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Research Article

Eccentric Connectivity Polynomial and Total Eccentricity Polynomial of NA_m^n Nanotube

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Abstract. Let G be a molecular graph with vertex set $V(G)$ and edge set $E(G)$. In chemical graph theory, for a molecular graph we have many invariant polynomials and topological indices. The length of a shortest path between two vertices of G is called distance. In a connected graph G, the *eccentricity* $\epsilon(v)$ of vertex v is the distance between v and a vertex farthest from v in G. In this paper, we consider NA_m^n nanotube and compute eccentric connectivity polynomial and total eccentricity polynomial. Furthermore, we also compute some eccentricity based Zagreb indices of NA_m^n.

Keywords. Eccentric connectivity polynomial; Total eccentricity polynomial; Nanotube

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1. Introduction and Preliminary Results

Mathematical chemistry is a branch of theoretical chemistry in which we discuss and predict the chemical structure by using mathematical tools and does not necessarily refer to the quantum mechanics. *Chemical graph theory* is a branch of mathematical chemistry in which we apply tools of graph theory to model the chemical phenomenon mathematically. This theory plays a prominent role in the fields of chemical sciences. Molecules and molecular compounds are often modeled by molecular graphs. A molecular graph is a graph in which vertices are atoms of a given molecule and edges are its chemical bonds.

A *topological index* is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. In more precise way, a topological index Top(G) of a graph, is a number with the property that for every graph H isomorphic to G, Top(H) = Top(G) [6]. There are some major classes of topological indices such as distance based topological indices, eccentricity based topological indices, degree based topological indices and counting related polynomials and indices of graphs.

Let G be a molecular graph with vertex set $V(G)$ and edge set $E(G)$. The vertices of G denotes atoms and an edge between two vertices denotes the chemical bond between these vertices. If no vertices in $u-v$ walk are repeated then it is called $u-v$ *path* in graph G. The length of a path is the number of edges in it. The distance $d(u,v)$ from vertex u to vertex v is the length of a shortest $u-v$ path in a graph G where $u,v \in G$. In a connected graph G, the *eccentricity* $\epsilon(v)$ of vertex v is the distance between v and a vertex farthest from v in G.

The oldest topological index is *Wiener index* which was introduced by Harold Wiener when he was working on boiling point of paraffin, named this index as *path number*. Later on, the path number was renamed as *Wiener index* defined as half sum of the distances between all the pairs of vertices in a graph [43].

Let G be a graph. Then the Wiener index of G is defined as

$$W(G) = \frac{1}{2}\sum_{(u,v)} d(u,v) \tag{1.1}$$

where (u,v) is any ordered pair of vertices in G and $d(u,v)$ is $u-v$ geodesic. An important eccentricity based topological index of a graph G is the eccentric-connectivity index $\xi(G)$ which was proposed by Sharma, Goswami, and Madan. The eccentric-connectivity index is defined as

$$\xi(G) = \sum_{u\in V(G)} d(u)\epsilon(u). \tag{1.2}$$

where $d(u)$ denotes degree of the vertex u and $\epsilon(u)$ denotes the eccentricity of the vertex u.

The eccentric connectivity polynomial is the polynomial version of the eccentric-connectivity index which was proposed by Alaeiyan, Mojarad and Asadpour. Now we define the eccentric connectivity polynomial of a graph G, which is defined as [1]

$$ECP(G,x) = \sum_{u\in V(G)} d(u)x^{\epsilon(u)}. \tag{1.3}$$

where $d(u)$ denotes degree of the vertex u and $\epsilon(u)$ denotes the eccentricity of the vertex u where value of x is greater than 1. The relationship between eccentric connectivity polynomial and eccentric-connectivity index is given by

$$ECP(G,x) = \xi(G,1)$$

where $\xi(G,1)$ is the first derivative of $ECP(G,x)$ [7, 28, 29]. When the vertex degrees are not taken into account, we obtain the total-eccentricity index of the graph G defined by

$$\varsigma(G) = \sum_{u \in V(G)} \epsilon(u). \tag{1.4}$$

where $\epsilon(u)$ denotes the eccentricity of the vertex u.

The total eccentricity polynomial is the polynomial version of the total-eccentricity index. Now we define the total eccentricity polynomial of a graph G, which is defined as

$$TEP(G,x) = \sum_{u \in V(G)} x^{\epsilon(u)}. \tag{1.5}$$

It is easy to see that the total-eccentricity index can be obtained from the corresponding polynomial by evaluating its first derivative at $x = 1$ [3].

