and Moraga-Gonzalez, firms strategically form bilateral collaborative links with other firms where the collaboration of firms is modeled as a three-stage game.

The first stage: Each firm chooses its research partners. Firms and the cooperative links together constitute a network of cooperation in R&D.

The second stage: Given the R&D network, each firm chooses the amounts of investment (effort) in R&D simultaneously and independently in order to reduce the cost of production.

The third stage: Given the R&D investments of each firm and the effective R&D effort (as determined by the R&D network), firms compete in the product market by setting quantities (Cournot competition) in order to maximize their profits.

• Cost Reduction

In Goyal and Moraga-Gonzalez [7], the effective R&D effort for each firm is defined by the following equation:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n, \tag{2.5}$$

[2]A symmetric (regular) network is a graph where each node has the same number of links. In an asymmetric network, the links distribution is heterogeneous.

[3]Stability of networks provides a preferable structure for firms to gain higher profits. Efficiency of networks matches the desires of firms and of consumers in an optimal structure.

where x_i denotes R&D effort of firm i, N_i is the set of firms participating in a joint venture with firm i and $\beta \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in a joint venture with firm i. The effective R&D effort reduces firm i's marginal cost ($\overline{c}$) of production

$$c_i = \overline{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n. \tag{2.6}$$

The effort is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is γx_i^2, where $\gamma > 0$ indicates the effectiveness of R&D expenditure [5]. The profit π_i for firm i is the difference between revenue and production cost minus the cost of R&D

$$\pi_i = \left(a - \sum_{i=1}^{n} q_i - \overline{c} + x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k \right) q_i - \gamma x_i^2, \quad i = 1, \dots, n, \tag{2.7}$$

where the marginal cost satisfies $a > \overline{c}$.

2.4 Nash Equilibria

We assume that the marginal cost function is constant and equal for all firms. Under Cournot and Bertrand competition, we identify the sub-game perfect Nash equilibrium by using backwards induction. Here, we show the reader how to calculate the equilibria and the final list of the equilibria is provided in the Appendix.

• Under Cournot Competition

Consider an industry comprised of n firms, each firm choosing an amount of output to produce. The firm i's output level is denoted as q_i. To find the equilibrium for the production quantity of firm i, we solve $\frac{\partial \pi_i}{\partial q_i} = 0$. This yields the best response function of quantity of good i:

$$q_i = \frac{a - c_i - \lambda \sum_{j \neq i} q_j}{2}. \tag{2.8}$$

Substituting the best response functions (equation 2.8 for each i) into each other yields the symmetric equilibrium for the production quantity:

$$q_i^* = \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)\big((n-1)\lambda + 2\big)}. \tag{2.9}$$

By substituting (2.9) into the profit function (2.3), the equilibrium profit is [4]

$$\pi_i^* = \left[\frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)\big((n-1)\lambda + 2\big)} \right]^2 - \gamma x_i^2, \tag{2.10}$$

where γ is the R&D effectiveness.[5] Calculating the equilibrium effort x_i depends on the structure of the R&D network. By knowing the structure, we find the cost function c_i to

[4]Note that the equilibrium profit function can be expressed in a more convenient form for practical calculation: $\pi_i^* = (q_i^*)^2$.

[5]To have suitable values of the effectiveness, the effort and cost functions should be non-negative and the second order condition for maximizing profit function ($\frac{\partial^2 \pi}{\partial x^2} < 0$) should be satisfied [7].

substitute it into the profit function (2.10). Then, we calculate the best response function of R&D effort for each firm i. By plugging them into each other, we have the symmetric equilibrium for the R&D effort.

• Under Bertrand Competition

Suppose the competition of n firms is by choosing the price of their product. Each firm maximizes the profit function by choosing a price, taken prices of other firms as given. As in Cournot competition, we calculate symmetric equilibria.

The firm i's price is denoted as p_i. To find the equilibrium for the price, we find the demand function for each good i that can be calculated by doing some substitutions on the inverse demand functions. Thus, the demand function for each good i is

$$q_i = \frac{(1-\lambda)a-(1+(n-2)\lambda)p_i+\lambda\sum\limits_{j\neq i}p_j}{(1-\lambda)(1+(n-1)\lambda)}\,. \tag{2.11}$$

The demand function (2.11) for $n=2$ firms is not well defined if the products are homogeneous or perfect complements. Also, the second order condition for maximizing the profit function is not satisfied if $\lambda \le \frac{1}{1-n}$ [13, 8, 9].

By substituting the demand function (2.11) into the profit function, then by calculating the first order condition ($\frac{\partial \pi_i}{\partial p_i}=0$) for good i, the best response function of price is

$$p_i = \frac{(1-\lambda)a+(1+(n-2)\lambda)c_i+\lambda\sum\limits_{j\neq i}p_j}{2(1+(n-2)\lambda)}\,. \tag{2.12}$$

The Nash equilibrium for the price of product i is found by substituting the best response functions into each other. This yields

$$p_i^* = \frac{(1-\lambda)(2+(2n-3)\lambda)a+(1+(n-2)\lambda)(2+(n-2)\lambda)c_i}{\big((2n-3)\lambda+2\big)\big((n-3)\lambda+2\big)} + \frac{\lambda(1+(n-2)\lambda)\sum\limits_{j\neq i}c_j}{\big((2n-3)\lambda+2\big)\big((n-3)\lambda+2\big)}\,. \tag{2.13}$$

By substituting equation (2.13) into the quantity function and profit function (equations 2.11 and 2.3, respectively), the equilibrium quantity and the profit-maximum are

$$q_i^* = \left(\frac{1+(n-2)\lambda}{(1-\lambda)\big(1+(n-1)\lambda\big)}\right)\left[\frac{(1-\lambda)\big(2+(2n-3)\lambda\big)a}{\big((2n-3)\lambda+2\big)\big((n-3)\lambda+2\big)} - \frac{\big(2+3(n-2)\lambda+(n^2-5n+5)\lambda^2\big)c_i-\lambda\big(1+(n-2)\lambda\big)\sum\limits_{j\neq i}c_j}{\big((2n-3)\lambda+2\big)\big((n-3)\lambda+2\big)}\right], \tag{2.14}$$

$$\pi_i^* = \left(\frac{(1-\lambda)\big(1+(n-1)\lambda\big)}{1+(n-2)\lambda}\right)q_i^{*2}\,. \tag{2.15}$$

3. The Outcomes under Different Strategies

In this section, we study the impact of the strategies of firms on the equilibrium outcomes. We combine the market structure, the market competition type and the market participation in R&D to conclude a strategy ensures high individual and social outcomes. We do the study for two firms. Then, we repeat the study for three firms to observe how the network concept contributes to the theoretical R&D literature.

3.1 Two-Player Game

The outcomes show that the behavior of the equilibria under Cournot and Bertrand competition with respect to the structures of the market and the network is consistent. The investment of firms in R&D leads to higher outcomes, regardless of the cooperation of firms. However, when considering the cooperation, the strategy of firms varies according to the market structure. In a weak competitive market (firms produce complementary or independent goods), the R&D cooperation always improves the equilibrium outcomes. This indicates that the R&D investment, production, profit and the total welfare are maximized when the two firms collaborate in R&D. In addition, non-cooperation involves a spillover to ensure knowledge flow between firms. Figure 1 shows that the spillover is a positive factor for all equilibria. As the R&D spillover rises, the individual and social outcomes increases.

