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Estimating Relative Attractiveness of Locations using Multiple Attribute Decision Making (MADM)

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Abstract. Competitive facility location problems involve identifying the best location of facility that can capture maximum market share in the presence of competition. One of the most popular models for competitive facility location, namely the Huff model is not considered very realistic and efforts have been made to improve the model by including additional factors. In this paper, an extension of Huff model to consider multiple factors using *multiple attribute decision making* (MADM) is proposed. MADM problem is a management science technique, which is popularly used to rank the priority of alternatives with respect to their competing attributes. Weights from the core of MADM: it is obvious that different weight lead to various evaluation results and decisions. The proposed model is applied for estimating the market share.

1. **Introduction**

The choice of a proper location of facility has significant implication on the fixed and operating cost of a firm. Hence, considerable research efforts have been directed towards studying location problems. Location problems can be classified into several categories and several authors have provided categorized surveys Nakanishi and Cooper [10]. In this paper, we focus on the competitive facility location theory that was first introduced by Hotellig [7]. The model suggested by Huff [8] forms one of the most researched models of competitive facility location. Huff [8] suggested that customers divide their patronage among competing facilities in direct proportion to the attractiveness of the distance from it. Thus only two factors of facility location, namely the attractiveness (normally measured by the square footage of facility) and distance, have been considered by the Huff model. This simple model has been further extended by several authors to make it more realistic. This paper also deals with an improvement in the Huff model by considering additional factors. When more than two factors have to be considered for a problem, it is normally difficult to identify the influence to these factors on

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the location choice. Many of the extensions to the Huff model relied on empirically estimating the influence of the factors, usually by conducting surveys. In this paper, we suggest a *multiple attribute decision making* (MADM). MADM problem is the process of finding the best option from all of the feasible alternatives. Technique for order preference by similarity to ideal solution (TOPSIS) approach has been dealt with. The proposed model is applied for estimating the market share. A brief review of some of extension to the Huff model is also provided in this section. Multiple attribute decision making problem (MADM) briefly discussed in section 3, TOPSIS model discussed and we illustrate our proposed method with an example in section 4 and the final section summary.

2. **The Huff model for competitive facility location**

The Huff model, Huff [8] is traditionally used for estimating the market shares of a discrete number of facilities located in a given area competing for customers, it is based on the gravity principle which proposes that probability that a customers selects a certain facility is proportional to its level of attraction and inversely proportional to a power of the distance to that facility. Normally, customers are grouped as cluster (such as cities, zip codes, census tracts) and calculation are carried out for each cluster rather than for individual customers. In the simple Huff model, only two factors one for measuring the level of attractiveness (usually the size of the retail facility) and one more for measuring the level of unattractiveness (usually some measure of distance) are employed. in the Huff model, the expected demand from population center i that will be attracted to retail location j is estimate as per the following steps:

Step 1: attractiveness of a facility is expressed as:

$$A_{ij} = \frac{S_j}{T_{ij}^{\lambda}}. \tag{1}$$

Where A_{ij} is the attraction to facility j for customers in the area i, S_j is the size of the store (e.g., square feet), T_{ij} is the travel time from area i to facility j and λ is a parameter reflecting propensity to travel ($\lambda > 0$) (e.g., $\lambda = 2$ for a trip to a shopping mall). Normally travel time is assumed to be proportional to the distance traveled.

Step 2: if there are several facilities competing for the patronage of the same set of customs, the probability P_{ij} of customers in an area i traveling to a particular facility j is computed as:

$$P_{ij} = \frac{A_{ij}}{\sum_{i=1}^{n} A_{ij}}. \tag{2}$$

Step 3: the annual customer expenditures E_{jk} for a product class k at a facility j is calculate as:

$$E_{jk} = \sum_{i=1}^{r} (P_{ij} C_i B_{ik}). \tag{3}$$

Where C_i is the number of customers in area i, B_{ik} is the annual budget for product class k for customers in area i and r is the number of customer areas in the region.