2. NA_m^n Nanotube

We consider the $m \times n$ quadrilateral section P_m^n with $m \geq 2$ hexagons on the top and bottom sides and $n \geq 2$ hexagons on the lateral sides cut from the regular hexagonal lattice L as shown in Figure 1. If we identify two lateral sides of P_m^n such that we identify the vertices u_0^j and u_m^j, for $j = 0,1,2,\ldots,n$ then we obtain the nanotube NA_m^n [6], [13, 14, 17, 30, 31, 37, 40, 44].

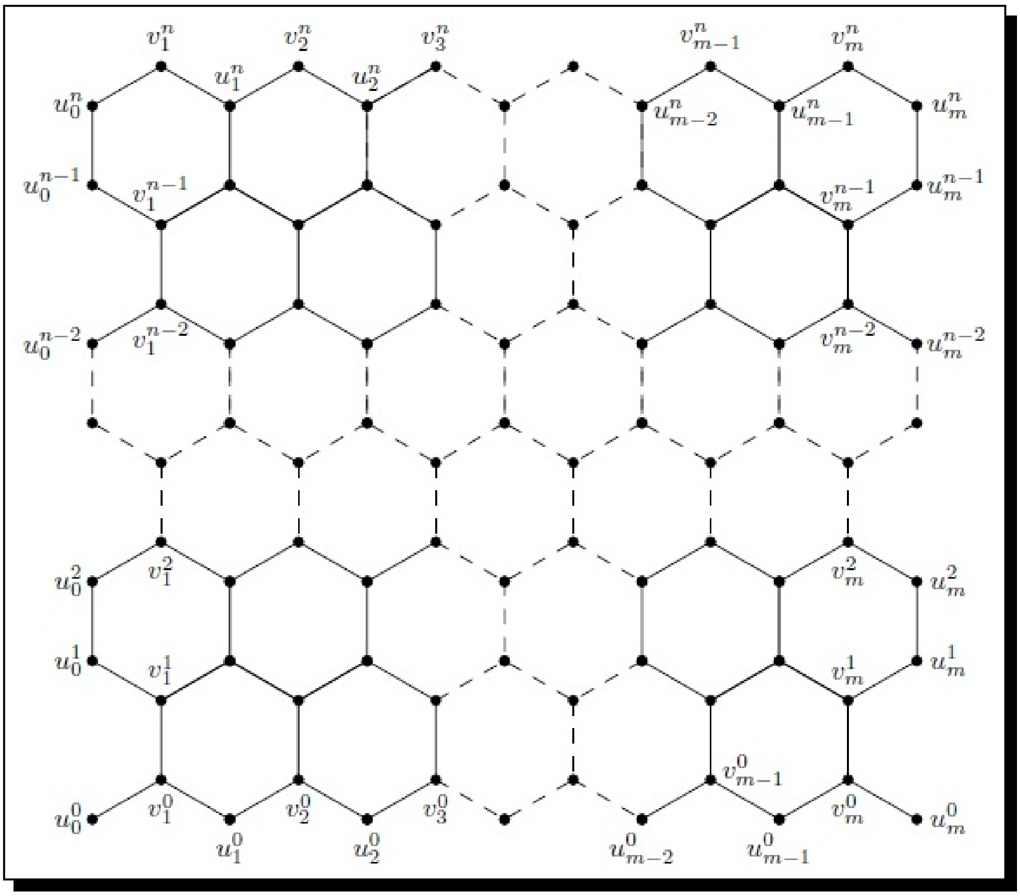

Figure 1. NA_m^n nanotube

In this paper, we consider NA_m^n nanotube with $n = m$ and we compute eccentric connectivity polynomial and total eccentricity polynomial of NA_m^n nanotube. For the eccentric connectivity polynomial and total eccentricity polynomial of NA_m^n nanotube we have two cases of n when $n \equiv 0(\mathrm{mod}\,2)$ and when $n \equiv 1(\mathrm{mod}\,2)$.