In a competitive market (firms produce substituted goods), the cooperation of firms and the R&D spillover do not always improve the equilibrium outcomes. For individuals, the R&D investment deceases with increasing the cooperation and the spillover. This also occurs to the quantity of production and to the total welfare, especially when the substitution rate between the products is high. This indicates that the R&D cooperation under the social perspective is not encouraged when the competition between firms increases. In contrast, the cooperation under the individual perspective is generally encouraged where firms obtain high profits with the cooperation.

Example 1. Consider two firms in a market. Figure 1 shows the equilibrium outcomes under cooperation and non-cooperation case in Cournot and Bertrand competition.

3.2 Three-Player Game

In this section, we show the importance of using the network concept. When studying cooperation of three firms, we need to consider all possible distinct R&D relationships[6]. For three firms, there are eight networks, but to describe the possible different R&D partnerships, we need only four networks where the other networks are equivalent to the present ones. In addition, for asymmetric interactions, the outcomes cannot be generalized since the equilibria vary according to the structure of the network[7]. Also, with increasing the market size n, the number of involved networks will increase and this makes the presentation of the outcomes difficult. Due to these reasons, we consider the case when there are three firms.

[6]For n firms, there are $2^{\binom{n}{2}}$ possible networks. They are refined to have only distinct networks.

[7]For symmetric interactions with ignoring the R&D spillover, [7] generalized the outcomes under Cournot for homogeneous and independent goods.

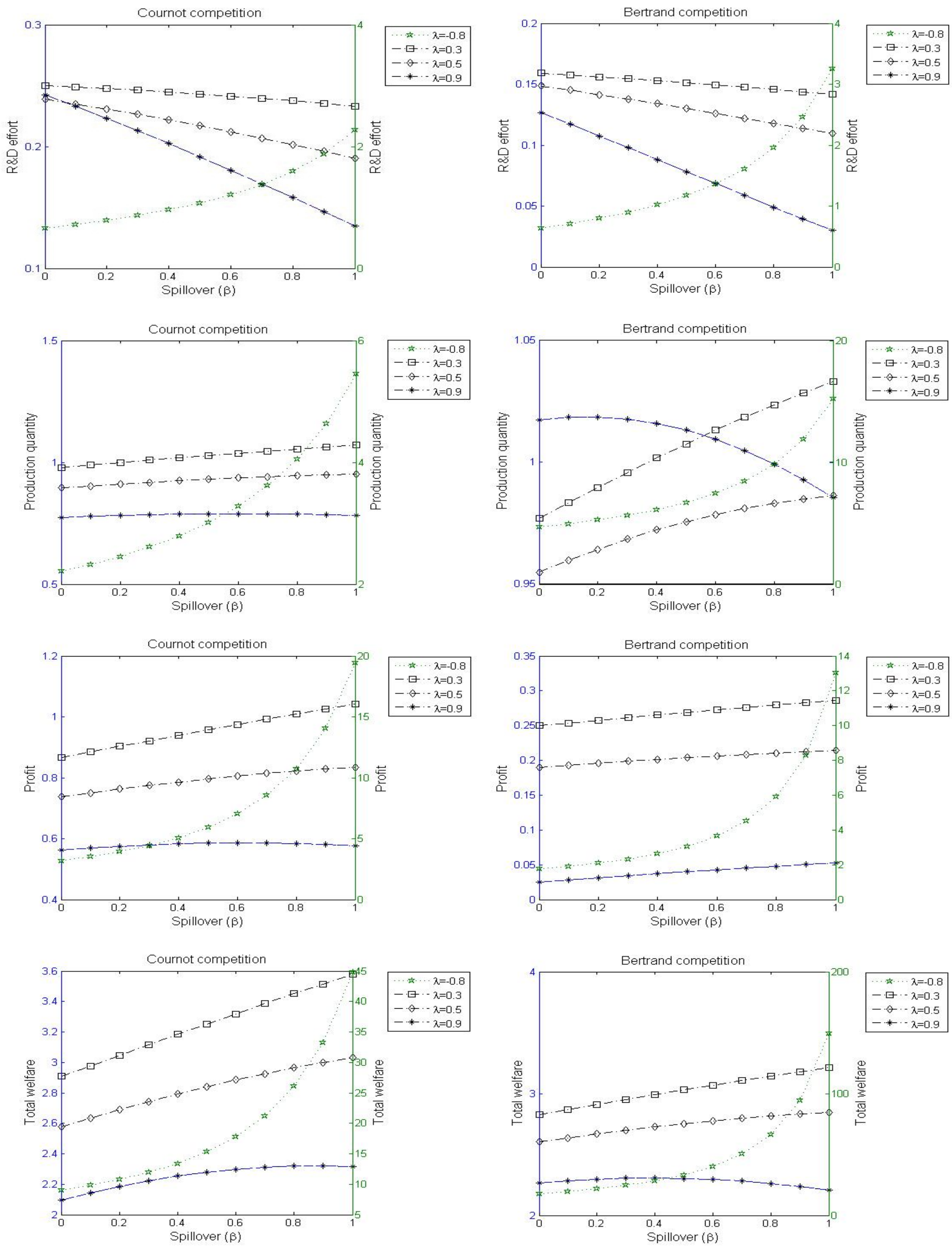

Figure 1. The equilibrium outcomes under Cournot and Bertrand competition for two firms in a market. The figure shows the R&D effort, production quantity, profit and total welfare. The parameters used to plot the graphs are $a = 12$, $\bar{c} = 10$ and $\gamma = 2$ ($\gamma = 3$) under Cournot (Bertrand).

Figure 2 displays the distinct R&D networks generated from cooperating three firms in a market. Figure 3 shows the equilibrium outcomes for those networks under Cournot and Bertrand competition. As mentioned in the two-player game, the two competitions are similar in terms of the behavior of the equilibria. In the sense that the effect of the R&D cooperation on the outcomes depends on the position of firms in the marketplace (the product type).

In a weak competitive market, increasing the cooperation leads to higher individual and social outcomes. This indicates that the R&D effort, profit and total welfare are maximized when firms form a complete network (G_1). Also, existing and increasing the R&D spillover between non-cooperative firms improves the outcomes. For example, the outcomes of firms 1 and 2 in network G_2 and the welfare in that network increase with increasing the spillover.

In a competitive market, the R&D cooperation does not have one effect as in a weak competitive market. First, the cooperation has a negative impact on the investment. When individuals form new cooperation, their R&D investment decreases. This indicates that the R&D investment is maximized in the empty network (G_2). Secondly, the cooperation positively affects the individual profit. The profit of firms increases with the cooperation and this makes the complete network G_1 a profitable structure. However, the cooperation of firms affects negatively on the profit of other firms in a network. For example, when comparing the profit of firm 1 in the networks G_3 and G_4, we find that the profit of that firm decreases when firms 2 and 3 cooperate. Finally, the cooperation is a social demand if the intensity of the competition is not high. This indicates that the complete network is profitable in the social perspective if the substitution between the products is not high. The effect of the spillover depends on the intensity of the cooperation. The R&D effort and the profit of highly connected firms (like firm 1 in G_3) decrease with increasing the spillover. In the social view, the spillover improves the welfare if the competition is not high.

