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Formal Solutions of Certain Dual and Triple Series Involving the Fox H-Functions

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Abstract. It this paper we show how certain classes of dual (triple) series equations involving the Fox H-functions can be reduced to some dual (triple) integral equations involving the Fox H-functions. Also, using the Erdelyi-Kober operators of fractional integration exact solutions of these dual (triple) integral equations are obtained.

1. Introduction

The formal solutions of dual and triple series equations containing the orthogonal polynomials such as Laguerre, Jacobi Polynomials have been obtained by many researchers. The applicable approach to finding the exact solutions od these equations are based on the main property of orthogonality for these polynomials see [1], [5], [6], [14], [19].

In this paper, in general case we consider the dual series equations involving the Fox H-functions as

$$\sum_{i=0}^{\infty} \alpha_i H_{n+p,m+q}^{m,n}\left[ix \,\middle|\, \begin{matrix} (a_j,A_j)_1^{n+p} \\ (b_k,B_k)_1^{m+q} \end{matrix}\right] = \phi(x), \quad 0 < x < x_0, \tag{1.1}$$

$$\sum_{i=0}^{\infty} \alpha_i H_{n+p,m+q}^{m,n}\left[ix \,\middle|\, \begin{matrix} (c_j,A_j)_1^{n+p} \\ (d_k,B_k)_1^{m+q} \end{matrix}\right] = \psi(x), \quad x > x_0, \tag{1.2}$$

where $\phi(x)$, $\psi(x)$ are known functions and try to find the unknown coefficients α_i. In this respect, by converting the introduced dual series equations into the dual integral equations and applying the Erd lyi-Kober operators to the new dual integral equations we get the exact solution of the dual series (1.1), (1.2) via an integral representation involving the Fox H-function. Our strategy in this paper is an extension of the methods of Saxena [11-13] for solving the dual integral equations. As a generalization of the dual series equations in section 3 we state

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a class of the triple series equations and by applying the introduced technique its formal solution is obtained. See also other techniques for solving dual and triple integral equations which may be applied for solving these equations [2], [9], [10], [15], [17], [18].

Before we start to solve the equations (1.1), (1.2) let us state the definition of the Fox H-function and main properties of this function applied in this paper. The Fox H-function is a generalized hypergeometric function defined by means of Mellin-Barens type contour integral as follows

$$H^{m,n}_{n+p,m+q}(z) = H^{m,n}_{n+p,m+q}\left[z \,\middle|\, \begin{array}{c} (a_j,A_j)_1^{n+p} \\ (b_k,B_k)_1^{m+q} \end{array} \right]$$
$$= \frac{1}{2\pi i}\int_L \mathscr{H}^{m,n}_{n+p,m+q}(s) z^s\, ds, \quad z \neq 0 \tag{1.3}$$

where the integrand $\mathscr{H}$ has the form in terms of the Gamma functions

$$\mathscr{H}^{m,n}_{n+p,m+q}(s) = \frac{\prod_{k=1}^{m}\Gamma(b_k - B_k s)\prod_{j=1}^{n}\Gamma(1-a_j+A_j s)}{\prod_{k=m+1}^{q}\Gamma(1-b_k+B_k s)\prod_{j=n+1}^{p}\Gamma(a_j - A_j s)} \tag{1.4}$$

and the orders (m,n,p,q) are non-negative integers such that $1 \leq m \leq q$, $0 \leq n \leq p$, the parameters $A_j > 0$, $B_k > 0$ are positive and a_j, b_k can be arbitrary complex such that the poles of the Gamma function entering the expressions $A(s)$, $B(s)$ are simple poles and do not coincide i.e.

$$A_j(b_k + l) \neq B_k(a_j - l' - 1), \quad l, l' = 0,1,2,\cdots, \quad j = 1,\cdots,n, \; k = 1,\cdots,m.$$

The contour L is chosen such that the poles of $\Gamma(b_k - B_k s)$, $k = 1,\cdots,m$ are separated from the poles of $\Gamma(1-a_j+A_j s)$, $j = 1,\cdots,n$.

Also if we set

$$\Omega = \sum_{j=1}^{n} A_j - \sum_{j=n+1}^{p} A_j + \sum_{k=1}^{m} B_k - \sum_{k=m+1}^{q} B_k$$

the integral (1.3) is absolutely convergent and makes the analytic in the sector $|\arg(z)| < \frac{\pi}{2}\Omega$.