3. Results for the eccentric connectivity polynomial of NA_m^n nanotube

Theorem 3.1. *For every $n \equiv 0(\mathrm{mod}\,2)$ consider the graph of $G \cong NA_m^n$ nanotube. Then the eccentric connectivity polynomial is equal to*

$$\begin{aligned}
ECP(G,x) &= 2x^{3n} + 4x^5 + \sum_{m=1}^{\frac{n}{2},n>2} 2tx^{3n-m} + 4x^6 + \sum_{m=\frac{n}{2}}^{n,n>2} 2tx^{2n+m} \\
&+ \sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\mathrm{mod}\,2)} (2tx^{3n-m-k} - 4x^{3n-m-k})\right] \\
&+ \sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)} \left(\frac{3}{2}t^2x^{3n-m-k} - \frac{3}{2}tx^{3n-m-k}\right)\right] \\
&+ \sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)} \left(\frac{3}{2}t^2x^{2n+m-k} - \frac{3}{2}tx^{2n+m-k}\right)\right] + 2x^{2n+1} \\
&+ \sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)} \left(\frac{3}{2}t^2x^{3n-m-k} + 3tx^{3n-m-k}\right)\right] \\
&+ \sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)} \left(\frac{3}{2}t^2x^{2n+m-k} - 3tx^{2n+m-k}\right)\right] \\
&+ \sum_{m=n}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)} 2tx^{2n+m-k}\right].
\end{aligned}$$

Proof. Let G be the graph of NA_m^n nanotube. The formula for the eccentric connectivity polynomial is equal to

$$ECP(G,x) = \sum_{u\in V(G)} d(u)x^{\epsilon(u)}$$

By using the values from Table 1, we get

$$\begin{aligned}
ECP(G,x) &= 1\times 2\times x^{3n} + 2\times 2\times x^5 + 2\times t\times \sum_{m=1}^{\frac{n}{2},n>2} x^{3n-m} + 2\times 2\times x^6 + 2\times t\times \sum_{m=\frac{n}{2}}^{n,n>2} x^{2n+m} \\
&+ 2\times(t-2)\times \sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\mathrm{mod}\,2)} x^{3n-m-k}\right] \\
&+ 3\times\left(\frac{t^2-t}{2}\right)\times \sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)} x^{3n-m-k}\right]
\end{aligned}$$

$$+3\times\left(\frac{t^2-t}{2}\right)\times\sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)}x^{2n+m-k}\right]+2\times 1\times x^{2n+1}$$

$$+3\times\left(\frac{t^2+2t}{2}\right)\times\sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}x^{3n-m-k}\right]$$

$$+3\times\left(\frac{t^2-2t}{2}\right)\times\sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}x^{2n+m-k}\right]$$

$$+2\times t\times\sum_{m=n}\left[\sum_{k=1}^{n-1}x^{2n+m-k}\right].$$

After an easy simplification, we get

$$ECP(G,x)=2x^{3n}+4x^5+\sum_{m=1}^{\frac{n}{2},n>2}2tx^{3n-m}+4x^6+\sum_{m=\frac{n}{2}}^{n,n>2}2tx^{2n+m}$$

$$+\sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\mathrm{mod}\,2)}(2tx^{3n-m-k}-4x^{3n-m-k})\right]$$

$$+\sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}-\frac{3}{2}tx^{3n-m-k}\right)\right]$$

$$+\sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}-\frac{3}{2}tx^{2n+m-k}\right)\right]$$

$$+2x^{2n+1}+\sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}+3tx^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}-3tx^{2n+m-k}\right)\right]$$

$$+\sum_{m=n}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}2tx^{2n+m-k}\right].\qquad\square$$

Theorem 3.2. *For every $n\equiv 1(\mathrm{mod}\,2)$ consider the graph of $G\cong NA_m^n$ nanotube. Then the eccentric connectivity polynomial is equal to*

$$ECP(G,x)=4x^{3n}+\sum_{m=1}^{\frac{n-1}{2}}(2tx^{3n-m}-2x^{3n-m})+\sum_{m=\frac{n+1}{2}}^{n-1}(2tx^{2n+m}-2x^{2n+m})$$

$$+\sum_{m=0}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}(2tx^{3n-m-k}-2x^{3n-m-k})\right]$$

$$+\sum_{m=n}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}(2tx^{2n+m-k}-2x^{2n+m-k})\right]$$

$$+\sum_{m=1}^{\frac{n-1}{2}}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}-3tx^{3n-m-k}+\frac{3}{2}x^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n-1}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}-3tx^{2n+m-k}+\frac{3}{2}x^{2n+m-k}\right)\right]$$

$$+\sum_{m=0}^{\frac{n-1}{2}}\left[\sum_{k=1}^{n,k\equiv 1(\text{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}+\frac{3}{2}tx^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n}\left[\sum_{k=1}^{n,k\equiv 1(\text{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}+\frac{3}{2}tx^{2n+m-k}\right)\right].$$