Example 2. Consider three firms in a market. Figure 2 shows the distinct networks generated from three firms. Figure 3 shows the equilibrium outcomes under Cournot and Bertrand competition.

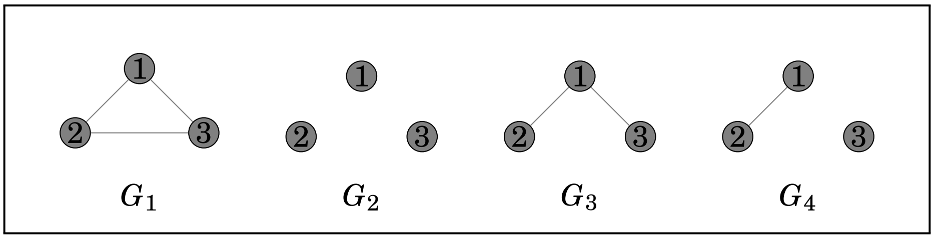

Figure 2. The distinct networks with three firms. In each of G_1 and G_2, there is one group of firms, but the other networks, there are two groups. In the network G_3, there is a hub (firm 1) and peripheries (firms 2 and 3) and in the network G_4, there are linked firms (firms 1 and 2) and an isolated firm (firm 3).

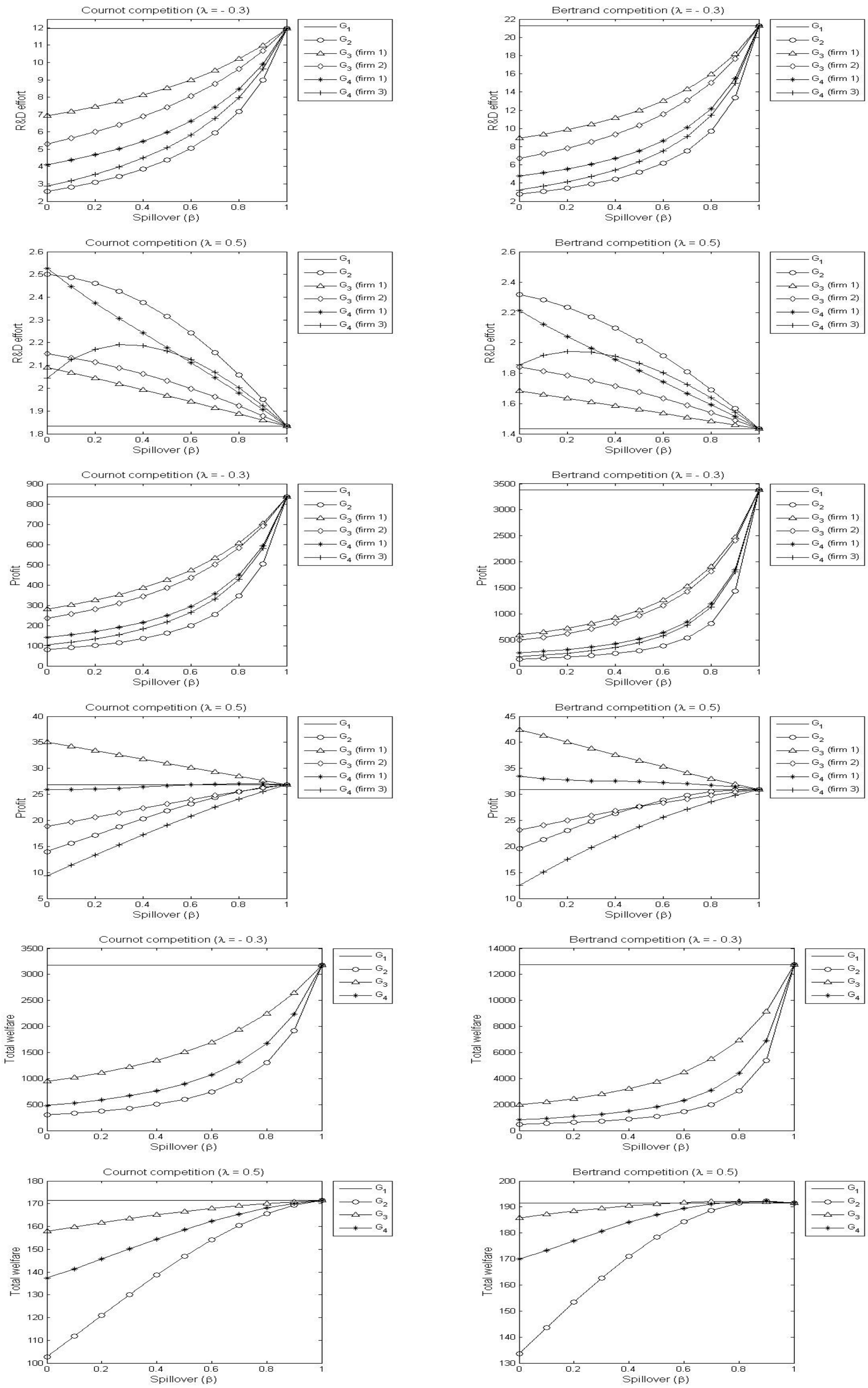

Figure 3. The equilibrium outcomes for the networks given in 2 under Cournot and Bertrand competition. The figure shows the effort, profit and total welfare under the two competitions. The parameters used to plot the graphs are $a = 12$, $\bar{c} = 10$ and $\gamma = 2$ ($\gamma = 1$) if $\lambda = -0.3$ ($\lambda = 0.5$).

3.3 Impact of the Differentiation Degree on the Contrast between Cournot and Bertrand

In the previous section, we examined the impact of the market and network structure under Cournot or Bertrand competition on the equilibrium outcomes. The common findings under the two competitions reveal the fundamental role of the market structure in the strategy of firms in building R&D relationships. In this section, we will explain the effect of the differentiation degree on the outcomes in order to discriminate between Cournot and Bertrand.

In economics, it is a matter for firms to know who is in a market whether or not competitor. This does not only impact the strategic interaction of firms in the marketplace in terms of setting their quantities or prices, but it also impacts the overall outcomes. When comparing the equilibria under Cournot and Bertrand competition, the finding suggests that the production quantity is higher and the price is lower under Bertrand competition than under Cournot competition. This indicates that the consumer surplus is higher under Bertrand competition than under Cournot competition ([13, 4, 12, 8, 9]). For the individual profit, Bertrand competition is profitable if goods are complements, but if they are substitutes, Cournot competition becomes profitable. Despite that the profitable competition depends on the market structure (goods are complements or substitutes), the social welfare is maximized under Bertrand competition for all product types. This indicates that the social benefit sometimes consists with the individual desire in terms of the competition type.