Step 4: finally, the market share M_{jk} captured by facility j of product class k is calculate as:

$$M_{jk} = \frac{E_{jk}}{\sum_{i=1}^{R} (C_i B_{ik})}. \tag{4}$$

2.1. *Extensions to the Huff model*

Prominent critique to the Huff model is that it is over-simplistic since it considers just two factors describe consumer patronage Bucklin [1], Serra and Colome [12]. Hence, extensions of the Huff model to consider additional factors are reported in the literature. Nakanishi and Cooper [10] extended the Huff model by including additional factors to represent a facility's attractiveness (instead of just the area as the only attractiveness attribute in the Huff model). Subsequently, more factors have been considered in the location model, such as consumer opinion of facility image, facility appearance, number of checkout counters and credit car services to capture the attractiveness, and travel distance and physical distance for the measure of unattractiveness (Jain and Mahajan [9], Ghosh and Mclafferty [6], Vandell and Carter [16], and Tan and Thang [15]). A more general model considering several factors for a competitive location problem is something known as the multiplication for a competitiveness interaction MCI model (Smith and Moses [14], Gonzalez-Benito *et al.* [5], and Serra and Colome [12]) and is generally formulated as follows.

$$P_{ij} = \frac{\prod_{l=1}^{q} X_{lij}^{\beta l}}{\sum_{j=1}^{n} \prod_{l=1}^{q} X_{lij}^{\beta l}}. \tag{5}$$

Where P_{ij} is the probability of customers living at area i patronising the facility of j, X_{lij} is the l-th factor describing the facility j attracting customers at area i and β_i is the estimated parameter reflecting sensitively of customers with respect to factor l. thus, MCI models differ from the original Huff model in calculating the probabilities. The impact of all the factors is assumed to be multiplicative; the numerator captures the total attraction by all the factors that add to the attractiveness, denominator captures the total unattractiveness of all the factors that discourage consumers from patronizing a facility. The extend of influence of factor l (i.e., β_l) is captured using revealed preference methods by conducting consumer surveys Colome and Serra [3]. The value of the influence parameters should be calibrated first before they can be used for addressing facility location problems.

3. **Multiple Attribute Decision Making (MADM)**

Multiple attribute decision making has been one of the fastest growing areas during the last decade depending on the changing. Decision marker(s) need a decision aid to decide between the alternatives and mainly excel less preferable alternatives fast. With the help of computers the decision making methods have found great acceptance in all area of the decision making processes. Since *multiple attribute decision making* (MADM) has found acceptance in area of operation research and management science, the discipline usage has increased significantly, the application of MADM methods has considerably become easier for the users the decision makers. In discrete alternative multiple attribute decision problems; the primary concern for the decision aid is the following:

1. choosing the most preferred alternative to the decision maker (DM),
2. ranking alternative in order of importance for selection problems, or
3. screening alternatives for the final decision.

The general concepts of domination structures and non-dominated solutions play an important role in describing the decision problems and the decision maker's revealed preferences describes above. So far, various approaches have been developed as the decision aid. That is, for many such problems, the decision maker wants to solve a multiple attribute decision making (MADM) problem. A MADM problem can be concisely expressed in matrix format as:

	C_1	C_2	$\cdots$	C_n
A_1	x_{11}	x_{12}	$\cdots$	x_{1n}
A_2	x_{21}	x_{22}	$\cdots$	x_{2n}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\cdots$	x_{mn}

$$W = \{w_1, w_2, \dots, w_n\}$$

Where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $x_1, x_2, \dots, x_n$ are attribute with which alternative performance are

measured, x_{ij} is the rating of alternatives A_i with respect to attribute x_j, w_j is the weight of attribute C_j.
The main steps of multiple attribute decision making are the following:

1. Establishing system evaluation attribute that relate system capabilities to goal.
2. Developing alternative systems attaining the goals (generating alternatives).
3. Evaluation alternatives in terms of attribute (the value of the attribute function).
4. Applying a normative multi attribute analysis method.
5. Accepting one alternative as "optimal"(preferred).
6. If the final solution is not accepted, gather new information and go into the next interaction of multi attribute optimization.

Step 1 and 5 are preformed at the upper level, where decision makers *h* ave the central role, and the other steps are mostly engineering task. For step 4, a decision maker should express his/her preference by similarity to ideal solution (TOPSIS), one of known classical MADM method, was first developed by Hwang and Yoon for solving a MADM problem. TOPSIS, known as one of the most classical MADM methods, is based on the idea, that the chosen alternative should have the shortest distance from the positive ideal solution and on the other side the farthest distance of the negative ideal solution. The TOPSIS-method will be applied to a case study, which is described in detail. In classical MADM methods, the rating and the weight of the attribute are known precisely. A survey of the methods has been presented in Hwang and Yoon [2]. In the process of TOPSIS, the performance rating and the weights of the attribute are given as exact values.