Proof. Let G be the graph of NA_m^n nanotube. The formula for the eccentric connectivity polynomial is equal to,

$$ECP(G,x)=\sum_{u\in V(G)}d(u)x^{\epsilon(u)}$$

By using the values from Table 2, we get

$$ECP(G,x)=1\times 4\times x^{3n}+2\times\left(\frac{2t-2}{2}\right)\times\sum_{m=1}^{\frac{n-1}{2}}x^{3n-m}+2\times\left(\frac{2t-2}{2}\right)\times\sum_{m=\frac{n+1}{2}}^{n-1}x^{2n+m}$$

$$+2\times(t-1)\times\sum_{m=0}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}x^{3n-m-k}\right]$$

$$+2\times(t-1)\times\sum_{m=n}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}x^{2n+m-k}\right]$$

$$+3\times\left(\frac{t^2-2t+1}{2}\right)\times\sum_{m=1}^{\frac{n-1}{2}}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}x^{3n-m-k}\right]$$

$$+3\times\left(\frac{t^2-2t+1}{2}\right)\times\sum_{m=\frac{n+1}{2}}^{n-1}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}x^{2n+m-k}\right]$$

$$+3\times\left(\frac{t^2+t}{2}\right)\times\sum_{m=0}^{\frac{n-1}{2}}\left[\sum_{k=1}^{n,k\equiv 1(\text{mod}\,2)}x^{3n-m-k}\right]$$

$$+3\times\left(\frac{t^2+t}{2}\right)\times\sum_{m=\frac{n+1}{2}}^{n}\left[\sum_{k=1}^{n,k\equiv 1(\text{mod}\,2)}x^{2n+m-k}\right].$$

After an easy simplification, we get

$$ECP(G,x)=4x^{3n}+\sum_{m=1}^{\frac{n-1}{2}}(2tx^{3n-m}-2x^{3n-m})+\sum_{m=\frac{n+1}{2}}^{n-1}(2tx^{2n+m}-2x^{2n+m})$$

$$+\sum_{m=0}\left[\sum_{k=2}^{n-1,k\equiv 0(\text{mod}\,2)}(2tx^{3n-m-k}-2x^{3n-m-k})\right]$$

$$+\sum_{m=n}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}(2tx^{2n+m-k}-2x^{2n+m-k})\right]$$

$$+\sum_{m=1}^{\frac{n-1}{2}}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}-3tx^{3n-m-k}+\frac{3}{2}x^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n-1}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}-3tx^{2n+m-k}+\frac{3}{2}x^{2n+m-k}\right)\right]$$

$$+\sum_{m=0}^{\frac{n-1}{2}}\left[\sum_{k=1}^{n,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{3n-m-k}+\frac{3}{2}tx^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n}\left[\sum_{k=1}^{n,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{3}{2}t^2x^{2n+m-k}+\frac{3}{2}tx^{2n+m-k}\right)\right]. \qquad \square$$

4. Results for the Total Eccentricity Polynomial of NA_m^n Nanotube

Theorem 4.1. *For every $n\equiv 0(\mathrm{mod}\,2)$ consider the graph of $G\cong NA_m^n$ nanotube. Then the total eccentricity polynomial is equal to*

$$TEP(G,x)=2x^{3n}+2x^5+\sum_{m=1}^{\frac{n}{2},n>2}tx^{3n-m}+2x^6+\sum_{m=\frac{n}{2}}^{n,n>2}tx^{2n+m}$$

$$+\sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\mathrm{mod}\,2)}(tx^{3n-m-k}-2x^{3n-m-k})\right]$$

$$+\sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{3n-m-k}-\frac{1}{2}tx^{3n-m-k}\right)\right]$$

$$+\sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{2n+m-k}-\frac{1}{2}tx^{2n+m-k}\right)\right]+x^{2n+1}$$

$$+\sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{3n-m-k}+tx^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{2n+m-k}-tx^{2n+m-k}\right)\right]$$

$$+\sum_{m=n}\left[\sum_{k=1}^{n-1,k\equiv 1(\mathrm{mod}\,2)}tx^{2n+m-k}\right].$$