In the following example, we compare between the outcomes under Cournot and Bertrand competition in a duopoly market. The comparison between Cournot and Bertrand competition can be summarized as follows. Firstly, for all types of the products, the price is higher and quantity is lower under Cournot competition than under Bertrand competition. Secondly, the R&D investment and the profit under Bertrand are higher than under Cournot if goods are complements; however, the opposite occurs when goods are substitutes. Finally, the total welfare under Bertrand competition is higher than under Cournot competition, regardless of the product type.

Example 3. Consider two firms in a market. Figure 4 compares between the equilibrium outcomes under Cournot and Bertrand competition.

In the following, we identify the gap between Cournot and Bertrand competition. We examine the contrast between the two competitions with respect to the differentiation degree. To simplify the outcomes, we do the study for two firms. Also, we do the study without R&D stage since the comparison between the two competitions is not affected by the R&D investment.

The following functions compares between the equilibria of Cournot and Bertrand competition.

$$\Delta_q = \frac{\lambda^2(a-c)}{(4-\lambda^2)(1+\lambda)}, \tag{3.1a}$$

$$\Delta_p = \frac{\lambda^2(a-c)}{4-\lambda^2}, \tag{3.1b}$$

$$\Delta_\pi = \frac{2\lambda^3(a-c)^2}{(4-\lambda^2)^2(1+\lambda)}, \tag{3.1c}$$

$$\Delta_{TW} = \frac{\lambda^2(4-2\lambda-\lambda^2)(a-c)^2}{(4-\lambda^2)^2(1+\lambda)} \ . \tag{3.1d}$$

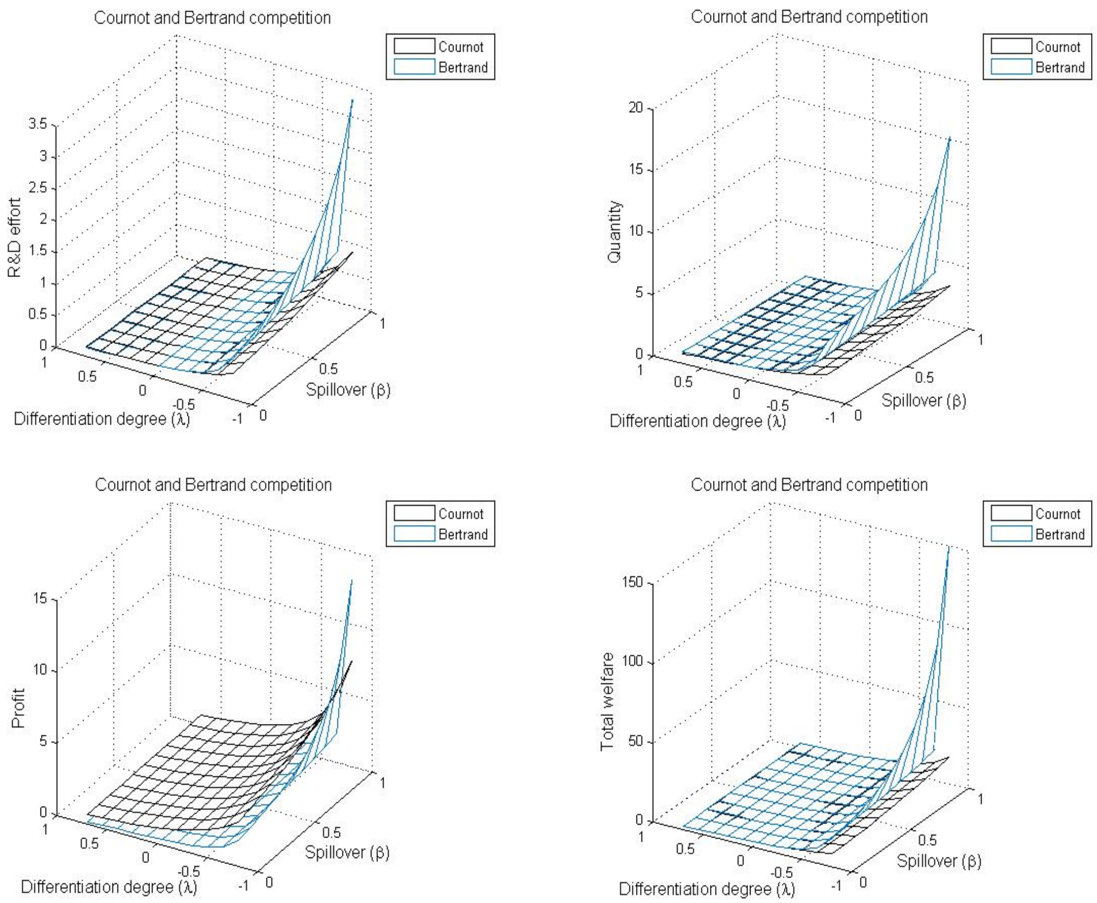

Figure 4. Comparison of Cournot and Bertrand for two firms in a market. The figure shows the R&D effort, production quantity, profit and total welfare. The parameters used to plot the graphs are $a = 12$, $\overline{c} = 10$ and $\gamma = 3$.

It can be observed that the contrast between Cournot and Bertrand is determined by three main values: the intercept demand a, the marginal cost c and the differentiation degree λ. As seen in this paper, the marginal cost can be reduced by investing firms in R&D (equation 2.5). This increases the difference between a and c which in turn increases the contrast between the two competitions.

The upcoming results show the effect of the differentiation degree on the equilibrium outcomes under Cournot and Bertrand competition. Proposition 1 states that the functions Δ_q and Δ_p are non-monotonic with respect to the differentiation degree λ if all values are considered i.e., $\lambda \in (-1,1)\setminus\{0\}$. However, if one type of goods is considered, the two functions are monotonic decreasing (increasing) when goods are complements (substitutes). This indicates that the gap between Cournot and Bertrand in terms of the quantities and the prices is minimized when the differentiation degree λ approaches zero value. The opposite occurs when the degree approaches its minimum or maximum value i.e., $\lambda \to -1$ or 1.

Proposition 1. *Consider two firms in a market. If $\lambda < 0$ ($\lambda > 0$), the functions Δ_q and Δ_p are monotonic decreasing (increasing) with respect to λ.*

The proof is given in the Appendix C.

The relationship between the quantities and prices under Cournot and Bertrand competition can be summarized in the following expression:

$$\Delta_p = (1+\lambda)\Delta_q \,. \tag{3.2}$$

The equation indicates that the contrast between Cournot and Bertrand in terms of the quantities and the prices is equal if the differentiation degree $\lambda = 0$. Meaning that, firms do not have a preferable strategy in terms of choosing the competition type if goods are independent. Moreover, the equation shows that as the differentiation degree approaches -1, the function Δ_q is higher than the function Δ_p; whereas the opposite occurs when λ approaches 1.

Proposition 2 states that if goods are complements (substitutes), the function Δ_π decreases (increases) with respect to the differentiation degree. This indicates that the strategy of firms in interacting in the marketplace is strongly affected by the toughness of competition. In the sense that when the competition is very weak (strong), setting prices (quantities) in a market generates a huge contrast in the profits.

Proposition 2. *Consider two firms in a market. If $\lambda < 0$ ($\lambda > 0$), the function Δ_π is monotonic decreasing (increasing) with respect to λ.*

The proof is given in the Appendix C.