4. **TOPSIS Method**

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang [13], with reference to Hwang and Yoon. TOPSIS is a multiple attribute method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The procedure of TOPSIS can be expressed in a series of step:

1. Calculate the normalize decision matrix. The normalize value n_{ij} is calculated as:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, \dots, m, \; j = 1, \dots, n.$$

2. Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as:

$$v_{ij} = w_j n_{ij}, \qquad i = 1, \ldots, m, \; j = 1, \ldots, n.$$

Where w_{ij} is the weight of the i-th attribute, and $\sum_{j=1}^{n} w_j = 1$.

3. Determine the positive ideal and negative ideal solution:

$$A^+ = \{v_1^+, \ldots, v_n^+\} = \left\{ \left(\max_j v_{ij} / i \in I \right), \left(\min_j v_{ij} / i \in J \right) \right\}$$

$$A^- = \{v_1^-, \ldots, v_n^-\} = \left\{ \left(\min_j v_{ij} / i \in I \right), \left(\max_j v_{ij} / i \in J \right) \right\}$$

Where I is associated with benefit attribute, and J is associated with cost attribute.

4. Calculate the separation from the negative ideal solution is given as:

$$d_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{\frac{1}{2}}, \quad i = 1, \ldots, m.$$

Similarity, the separation from the negative ideal solution is given as:

$$d_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{\frac{1}{2}}, \quad i = 1, \ldots, m.$$

5. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^+ is defined as:

$$R_i = \frac{d_i^-}{(d_i^+ + d_i^-)}, \quad i = 1, \ldots, m.$$

Since $d_i^- \geq 0$ and $d_i^+ \geq 0$, then, clearly, $R_i \in [0, 1]$.

6. Rank the preference order, for ranking alternatives using this index; we can rank alternatives in decreasing order.

The basic principle of the TOPSIS method is that the chosen alternative should have the "shortest distance" from the positive ideal solution and "farthest distance" from the negative ideal solution. The TOPSIS method introduces two "reference" points, but it does not consider the relative importance of the distances from these points.

4.1. *Application of the Proposed Model for Estimating the Market Share of Tourist Destination in Oman*

While the proposed model is equally applicable to any conventional retail applications (such as locating a retail branch of a supermarket), the application of model is illustrated by a problem in the tourist sector. This choice of tourism problem is motivated by the need for an appropriate method for choosing the right tourist destination in the national context and a variety of factors can be incorporated when applied to the tourism problem to highlight the benefits of the propose model.

Only the details that are relevant for applying the proposed model are provided here. No elaboration is provided for the rationale behind the choice of locations. Data availability and estimation of data related to population, customer expenditures, etc. these details are discussed in separate report Ramanathan [11].

Tourism plays an important role in contributing to the national income of a nation. Hence, nations all over the world have stressed the need to develop the tourism sector. Tourism is special importance to the Sultanate of Oman because:

A. Oman is endowed with many natural and scenic places that are yet to be developed to utilize their full potential
B. Being one of the countries in the Middle East with relatively modest oil reserves, Oman intends to diversify its economy away from oil-based resources and tourism provides a big opportunity in this regard.

There are several places of tourist interest in Oman and estimating the relative attractiveness of these locations can help in deciding about allocating resources for their development. Capital of Oman and is a popular destination for inland tourists, especially in summer. The average consumer expenditure on tourism is obtained from the results of a study of inbound tourism to Oman for the year 2002 (Tourism Survey, 2003).

Choice of a proper tourist location obviously requires consideration of many factors. The following factors are considered here. Since Oman is situated in a hot region, locations that receive more rainfall and that register the lowest maximum temperature and the lowest maximum humidity are preferred by the domestic tourists. Accordingly, factors that will increase the attractiveness of a tourist location are:

1. number of tourist attractions in a tourist location
2. rainfall
3. availability of accommodation facilities
4. water availability
5. availability of medical facilities.