Proof. Let G be the graph of NA_m^n nanotube. The formula for the total eccentricity polynomial is equal to

$$TEP(G,x)=\sum_{u\in V(G)}x^{\epsilon(u)}$$

By using the values from Table 1, we get

$$TEP(G,x) = 2\times x^{3n} + 2\times x^5 + t\times \sum_{m=1}^{\frac{n}{2},n>2} x^{3n-m} + 2\times x^6 + t\times \sum_{m=\frac{n}{2}}^{n,n>2} x^{2n+m}$$
$$+(t-2)\times \sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\text{mod }2)} x^{3n-m-k}\right]$$
$$+\left(\frac{t^2-t}{2}\right)\times \sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\text{mod }2)} x^{3n-m-k}\right]$$
$$+\left(\frac{t^2-t}{2}\right)\times \sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\text{mod }2)} x^{2n+m-k}\right] + 1\times x^{2n+1}$$
$$+\left(\frac{t^2+2t}{2}\right)\times \sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} x^{3n-m-k}\right]$$
$$+\left(\frac{t^2-2t}{2}\right)\times \sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} x^{2n+m-k}\right]$$
$$+t\times \sum_{m=n}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} x^{2n+m-k}\right].$$

After an easy simplification, we get

$$TEP(G,x) = 2x^{3n} + 2x^5 + \sum_{m=1}^{\frac{n}{2},n>2} tx^{3n-m} + 2x^6 + \sum_{m=\frac{n}{2}}^{n,n>2} tx^{2n+m}$$
$$+\sum_{m=0}\left[\sum_{k=2}^{n-2,n>2,k\equiv 0(\text{mod }2)} (tx^{3n-m-k} - 2x^{3n-m-k})\right]$$
$$+\sum_{m=1}^{\frac{n}{2}}\left[\sum_{k=2}^{n,k\equiv 0(\text{mod }2)} \left(\frac{1}{2}t^2x^{3n-m-k} - \frac{1}{2}tx^{3n-m-k}\right)\right]$$
$$+\sum_{m>\frac{n}{2}}^{n}\left[\sum_{k=2}^{n,k\equiv 0(\text{mod }2)} \left(\frac{1}{2}t^2x^{2n+m-k} - \frac{1}{2}tx^{2n+m-k}\right)\right] + x^{2n+1}$$
$$+\sum_{m=0}^{\frac{n}{2}}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} \left(\frac{1}{2}t^2x^{3n-m-k} + tx^{3n-m-k}\right)\right]$$
$$+\sum_{m=\frac{n}{2}}^{n-1}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} \left(\frac{1}{2}t^2x^{2n+m-k} - tx^{2n+m-k}\right)\right]$$
$$+\sum_{m=n}\left[\sum_{k=1}^{n-1,k\equiv 1(\text{mod }2)} tx^{2n+m-k}\right].$$

□

Table 1. Vertices partition of NA_m^n nanotube based on degree and eccentricity of each vertex when $n \equiv 0(\text{mod}\,2)$.