As discussed in this section, the total welfare under Bertrand competition is higher than under Cournot competition, regardless of the production type. The following proposition shows that the discrimination between the two competitions in terms of the total welfare is maximized when the differentiation degree departs from zero.

Proposition 3. *Consider two firms in a market. If $\lambda < 0$ ($\lambda > 0$), the function Δ_{TW} is monotonic decreasing (increasing) with respect to λ.*

The proof is given in the Appendix C.

The following example illustrates the previous propositions.

Example 4. Consider two firms in a market. Figure 5 shows the functions given in (3.1).

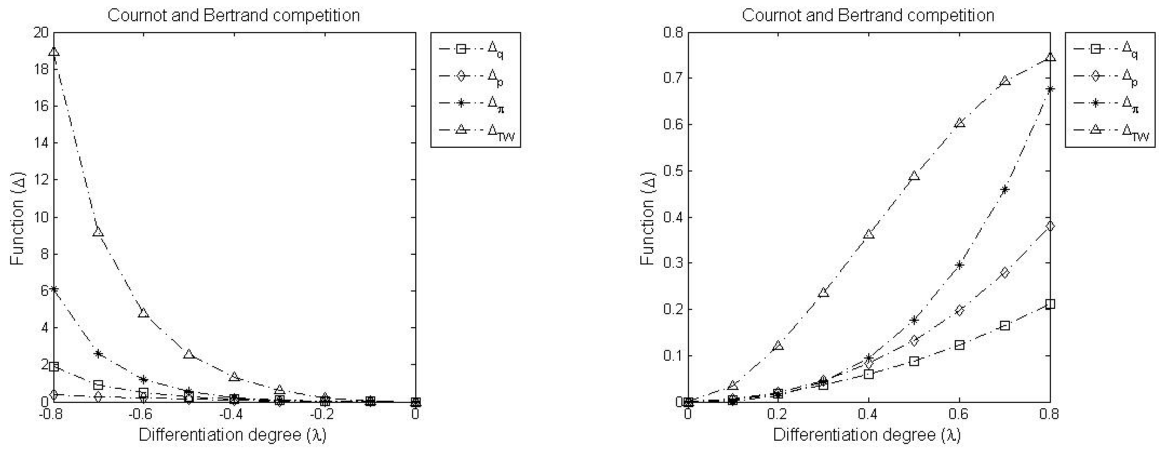

Figure 5. The equations (3.1) with respect to the differentiation degree λ. The parameters used to plot the figures are $a = 12$ and $c = 10$.

4. Conclusion

In this paper, we examined the impact of the market and R&D network structure and the competition type on the equilibrium outcomes. The results suggest that Cournot and Bertrand competitions are consistent in terms of the behavior of the equilibria with respect to the structure of the market and the network. Firstly, the investment in R&D has a role in enhancing the individual and social outcomes. Secondly, the role of the cooperation in R&D relies on the toughness of competition. In a weak competitive market, there is a positive relationship between the R&D cooperation and the individual and social outcomes. When the competition increases the benefit behind the cooperation is mostly limited to the individual profits.

In addition, the results show that when comparing Cournot and Bertrand competition, there are contrasts between the equilibria. In this part, we highlighted the effect of the market structure on this contrast. The outcomes suggest that the conflict between Cournot and Bertrand competition is managed by the position of firms in the market. When the products of firms are complements or substitutes, the firms' choice for the competition type has a significant impact on the individual and social outcomes. The opposite occurs when the products are independent where firms do not have a preference in choosing Cournot or Bertrand competition.

Appendix A

Equilibria under Cournot

1. Two-Player Game:

- Without R&D investment:

$$q = \frac{a-c}{\lambda+2}, \tag{A.1a}$$

$$p = \frac{a+(1+\lambda)c}{\lambda+2}, \tag{A.1b}$$

$$\pi = \left(\frac{a-c}{\lambda+2}\right)^2, \tag{A.1c}$$

$$TW = \frac{(3+\lambda)(a-c)^2}{(\lambda+2)^2}\ . \tag{A.1d}$$

- With R&D investment:

Cooperation case:

$$x = \frac{(a-\bar{c})}{\gamma(2+\lambda)^2-2}, \tag{A.2a}$$

$$q = \frac{\gamma(2+\lambda)(a-\bar{c})}{\gamma(2+\lambda)^2-2}, \tag{A.2b}$$

$$\pi = \frac{\gamma[\gamma(2+\lambda)^2-1](a-\bar{c})^2}{(\gamma(2+\lambda)^2-2)^2}, \tag{A.2c}$$

$$TW = \frac{\gamma[\gamma(2+\lambda)^2(3+\lambda)-2](a-\bar{c})^2}{(\gamma(2+\lambda)^2-2)^2}\ . \tag{A.2d}$$

Non-cooperation case:

$$x = \frac{(a-\bar{c})(2-\lambda\beta)}{\gamma(2+\lambda)^2(2-\lambda)-(1+\beta)(2-\lambda\beta)}, \tag{A.3a}$$

$$q = \frac{\gamma(4-\lambda^2)(a-\bar{c})}{\gamma(2+\lambda)^2(2-\lambda)-(1+\beta)(2-\lambda\beta)}, \tag{A.3b}$$

$$\pi = \frac{\gamma[\gamma(4-\lambda^2)^2-(2-\lambda)^2](a-\bar{c})}{\left(\gamma(2+\lambda)^2(2-\lambda)-(1+\beta)(2-\lambda\beta)\right)^2}, \tag{A.3c}$$

$$TW = \frac{\gamma[\gamma(4-\lambda^2)^2(3+\lambda)-2(2-\lambda\beta)^2](a-\bar{c})}{\left(\gamma(2+\lambda)^2(2-\lambda)-(1+\beta)(2-\lambda\beta)\right)^2}\ . \tag{A.3d}$$

2. Three-Player Game:

$$x_{G_1} = \frac{(a-\bar{c})}{((4\lambda^2+8\lambda+4)\gamma-3)}, \tag{A.4a}$$

$$q_{G_1} = \frac{(2\gamma(\lambda+1)(a-\bar{c}))}{((4\lambda^2+8\lambda+4)\gamma-3)}\ , \tag{A.4b}$$

$$x_{G_2} = \frac{(a-\bar{c})(\lambda(2\beta-1)-2)}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3}, \tag{A.5a}$$

$$x_{G_2} = \frac{(2\gamma(a-\bar{c})(\lambda^2-\lambda-2))}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3}\ , \tag{A.5b}$$

$$x_{G_3}(\text{firm 1}) = \frac{(a-\bar{c})(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2)}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{A.6a}$$

$$q_{G_3}(\text{firm 2}) = \frac{(2\gamma(a-\bar{c})(\lambda+1)(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2))}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{A.6b}$$

$$x_{G_3}(\text{firm 1}) = \frac{(2\gamma(\beta\lambda-2)(\lambda+1)(\lambda-2))(a-\overline{c})}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{A.6c}$$

$$q_{G_3}(\text{firm 2}) = \frac{4\gamma^2(a-\overline{c})(\lambda-\lambda^2+2)^2}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{A.6d}$$