One of the important property of this function applied in next section is the Laplace transform of it [3]

$$\mathscr{L}\left\{ \frac{1}{z} H^{m,n}_{n+p,m+q+1}\left[az^{\sigma} \,\middle|\, \begin{array}{c} (a_j,A_j)_1^{n+p} \\ (b_k,B_k)_1^{m+q},(0,\sigma) \end{array} \right]; s \right\}$$
$$= H^{m,n}_{n+p,m+q}\left[as^{-\sigma} \,\middle|\, \begin{array}{c} (a_j,A_j)_1^{n+p} \\ (b_k,B_k)_1^{m+q} \end{array} \right], \quad a,\sigma \in \mathbb{C}. \tag{1.5}$$

For other properties and details about this functions such as convergency, analytic continuation and their application in applied sciences the reader is referred to [3, 4] and [7, 8], [16].

2. **Dual Series Equations Involving the Fox H-Function**

In this section we find the unknown coefficients α_i, $i = 0, 1, \cdots$ in the dual series (1.1), (1.2). In this regard, by definition of the Laplace transform

$$\mathscr{L}\{H(x); s\} = \int_0^\infty e^{-sx} H(x) dx, \quad s \in \mathbb{C}$$

we can consider the following relation for obtaining the infinite series through the infinite integral with the inversion of the Laplace transform kernel

$$\sum_{i=0}^{\infty} \alpha_i H(ix) = \int_0^\infty \mathscr{L}^{-1}\{H(ix); u\} f(u) du \tag{2.1}$$

where

$$f(u) = \sum_{i=0}^{\infty} \alpha_i e^{-iu}. \tag{2.2}$$

It is obvious that, by finding the function $f(u)$ the coefficients α_i can be easily obtained as the coefficients of power series as follows

$$\alpha_i = \frac{1}{i!} \frac{d^i}{du^i} f\left(\ln\left(\frac{1}{u}\right)\right)\Bigg|_{u=0}, \quad i = 0, 1, 2, \cdots. \tag{2.3}$$

Now, by applying the identity (2.1) for the Laplace transform inversion of the Fox H-function (1.5) we reduce the dual series (1.1), (1.2) into the following dual integral equations

$$\int_0^\infty \frac{1}{u} H^{m,n}_{n+p+1,m+q}\left[\frac{x}{u}\,\middle|\, \begin{matrix} (a_j, A_j), (0, 1) \\ (b_k, B_k) \end{matrix}\right] f(u) du = \phi(x), \quad 0 < x < x_0, \tag{2.4}$$

$$\int_0^\infty \frac{1}{u} H^{m,n}_{n+p+1,m+q}\left[\frac{x}{u}\,\middle|\, \begin{matrix} (c_j, A_j), (0, 1) \\ (d_k, B_k) \end{matrix}\right] f(u) du = \psi(x), \quad x > x_0, \tag{2.5}$$

where the function $f(x)$ should be determined. For the above equations if we use the Mellin transform convolution

$$\int_0^\infty h\left(\frac{x}{u}\right) f(u) \frac{du}{u} = \frac{1}{2\pi i} \int_L \mathscr{H}(s) F(s) x^{-s}\, ds \tag{2.6}$$

$$\mathscr{M}\{h(x)\} = H(s) \text{ and } \mathscr{M}\{f(x)\} = F(s) \tag{2.7}$$

then we can rewrite the dual integral equations in terms of Bromwich integrals as

$$\frac{1}{2\pi i} \int_L \frac{\prod_{k=1}^{m} \Gamma(b_k + B_k s) \prod_{j=1}^{n} \Gamma(a_j - A_j s)}{\prod_{k=1}^{q} \Gamma(b_{k+m} - B_{k+m} s) \prod_{j=1}^{p+1} \Gamma(a_{n+j} + A_{n+j} s)} F(s) x^{-s}\, ds = \phi(x), \quad 0 < x < x_0, \tag{2.8}$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(d_k+B_ks)\prod_{j=1}^{n}\Gamma(c_j-A_js)}{\prod_{k=1}^{q}\Gamma(d_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(c_{n+j}+A_{n+j}s)}F(s)x^{-s}\,ds=\psi(x),\quad x>x_0, \tag{2.9}$$

which for the above equations we write the Mellin transform of the Fox H-functions from the definition (1.3) as the coefficient of z^{-s}