Representatives	Degree	Eccentricity	Range	Frequency
$u_i^m = u_{n-i}^m$	1	$3n$	$i = 0$, $m = 0$.	2
$u_i^m = u_{n-i}^m$	2	$3n - m$	$m = 1$ when $n = 2$, $1 \le m \le \frac{n}{2}$ when $n > 2$, $i = 0$.	t, where $t = n$ or $t \equiv 0(\text{mod}\,2)$ and $t \ne 2$.
$u_i^m = u_{n-i}^m$	2	$2n + m$	$m = 2$ when $n = 2$, $\frac{n}{2} \le m \le n$ when $n > 2$, $i = 0$.	t, where $t = n$ or $t \equiv 0(\text{mod}\,2)$ and $t \ne 2$.
$u_i^0 = u_{n-i}^0$	2	$3n - m - k$	$m = 0$, $2 \le k \le n-2$, $n \ne 2$ where $k \equiv 0(\text{mod}\,2)$, $1 \le i \le \frac{n}{2} - 1$, $n \ne 2$.	$t - 2$, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	3	$3n - m - k$	$1 \le m \le \frac{n}{2}$, $1 \le i \le \frac{n}{2}$ for all m, $2 \le k \le n$ where $k \equiv 0(\text{mod}\,2)$ for all m.	$\frac{t^2-t}{2}$, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	3	$2n + m - k$	$\frac{n}{2} \le m \le n$, $1 \le i \le \frac{n}{2}$ for all m, $2 \le k \le n$ where $k \equiv 0(\text{mod}\,2)$ for all m.	$\frac{t^2-t}{2}$, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.
$u_{\frac{n}{2}}^0$	2	$2n + 1$	$m = 0, n \equiv 0(\text{mod}\,2)$	1
$v_i^m = v_{n-i+1}^m$	3	$3n - m - k$	$0 \le m \le \frac{n}{2}$, $1 \le i \le \frac{n}{2}$ for all m, $1 \le k \le n-1$ where $k \equiv 1(\text{mod}\,2)$ for all m.	$\frac{t^2+2t}{2}$, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.
$v_i^m = v_{n-i+1}^m$	3	$2n + m - k$	$\frac{n}{2} \le m \le n-1$, $1 \le i \le \frac{n}{2}$ for all m, $1 \le k \le n-1$ where $k \equiv 1(\text{mod}\,2)$ for all m.	$\frac{t^2-2t}{2}$, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.
$v_i^m = v_{n-i+1}^m$	2	$2n + m - k$	$m = n \equiv 0(\text{mod}\,2)$ $1 \le i \le \frac{n}{2}$ for all m , $1 \le k \le n-1$ where $k \equiv 1(\text{mod}\,2)$ for all m.	t, where $t = n$ or $t \equiv 0(\text{mod}\,2)$.

Table 2. Vertices partition of NA_m^n nanotube based on degree and eccentricity of each vertex when $n \equiv 1(\text{mod}\,2)$.

Representatives	Degree	Eccentricity	Range	Frequency
$u_i^m = u_{n-i}^m$	1	$3n$	$i = 0$ when $m = 0$ and $m = n$	4
$u_i^m = u_{n-i}^m$	2	$3n - m$	$1 \le m \le \frac{n-1}{2}$, $i = 0 for$ all m.	$\frac{2t-2}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	2	$2n + m$	$\frac{n+1}{2} \le m \le n-1$, $i = 0$ for all m.	$\frac{2t-2}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$u_i^0 = u_{n-i}^0$	2	$3n - m - k$	$m = 0$, $1 \le i \le \frac{n-1}{2}$ for all m, $2 \le k \le n-1$ for all m where $k \equiv 0(\text{mod}\,2)$.	$t - 1$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	2	$2n + m - k$	$m = n$, $1 \le i \le \frac{n-1}{2}$ for all m, $2 \le k \le n-1$ for all m where $k \equiv 0(\text{mod}\,2)$.	$t - 1$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	3	$3n - m - k$	$1 \le m \le \frac{n-1}{2}$, $1 \le i \le \frac{n-1}{2}$ for all m, $2 \le k \le n-1$ for all m where $k \equiv 0(\text{mod}\,2)$.	$\frac{t^2-2t+1}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$u_i^m = u_{n-i}^m$	3	$2n + m - k$	$\frac{n+1}{2} \le m \le n-1$, $1 \le i \le \frac{n-1}{2}$ for all m, $2 \le k \le n-1$ for all m where $k \equiv 0(\text{mod}\,2)$.	$\frac{t^2-2t+1}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$v_i^m = v_{n-i+1}^m$	3	$3n - m - k$	$0 \le m \le \frac{n-1}{2}$, $1 \le i \le \frac{n+1}{2}$ for all m, $1 \le k \le n$ for all m where $k \equiv 1(\text{mod}\,2)$.	$\frac{t^2+t}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.
$v_i^m = v_{n-i+1}^m$	3	$2n + m - k$	$\frac{n+1}{2} \le m \le n$, $1 \le i \le \frac{n+1}{2}$ for all m, $1 \le k \le n$ for all m where $k \equiv 1(\text{mod}\,2)$.	$\frac{t^2+t}{2}$, where $t = n$ or $t \equiv 1(\text{mod}\,2)$.