$$x_{G_4}(\text{firm 1}) = \frac{(\beta\lambda-2)(a-\overline{c})(2\beta^2\lambda-3\beta\lambda-2\beta-2\gamma\lambda^3+6\gamma\lambda^2+\lambda-8\gamma+2)}{2(-4\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1))}, \tag{A.7a}$$

$$q_{G_4}(\text{firm 1}) = \frac{(\gamma(a-\overline{c})(\lambda-\lambda^2+2)(3\beta\lambda-2\beta^2\lambda+2\beta+2\gamma\lambda^3-6\gamma\lambda^2-\lambda+8\gamma-2))}{-4\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1)}, \tag{A.7b}$$

$$x_{G_4}(\text{firm 3}) = \frac{(a-\overline{c})(\lambda-2\beta\lambda+2)(\beta\lambda-\beta^2\lambda+2\beta+\gamma\lambda^3-3\gamma\lambda^2+4\gamma-2)}{-4\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1)}, \tag{A.7c}$$

$$x_{G_4}(\text{firm 3}) = \frac{(2\gamma(a-\overline{c})(\lambda-\lambda^2+2)(\beta\lambda-\beta^2\lambda+2\beta+\gamma\lambda^3-3\gamma\lambda^2+4\gamma-2))}{-4\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1)}, \tag{A.7d}$$

where $S_1 = 2(20\gamma^2+(2\beta+1)\gamma)$, $S_2 = 2(4\gamma^2+(2\beta^2+7)\gamma)$, $S_3 = 64\gamma^2+4\beta(\beta-1)(4\gamma-1)$, $S_4 = 12\gamma^2+(6\beta^2-4\beta+1)\gamma$, $S_5 = -44\gamma^2-(6\beta^2+12\beta-3)\gamma$, $S_6 = (6+24\beta-12\beta^2)\gamma-24\gamma^2-\beta(\beta^2-1)(2\beta-1)$, $S_7 = 2(\beta(3\beta^2-\beta-3)+24\gamma^2-(10-16\beta)\gamma+1)$.

Appendix B

Equilibria Under Bertrand

1. Two-Player Game:

- Without R&D investment:

$$p = \frac{(1-\lambda)a+c}{2-\lambda}, \tag{B.1a}$$

$$q = \frac{a-c}{(2-\lambda)(1+\lambda)}, \tag{B.1b}$$

$$\pi = \frac{(1-\lambda)(a-c)^2}{(2-\lambda)^2(1+\lambda)}, \tag{B.1c}$$

$$TW = \frac{(3-2\lambda)(a-c)^2}{(2-\lambda)^2(1+\lambda)}. \tag{B.1d}$$

- With R&D investment:

Cooperation case:

$$x = \frac{(1-\lambda)(a-\overline{c})}{\gamma(1+\lambda)(2-\lambda)^2-2(1-\lambda)}, \tag{B.2a}$$

$$q = \frac{\gamma(2-\lambda)(a-\overline{c})}{\gamma(1+\lambda)(2-\lambda)^2-2(1-\lambda)}, \tag{B.2b}$$

$$\pi = \frac{\gamma(1-\lambda)[\gamma(1+\lambda)(2-\lambda)^2-(1-\lambda)](a-\overline{c})^2}{\left(\gamma(1+\lambda)(2-\lambda)^2-2(1-\lambda)\right)^2}, \tag{B.2c}$$

$$TW = \frac{\gamma[\gamma(3-2\lambda)(1+\lambda)(2-\lambda)^2 - 2(1-\lambda)^2](a-\bar{c})^2}{\left(\gamma(1+\lambda)(2-\lambda)^2 - 2(1-\lambda)\right)^2}\ . \tag{B.2d}$$

Non-cooperation case:

$$x^* = \frac{(a-\bar{c})(2-\lambda\beta\lambda^2)}{(1+\lambda)(2+\lambda)(2-\lambda)^2\gamma - (\beta+1)(2-\lambda\beta-\lambda^2)}, \tag{B.3a}$$

$$q^* = \frac{(a-\bar{c})(4-\lambda^2)\gamma}{(1+\lambda)(2+\lambda)(2-\lambda)^2\gamma - (\beta+1)(2-\lambda\beta-\lambda^2)}, \tag{B.3b}$$

$$\pi^* = \frac{\gamma(a-\bar{c})^2\left[(1-\lambda)(4-\lambda^2)^2\gamma - (2-\lambda\beta-\lambda^2)^2\right]}{\left[(1+\lambda)(2+\lambda)(2-\lambda)^2\gamma - (\beta+1)(2-\lambda\beta-\lambda^2)\right]^2}, \tag{B.3c}$$

$$TW^* = \frac{\gamma(a-\bar{c})^2\left[\gamma(4-\lambda^2)^2(1+\lambda)(3-2\lambda) - 2(2-\lambda\beta-\lambda^2)\right]}{\left[(1+\lambda)(2+\lambda)(2-\lambda)^2\gamma - (\beta+1)(2-\lambda\beta-\lambda^2)\right]^2}\ . \tag{B.3d}$$

2. Three-Player Game:

$$x_{G_1} = \frac{(1-\lambda^2)(a-\bar{c})}{4(2\lambda+1)\gamma - 3(1-\lambda^2)}, \tag{B.4a}$$

$$q_{G_1} = \frac{2\gamma(a-\bar{c})(1+\lambda)}{4(2\lambda+1)\gamma - 3(1-\lambda^2)}\ , \tag{B.4b}$$

$$x_{G_2} = \frac{(a-\bar{c})(\lambda+1)((2-(2\beta-3)\lambda) - (2\beta+1)\lambda^2)}{V_1\lambda^3 + V_2\lambda^2 + V_3\lambda + 2(4\gamma-2\beta-1)}, \tag{B.5a}$$

$$q_{G_2} = \frac{2\gamma(3\lambda+2)(a-\bar{c})(\lambda+1)}{V_1\lambda^3 + V_2\lambda^2 + V_3\lambda + 2(4\gamma-2\beta-1)}\ , \tag{B.5b}$$

$$x_{G_3}(\text{firm } 1) = \frac{(\lambda^2-1)(a-\bar{c})\left(V_{12} - \gamma V_{13} - 2(\beta-1)\right)}{\lambda^4 V_7 - \lambda^2 V_5 + \lambda V_4 + \lambda^5 V_8 - \lambda^3 V_6 + 2(1-\beta+(8+4\beta)\gamma - 16\gamma^2)}, \tag{B.6a}$$

$$q_{G_3}(\text{firm } 1) = \frac{2\gamma(a-\bar{c})(\lambda+1)(2(\beta-4\gamma-1) - V_9\lambda^3 - V_{10}\lambda^2 - V_{11}\lambda)}{\lambda^4 V_7 - \lambda^2 V_5 + \lambda V_4 + \lambda^5 V_8 - \lambda^3 V_6 + 2(1-\beta+(8+4\beta)\gamma - 16\gamma^2)}, \tag{B.6b}$$