$$\mathscr{M}\left\{H^{m,n}_{n+p+1,m+q}\left[x\left|\begin{array}{c}(a_j,A_j)\\(b_k,B_k)\end{array}\right.\right];s\right\}=\frac{\prod_{k=1}^{m}\Gamma(b_k+B_ks)\prod_{j=1}^{n}\Gamma(a_j-A_js)}{\prod_{k=1}^{q}\Gamma(b_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j}+A_{n+j}s)} \tag{2.10}$$

and use the following notations for convenience in calculations

$$b_k:=b_k,\quad k=1,\cdots,m,\qquad a_j:=1-a_j,\quad j=1,\cdots,n \tag{2.11}$$

$$b_{k+m}=1-b_k,\quad k=1,\cdots,q,\qquad a_{n+j}:=a_j,\quad j=1,\cdots,p+1. \tag{2.12}$$

In this stage by using the Erd lyi-Kober operators we transform the equations (2.8), (2.9) into two others with the same kernel. In this regard we use the Erd lyi-Kober operator $\mathscr{I}$

$$\mathscr{I}(\alpha,\eta;m:f(x))=\frac{m}{\Gamma(\alpha)}x^{-\eta-m\alpha+m-1}\int_0^x t^{\eta}(x^m-t^m)^{\alpha-1}f(t)dt \tag{2.13}$$

to transform $\dfrac{\prod_{j=1}^{n}\Gamma(a_j-A_js)}{\prod_{k=1}^{q}\Gamma(b_{k+m}-B_{k+m}s)}$ into $\dfrac{\prod_{j=1}^{n}\Gamma(c_j-A_js)}{\prod_{k=1}^{q}\Gamma(d_{k+m}-B_{k+m}s)}$ and use the Erd lyi-Kober operator $\mathscr{K}$

$$\mathscr{K}(\alpha,\xi;m:f(x))=\frac{mx^{\xi}}{\Gamma(\alpha)}\int_x^{\infty}t^{-\xi-m\alpha+m-1}(t^m-x^m)^{\alpha-1}f(t)dt \tag{2.14}$$

to transform $\dfrac{\prod_{k=1}^{m}\Gamma(d_k+B_ks)}{\prod_{j=1}^{p+1}\Gamma(c_{n+j}+A_{n+j}s)}$ into $\dfrac{\prod_{k=1}^{m}\Gamma(b_k+B_ks)}{\prod_{j=1}^{p+1}\Gamma(a_{n+j}+A_{n+j}s)}$.

By considering the contracted forms of the Erd lyi-Kober operators in following forms

$$\mathscr{I}_j(\phi(x))=\mathscr{I}\left((a_j-c_j),\frac{c_j}{A_j};\frac{1}{A_j}:\phi(x)\right),\quad j=1,\cdots,n \tag{2.15}$$

$$\mathscr{I}^*_k(\phi(x))=\mathscr{I}^*\left((d_{m+k}-b_{m+k}),\frac{b_{m+k}}{B_{m+k}}-1;\frac{1}{B_{m+k}}:\phi(x)\right),\quad k=1,\cdots,q \tag{2.16}$$

$$\mathscr{K}_l(\psi(x)) = \mathscr{K}\left((d_l - b_l), \frac{b_l}{B_l}; \frac{1}{B_l} : \psi(x)\right), \quad l = 1, \cdots, m \tag{2.17}$$

$$\mathscr{K}_h^*(\psi(x)) = \mathscr{K}^*\left((a_{n+h} - c_{n+h}), \frac{c_{n+h}}{A_{n+h}} - 1; \frac{1}{A_{n+h}} : \psi(x)\right), \;\; h = 1, \cdots, p+1 \tag{2.18}$$

and applying the operators $\mathscr{I}_j$, $\mathscr{I}_k^*$ on the equation (2.8) successively we can find

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k + B_k s)\prod_{j=1}^{n}\Gamma(c_j - A_j s)}{\prod_{k=1}^{q}\Gamma(d_{k+m} - B_{k+m} s)\prod_{j=1}^{p+1}\Gamma(a_{n+j} + A_{n+j} s)} F(s) x^{-s}\, ds$$
$$= \mathscr{I}_1^* \cdots \mathscr{I}_q^* \mathscr{I}_1 \cdots \mathscr{I}_n \phi(x), \quad 0 < x < x_0 . \tag{2.19}$$