Factors that will reduce the customer areas

1. Distance of the tourist location from the customer area
2. maximum temperature
3. maximum humidity
4. maximum altitude
5. number of traffic accidents
6. Number of crime in the location.

The performance data of the three locations in terms of these factors and the distances of the tourist locations are given in Table 1.

Table 1. The decision matrix and weights of three alternatives

	Number of attractions	Rainfall (mm)	Accommodation (number of hotels)	Water distribution (million gallon)	Medical and clinics	max temp (°C)	max humidity (%)	max altitude (metres)	Number of accidents	Total number of crimes	Distances from capital (Muscat)
A_1	6	75	11	3432.8	46	40	96.2	10	616	351	1023
A_2	5	234.2	1	836.4	48	34.6	100	3009	587	485	221
A_3	4	31	7	1084	56	47.4	98.6	200	1152	653	337

$W = (w_1, \ldots, w_{11}) = (0.1, 0.1, 0.03, 0.03, 0.1, 0.1, 0.2, 0.2, 0.1, 0.02, 0.02)$

Table 2. The normalized decision matrix

	Number of attractions	Rainfall (mm)	Accommodation (number of hotels)	Water distribution (million gallon)	Medical and clinics	max temp (°C)	max humidity (%)	max altitude (metres)	Number of accidents	Total number of crimes	Distances from capital (Muscat)
A_1	0.6841	0.3025	0.8416	0.9288	0.5291	0.5632	0.5651	0.0033	0.4301	0.4062	0.9304
A_2	0.5701	0.9448	0.0765	0.2263	0.5521	0.4871	0.5874	0.9977	0.4098	0.5613	0.2009
A_3	.04561	0.1250	0.5355	0.2933	0.6442	0.6674	0.5792	0.0663	0.8043	0.7210	0.3064

Table 3. The normalized decision matrix

	Number of attractions	Rainfall (mm)	Accommodation (number of hotels)	Water distribution (million gallon)	Medical and clinics	max temp (°C)	max humidity (%)	max altitude (metres)	Number of accidents	Total number of crimes	Distances from capital (Muscat)
A_1	0.06841	0.03025	0.02524	0.02786	0.05291	0.02253	0.11302	0.000662	0.012903	0.008124	0.018608
A_2	0.05701	0.09448	0.00139	0.00678	0.05521	0.01948	0.11748	0.19954	0.012294	0.011226	0.004018
A_3	0.00456	0.0125	0.01606	0.00879	0.06442	0.02670	0.11584	0.01326	0.024129	0.01442	0.006128

We work out a numerical example to illustrate the TOPSIS method for decision-making problem. Suppose that we have three alternatives A_1, A_2 and A_3 among which decision makers have to choose and, also, 11 benefits $C_1, \dots, C_{11}$ are identified as the evaluation attribute for these alternatives.

$$
\begin{aligned}
A^+ &= \{0.06841, 0.09448, 0.02524, 0.02786, 0.06442, 0.02670, 0.11748,\\
&\qquad 0.1995, 0.024129, 0.01442, 0.018608\},\\
A^- &= \{0.004561, 0.0125, 0.001395, 0.006789, 0.05291, 0.01948,\\
&\qquad 0.11302, 0.000662, .012294, 0.008124, 0.004018\},\\
d_1^+ &= \sqrt{0.044013} = 0.209793,\\
d_2^+ &= \sqrt{0.001642} = 0.040529,\\
d_3^+ &= \sqrt{0.46103} = 0.214717,\\
d_1^- &= \sqrt{0.005626} = 0.075012,\\
d_2^- &= \sqrt{0.049058} = 0.022149,\\
d_3^- &= \sqrt{0.000754} = 0.0027468,\\
R_1 &= 0.263380, \quad R_2 = 0.353377, \quad R_3 = 0.0113417.
\end{aligned}
$$

Table 4. Ranking

	R_i $(i = 1, 2, 3)$	Rank
A1	0.263380	2
A2	0.353377	1
A3	0.113417	3

5. **Summary**

Decision making problem is the process of finding the best option from all of the feasible alternatives. In this paper, multiple attribute models for the most preferable choice, technique for order preference by similarity to deal solution (TOPSIS) approach has been dealt with. The data (attributes) are often not so deterministic; the aim of this paper used the TOPSIS method and decision making problem for competitive facility location problem.

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