$$x_{G_3}(\text{firm } 2) = \frac{(2\gamma(a-\bar{c})(1-\lambda^2)(3\lambda+2)(\beta\lambda - 2\lambda + \beta\lambda^2 + 2\lambda^2 - 2))}{\lambda^4 V_7 - \lambda^2 V_5 + \lambda V_4 + \lambda^5 V_8 - \lambda^3 V_6 + 2(1-\beta+(8+4\beta)\gamma - 16\gamma^2)}, \tag{B.6c}$$

$$q_{G_3}(\text{firm } 2) = \frac{4\gamma^2(3\lambda+2)^2(a-\bar{c})(\lambda^2-1)}{\lambda^4 V_7 - \lambda^2 V_5 + \lambda V_4 + \lambda^5 V_8 - \lambda^3 V_6 + 2(1-\beta+(8+4\beta)\gamma - 16\gamma^2)}, \tag{B.6d}$$

$$x_{G_4}(\text{firm } 1) = \frac{(\lambda+1)(a-\bar{c})(V_{24}\lambda^3 + V_{25}\lambda^2 + V_{26}\lambda - 2(\beta+4\gamma-1))V_{23}}{2(V_{19}\lambda^6 + V_{18}\lambda^5 + V_{17}\lambda^4 - V_{16}\lambda^3 + V_{15}\lambda^2 - V_{14}\lambda + 4\beta^2 - 4(8\gamma^2 - 6\gamma + 1)}, \tag{B.7a}$$

$$q_{G_4}(\text{firm } 1) = \frac{\gamma(3\lambda+2)(a-\bar{c})(\lambda+1)((2\beta^2 - \beta + 18\gamma - 1)\lambda^3}{(V_{19}\lambda^6 + V_{18}\lambda^5 + V_{17}\lambda^4 - V_{16}\lambda^3 + V_{15}\lambda^2 - V_{14}\lambda + 4\beta^2 - 4(8\gamma^2 - 6\gamma + 1)}$$

$$+\frac{(4\beta^2-6\beta+6\gamma+2)\lambda^2+(2\beta^2-7\beta-16\gamma+5)\lambda-2\beta-8\gamma+2)}{(V_{19}\lambda^6+V_{18}\lambda^5+V_{17}\lambda^4-V_{16}\lambda^3+V_{15}\lambda^2-V_{14}\lambda+4\beta^2-4(8\gamma^2-6\gamma+1)}, \tag{B.7b}$$

$$x_{G_4}(\text{firm }3)=\frac{(a-\overline{c})(\lambda+1)(2+3\lambda-\lambda^2-2\beta\lambda(1+\lambda))((\beta^2+\beta+9\gamma-2)\lambda^3}{(V_{19}\lambda^6+V_{18}\lambda^5+V_{17}\lambda^4-V_{16}\lambda^3+V_{15}\lambda^2-V_{14}\lambda+4\beta^2-4(8\gamma^2-6\gamma+1)}$$
$$+\frac{(2\beta^2-2\beta+3\gamma)\lambda^2+(\beta^2-5\beta-8\gamma+4)\lambda-2(\beta+2\gamma-1))}{(V_{19}\lambda^6+V_{18}\lambda^5+V_{17}\lambda^4-V_{16}\lambda^3+V_{15}\lambda^2-V_{14}\lambda+4\beta^2-4(8\gamma^2-6\gamma+1)}, \tag{B.7c}$$

$$q_{G_4}(\text{firm }3)=\frac{\big(2\gamma(3\lambda+2)(a-\overline{c})(\lambda+1)\big)((\beta^2+\beta+9\gamma-2)\lambda^3}{(V_{19}\lambda^6+V_{18}\lambda^5+V_{17}\lambda^4-V_{16}\lambda^3+V_{15}\lambda^2-V_{14}\lambda+4\beta^2-4(8\gamma^2-6\gamma+1)}$$
$$+\frac{(2\beta^2-2\beta+3\gamma)\lambda^2+(\beta^2-5\beta-8\gamma+4)\lambda-2\beta-4\gamma+2)}{(V_{19}\lambda^6+V_{18}\lambda^5+V_{17}\lambda^4-V_{16}\lambda^3+V_{15}\lambda^2-V_{14}\lambda+4\beta^2-4(8\gamma^2-6\gamma+1)}, \tag{B.7d}$$

where $V_1=(2\beta+1)^2$, $V_2=2(4\beta^2+12\gamma-1)$, $V_3=4(\beta^2-2\beta+7\gamma)-5$, $V_4=\beta^2(1-4\gamma)+5\beta(4\beta\gamma-1)-4(32\gamma^2-8\gamma-1)$, $V_5=\beta^2(12\gamma-2)+104\gamma^2+30\gamma+2$, $V_6=2\gamma(4\beta^2+10\beta+37)-6\beta-120\gamma^2+6$, $V_7=\beta^2(4\gamma-2)+2\beta(4\gamma+1)+144\gamma^2+6\gamma$, $V_8=34\gamma-\beta(1-16\gamma)+\beta^2(4\gamma-1)+2$, $V_9=\beta(\beta+1)-18\gamma-2$, $V_{10}=2(\beta(\beta-1)-3\gamma)$, $V_{11}=\beta(\beta-5)+16\gamma+4$, $V_{12}=\lambda^3(\beta^2+\beta-2)+2\beta\lambda^2(\beta-1)+\lambda(\beta-1)(\beta-4)$, $V_{13}=2(\lambda-1)(3\lambda+2)^2$, $V_{14}=6\beta^3-18\beta^2+2\beta(16\gamma-3)+2(4\gamma-1)(22\gamma-9)$, $V_{15}=2\beta^4-25\beta^3+2\beta^2(6\gamma+11)-\beta(152\gamma-25)-2(4\gamma-1)(37\gamma-12)$, $V_{16}=34\beta^3-8\beta^4-\beta^2(54\gamma-6)+\beta(244\gamma-34)+36\gamma^2-25\gamma+2$, $V_{17}=12\beta^4-12\beta^3+\beta^2(90\gamma-26)-\beta(124\gamma-12)+324\gamma^2-101\gamma+14$, $V_{18}=8\beta^4+8\beta^3+\beta^2(66\gamma-12)+4\beta(9\gamma-2)+216\gamma^2-21\gamma+4$, $V_{19}=9\gamma(3+2\beta^2)-\beta(5-36\gamma)+5\beta^3+2\beta^4-2$, $V_{20}=2\beta^2-7\beta-16\gamma+5$, $V_{21}=4\beta^2-6\beta+6\gamma+2$, $V_{22}=2\beta^2-\beta+18\gamma-1$, $V_{23}=2-(\beta-2)\lambda-(\beta+2)\lambda^2$, $V_{24}=2\beta^2-\beta+18\gamma-1$, $V_{25}=4\beta^2-6\beta+6\gamma+2$, $V_{26}=2\beta^2-7\beta-16\gamma+5$.

Appendix C

Proof of Proposition 1. We want to show that if $\lambda<0$ ($\lambda>0$), as λ increases, the functions Δ_q and Δ_p decrease (increase).