In similar manner, by applications of the operators $\mathscr{K}_l$, $\mathscr{K}_h^*$ the equation (2.9) can be transformed to

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k + B_k s)\prod_{j=1}^{n}\Gamma(c_j - A_j s)}{\prod_{k=1}^{q}\Gamma(d_{k+m} - B_{k+m} s)\prod_{j=1}^{p+1}\Gamma(a_{n+j} + A_{n+j} s)} F(s) x^{-s}\, ds$$
$$= \mathscr{K}_1^* \cdots \mathscr{K}_{p+1}^* \mathscr{K}_1 \cdots \mathscr{K}_m \psi(x), \quad x > x_0, \tag{2.20}$$

which for the evaluating the above integrals we use the well known beta function formulas

$$\int_0^x t^{cb-1}(x^c - t^c)^{a-b-1} dt = \frac{\Gamma(a-b)\Gamma(b)}{c\Gamma(a)} x^{ca-c} \tag{2.21}$$

$$\int_x^\infty t^{d-d\lambda-1}(x^d - t^d)^{\lambda-\mu-1} dt = \frac{\Gamma(\lambda-\mu)\Gamma(\mu)}{d\Gamma(\lambda)} x^{-d\mu}. \tag{2.22}$$

Finally by setting

$$c(x) = \begin{cases} \mathscr{I}_1^* \cdots \mathscr{I}_q^* \mathscr{I}_1 \cdots \mathscr{I}_n \phi(x), & 0 < x < x_0, \\ \mathscr{K}_1^* \cdots \mathscr{K}_{p+1}^* \mathscr{K}_1 \cdots \mathscr{K}_m \psi(x), & x > x_0, \end{cases} \tag{2.23}$$

we can reduce the equations (2.8), (2.9) into a compact form with a common kernel

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k + B_k s)\prod_{j=1}^{n}\Gamma(c_j - A_j s)}{\prod_{k=1}^{q}\Gamma(d_{k+m} - B_{k+m} s)\prod_{j=1}^{p+1}\Gamma(a_{n+j} + A_{n+j} s)} F(s) x^{-s}\, ds = c(x),$$

and the required function $f(x)$ can be obtained by the inversion of the H-transform [3, 16] as

$$f(x)=\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{q}\Gamma(d_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j}+A_{n+j}s)}{\prod_{k=1}^{m}\Gamma(b_k+B_ks)\prod_{j=1}^{n}\Gamma(c_j-A_js)}C(s)x^{-s}\,ds,$$

where $C(s)$ is the Mellin transform of the function $c(x)$.

The formal solution $f(x)$ for the dual integral equations (1.1), (1.2) may be rewritten in other form by using the Mellin convolution (2.6) as follows

$$f(x)=\int_0^\infty H^{q,p}_{n+p+1,m+q}\left[\frac{x}{u}\left|\begin{array}{c}(a_{n+j},A_{n+j}),(c_j,A_j)\\(d_{k+m},B_{k+m}),(b_k,B_k)\end{array}\right.\right]c(u)\frac{du}{u},$$

which by implementation of relation (2.3) required coefficients $\alpha_i, i=0,1,2,\cdots$ are finally written as

$$\begin{aligned}\alpha_i=&\frac{1}{i!}\frac{d^i}{dx^i}\Bigg[\int_0^{x_0}H^{q,p}_{n+p+1,m+q}\left[\frac{\ln(\frac{1}{x})}{u}\left|\begin{array}{c}(a_{n+j},A_{n+j}),(c_j,A_j)\\(d_{k+m},B_{k+m}),(b_k,B_k)\end{array}\right.\right]\\&\times\mathscr{I}_1^*\cdots\mathscr{I}_q^*\mathscr{I}_1\cdots\mathscr{I}_n\phi(u)\frac{du}{u},\Bigg]_{x=0}\\&+\frac{1}{i!}\frac{d^i}{dx^i}\Bigg[\int_{x_0}^{\infty}H^{q,p}_{n+p+1,m+q}\left[\frac{\ln(\frac{1}{x})}{u}\left|\begin{array}{c}(a_{n+j},A_{n+j}),(c_j,A_j)\\(d_{k+m},B_{k+m}),(b_k,B_k)\end{array}\right.\right]\\&\times\mathscr{K}_1^*\cdots\mathscr{K}_{p+1}^*\mathscr{K}_1\cdots\mathscr{K}_m\psi(u)\frac{du}{u},\Bigg]_{x=0}.\end{aligned}\tag{2.24}$$