Theorem 4.2. *For every $n \equiv 1 (\bmod 2)$ consider the graph of $G \cong NA_m^n$ nanotube. Then the total eccentricity polynomial is equal to*

$$TEP(G,x) = 4x^{3n} + \sum_{m=1}^{\frac{n-1}{2}} (tx^{3n-m} - x^{3n-m}) + \sum_{m=\frac{n+1}{2}}^{n-1} (tx^{2n+m} - x^{2n+m})$$
$$+ \sum_{m=0} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} (tx^{3n-m-k} - x^{3n-m-k}) \right]$$
$$+ \sum_{m=n} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} (tx^{2n+m-k} - x^{2n+m-k}) \right]$$
$$+ \sum_{m=\frac{n+1}{2}}^{n-1} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} \left(\frac{1}{2} t^2 x^{2n+m-k} - tx^{2n+m-k} + \frac{1}{2} x^{2n+m-k} \right) \right]$$
$$+ \sum_{m=0} \left[\sum_{k=1}^{n, k\equiv 1(\bmod 2)} \left(\frac{1}{2} t^2 x^{3n-m-k} + \frac{1}{2} tx^{3n-m-k} \right) \right]$$
$$+ \sum_{m=\frac{n+1}{2}}^{n} \left[\sum_{k=1}^{n, k\equiv 1(\bmod 2)} \left(\frac{1}{2} t^2 x^{2n+m-k} + \frac{1}{2} tx^{2n+m-k} \right) \right].$$

Proof. Let G be the graph of NA_m^n nanotube. The formula for the total eccentricity polynomial is equal to

$$TEP(G,x) = \sum_{u \in V(G)} x^{\epsilon(u)}$$

By using the values from Table 2 we get

$$TEP(G,x) = 4 \times x^{3n} + \left(\frac{2t-2}{2} \right) \times \sum_{m=1}^{\frac{n-1}{2}} x^{3n-m} + \left(\frac{2t-2}{2} \right) \times \sum_{m=\frac{n+1}{2}}^{n-1} x^{2n+m}$$
$$+ (t-1) \times \sum_{m=0} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} x^{3n-m-k} \right]$$
$$+ (t-1) \times \sum_{m=n} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} x^{2n+m-k} \right]$$
$$+ \left(\frac{t^2-2t+1}{2} \right) \times \sum_{m=1}^{\frac{n-1}{2}} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} x^{3n-m-k} \right]$$
$$+ \left(\frac{t^2-2t+1}{2} \right) \times \sum_{m=\frac{n+1}{2}}^{n-1} \left[\sum_{k=2}^{n-1, k\equiv 0(\bmod 2)} x^{2n+m-k} \right]$$
$$+ \left(\frac{t^2+t}{2} \right) \times \sum_{m=0}^{\frac{n-1}{2}} \left[\sum_{k=1}^{n, k\equiv 1(\bmod 2)} x^{3n-m-k} \right]$$

$$+\left(\frac{t^2+t}{2}\right)\times\sum_{m=\frac{n+1}{2}}^{n}\left[\sum_{k=1}^{n,k\equiv 1(\mathrm{mod}\,2)}x^{2n+m-k}\right].$$

After an easy simplification, we get

$$TEP(G,x)=4x^{3n}+\sum_{m=1}^{\frac{n-1}{2}}(tx^{3n-m}-x^{3n-m})+\sum_{m=\frac{n+1}{2}}^{n-1}(tx^{2n+m}-x^{2n+m})$$

$$+\sum_{m=0}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}(tx^{3n-m-k}-x^{3n-m-k})\right]$$

$$+\sum_{m=n}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}(tx^{2n+m-k}-x^{2n+m-k})\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n-1}\left[\sum_{k=2}^{n-1,k\equiv 0(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{2n+m-k}-tx^{2n+m-k}+\frac{1}{2}x^{2n+m-k}\right)\right]$$

$$+\sum_{m=0}\left[\sum_{k=1}^{n,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{3n-m-k}+\frac{1}{2}tx^{3n-m-k}\right)\right]$$

$$+\sum_{m=\frac{n+1}{2}}^{n}\left[\sum_{k=1}^{n,k\equiv 1(\mathrm{mod}\,2)}\left(\frac{1}{2}t^2x^{2n+m-k}+\frac{1}{2}tx^{2n+m-k}\right)\right].$$

□

5. Conclusion

In this paper, we discuss the Eccentric connectivity polynomial, Total eccentricity polynomial and their relationship with Eccentric connectivity index, Total eccentricity index. We consider the molecular graph of NA_m^n nanotube and we compute Eccentric connectivity polynomial and Total eccentricity polynomial.

Competing Interests

Author declares that he has no competing interests.

Authors' Contributions

Author wrote, read and approved the final manuscript.

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