Assume λ_1 and λ_2 are two differentiation degrees such that $\lambda_1<\lambda_2$. Then,

$$\Delta_q(\lambda_1)-\Delta_q(\lambda_2)=\frac{(\lambda_1-\lambda_2)\big((\lambda_1\lambda_2)^2+4\lambda_1\lambda_2+4(\lambda_1+\lambda_2)\big)}{(4-\lambda_1^2)(4-\lambda_2^2)(\lambda_1+1)(\lambda_2+1)},$$

$$\Delta_p(\lambda_1)-\Delta_p(\lambda_2)=\frac{4(\lambda_1^2-\lambda_2^2)}{(4-\lambda_1^2)(4-\lambda_2^2)(\lambda_1+1)(\lambda_2+1)}.$$

Suppose $\lambda_1<\lambda_2<0$, then $\lambda_1-\lambda_2<0$ and $(\lambda_1\lambda_2)^2+4\lambda_1\lambda_2+4(\lambda_1+\lambda_2)<0$. This implies $\Delta_q(\lambda_1)>\Delta_q(\lambda_2)$. Similarly, we can find that $\Delta_p(\lambda_1)>\Delta_p(\lambda_2)$ since $\lambda_1^2-\lambda_2^2>0$.

Suppose $0<\lambda_1<\lambda_2$. Then $\Delta_q(\lambda_1)>\Delta_q(\lambda_2)$ since $\lambda_1-\lambda_2<0$ and $(\lambda_1\lambda_2)^2+4\lambda_1\lambda_2+4(\lambda_1+\lambda_2)>0$. Similarly, if $0<\lambda_1<\lambda_2$, then $\lambda_1^2-\lambda_2^2<0$ which implies $\Delta_p(\lambda_1)<\Delta_p(\lambda_2)$.

An equivalent way to prove the previous result is by the first order condition ($d\Delta_q/d\lambda=0$). We have

$$\frac{d\Delta_q}{d\lambda}=\frac{\lambda(\lambda^3+4\lambda+8)(a-\overline{c})}{(-\lambda^3-\lambda^2+4\lambda+4)^2}.$$

The fraction $d\Delta_q/d\lambda = 0$ if $\lambda = 0$. For any $\lambda < 0$, we find that $d\Delta_q/d\lambda < 0$ (Δ_q is a decreasing function). The opposite occurs for any $\lambda > 0$ (Δ_q is an increasing function). Similarly, for the function Δ_p where $d\Delta_q/d\lambda = 8\lambda(a-\overline{c})/(\lambda^2-4)^2$. □

Proof of Proposition 2. We prove that if $\lambda < 0$ ($\lambda > 0$), as λ increases, the function Δ_π decreases (increases).

Assume λ_1 and λ_2 are two differentiation degrees such that $\lambda_1 < \lambda_2$. Compare the function Δ_π for λ_1 and λ_2,

$$\Delta_\pi(\lambda_1) - F_\pi(\lambda_2) = \frac{(\lambda_2-\lambda_1)\big(16(\lambda_1^2+\lambda_2^2)+\lambda_1\lambda_1(16-\lambda_1^2\lambda_2^2)(\lambda_1+\lambda_2+1)-8\lambda_1^2\lambda_1^2\big)}{(\lambda_1+2)^2(\lambda_1+1)(2-\lambda_1)^2(\lambda_2+2)^2(\lambda_2+1)(2-\lambda_2)^2}.$$

Let $\lambda_1 < \lambda_2 < 0$. The expression $\lambda_1\lambda_1(16-\lambda_1^2\lambda_2^2)(\lambda_1+\lambda_2+1) < 0$ if $\lambda_1+\lambda_2 < -1$. However, the previous expression is small since $\lambda_1\lambda_2 < 1$ (note that $\lambda \neq -1$ in Bertrand competition). Thus, $16(\lambda_1^2+\lambda_2^2)+\lambda_1\lambda_1(16-\lambda_1^2\lambda_2^2)(\lambda_1+\lambda_2+1) > 0$. Since $\lambda_2-\lambda_1 > 0$, $\Delta_\pi(\lambda_1)-\Delta_\pi(\lambda_2) > 0$ which implies Δ_π decreases as λ approaches 0.

Now, let $0 < \lambda_1 < \lambda_2$. The expression $16(\lambda_1^2+\lambda_2^2)+\lambda_1\lambda_1(16-\lambda_1^2\lambda_2^2)(\lambda_1+\lambda_2+1) > 0$. Since $\lambda_2-\lambda_1 < 0$, $\Delta_\pi(\lambda_1)-\Delta_\pi(\lambda_2) < 0$ which implies Δ_π increases as λ approaches 1. □

Proof of Proposition 3. Let $F_{TW}(\lambda_1)$ and $F_{TW}(\lambda_2)$ be two functions that are associated with the two differentiation degrees λ_1 and λ_2. Then,

$$\begin{aligned}\text{sing}\big(F_{TW}(\lambda_1)-F_{TW}(\lambda_2)\big) =& \text{sing}\big(\lambda_1^4\lambda_2^4+(\lambda_1+\lambda_2)(2\lambda_1^3\lambda_2^3+4\lambda_1^2\lambda_2^2-48\lambda_1\lambda_2 \\ & -16(\lambda_1^2-\lambda_1\lambda_2+\lambda_2^2)+64)-(\lambda_1^2+\lambda_2^2)(4\lambda_1^2\lambda_2^2+16\lambda_1\lambda_2+32) \\ & +2\lambda_1\lambda_2(3\lambda_1^2\lambda_2^2+16\lambda_1\lambda_2+16)\big).\end{aligned}$$

Note that for any λ_1 and λ_2, the following expressions are positive $2\lambda_1^3\lambda_2^3+4\lambda_1^2\lambda_2^2-48\lambda_1\lambda_2-16(\lambda_1^2-\lambda_1\lambda_2+\lambda_2^2)+64$, $4\lambda_1^2\lambda_2^2+16\lambda_1\lambda_2+32$ and $3\lambda_1^2\lambda_2^2+16\lambda_1\lambda_2+16$.

Assume $\lambda_1 < \lambda_2 < 0$. Then $(\lambda_1+\lambda_2)(2\lambda_1^3\lambda_2^3+4\lambda_1^2\lambda_2^2-48\lambda_1\lambda_2-16(\lambda_1^2-\lambda_1\lambda_2+\lambda_2^2)+64) < 0$. This yields $F_{TW}(\lambda_1)-F_{TW}(\lambda_2) > 0$.

Now, assume $0 < \lambda_1 < \lambda_2$. Then the previous expression becomes positive which implies $F_{TW}(\lambda_1)-F_{TW}(\lambda_2) < 0$.

We can see this from the first order condition

$$\frac{dF_{TW}}{d\lambda} = 0 \Rightarrow \frac{\lambda(a-c)^2(\lambda^5+4\lambda^4+2\lambda^3+24\lambda^2+8\lambda-32)}{(\lambda^2-4)^3(\lambda+1)^2}.$$

Now, $dF_{TW}/d\lambda = 0$ if $\lambda = 0$. If $\lambda < 0$ ($\lambda > 0$), $dF_{TW}/d\lambda < 0$ ($dF_\pi/d\lambda > 0$). This means that if $\lambda < 0$ ($\lambda > 0$), the function F_{TW} is decreasing (increasing). □

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Competing Interests

Author declares that he has no competing interests.

Authors' Contributions

Author wrote, read and approved the final manuscript.

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