3. **Triple Series Equations involving the Fox H-Function**

As a generalization of dual series equations in previous section we consider the triple eries equations as

$$\sum_{i=0}^{\infty}\alpha_iH^{m,n}_{n+p,m+q}\left[ix\left|\begin{array}{c}(a_j,A_j)_1^{n+p}\\(b_k,B_k)_1^{m+q}\end{array}\right.\right]=\phi(x),\quad 0<x<x_0,\tag{3.1}$$

$$\sum_{i=0}^{\infty}\alpha_iH^{m,n}_{n+p,m+q}\left[ix\left|\begin{array}{c}(c_j,A_j)_1^{n+p}\\(d_k,B_k)_1^{m+q}\end{array}\right.\right]=\psi(x),\quad x_0<x<x_1,\tag{3.2}$$

$$\sum_{i=0}^{\infty}\alpha_iH^{m,n}_{n+p,m+q}\left[ix\left|\begin{array}{c}(e_j,A_j)_1^{n+p}\\(f_k,B_k)_1^{m+q}\end{array}\right.\right]=\xi(x),\quad x>x_1,\tag{3.3}$$

which according the relation (2.1) can be reduced to the triple integral equations in the following forms

$$\int_0^\infty\frac{1}{u}H^{m,n}_{n+p+1,m+q}\left[\frac{x}{u}\left|\begin{array}{c}(a_j,A_j),(0,1)\\(b_k,B_k)\end{array}\right.\right]f(u)du=\phi(x),\quad 0<x<x_0,\tag{3.4}$$

$$\int_0^\infty \frac{1}{u} H^{m,n}_{n+p+1,m+q}\left[\frac{x}{u}\,\middle|\,\begin{matrix}(c_j,A_j),(0,1)\\(d_k,B_k)\end{matrix}\right] f(u)du = \psi(x), \quad x_0 < x < x_1, \tag{3.5}$$

$$\int_0^\infty \frac{1}{u} H^{m,n}_{n+p+1,m+q}\left[\frac{x}{u}\,\middle|\,\begin{matrix}(e_j,A_j),(0,1)\\(f_k,B_k)\end{matrix}\right] f(u)du = \xi(x), \quad x > x_1, \tag{3.6}$$

After Reforming the above equations the with respect to Mellin transform convolution in terms of Bromwich integrals and applying the notations (2.11), (2.12)

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k+B_k s)\prod_{j=1}^{n}\Gamma(a_j-A_j s)}{\prod_{k=1}^{q}\Gamma(b_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j}+A_{n+j}s)} F(s)x^{-s}\,ds = \phi(x), \quad 0 < x < x_0, \tag{3.7}$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(d_k+B_k s)\prod_{j=1}^{n}\Gamma(c_j-A_j s)}{\prod_{k=1}^{q}\Gamma(d_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(c_{n+j}+A_{n+j}s)} F(s)x^{-s}\,ds = \psi(x), \quad x_0 < x < x_1, \tag{3.8}$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(f_k+B_k s)\prod_{j=1}^{n}\Gamma(e_j-A_j s)}{\prod_{k=1}^{q}\Gamma(f_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(e_{n+j}+A_{n+j}s)} F(s)x^{-s}\,ds = \xi(x), \quad x > x_1, \tag{3.9}$$

first and second equations (3.7), (3.8) are transformed into following equation with the same kernel

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k+B_k s)\prod_{j=1}^{n}\Gamma(c_j-A_j s)}{\prod_{k=1}^{q}\Gamma(d_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j}+A_{n+j}s)} F(s)x^{-s}\,ds = c(x), \quad 0<x<x_1 \tag{3.10}$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(f_k+B_k s)\prod_{j=1}^{n}\Gamma(e_j-A_j s)}{\prod_{k=1}^{q}\Gamma(f_{k+m}-B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(e_{n+j}+A_{n+j}s)} F(s)x^{-s}\,ds = \xi(x), \quad x>x_1, \tag{3.11}$$

where the function $c(x)$ is obtained by applying the Erd lyi-Kober operators (2.15)-(2.18) denoted by ${}_1\mathscr{I}_j, {}_1\mathscr{I}^*_k, {}_1\mathscr{K}_l, {}_1\mathscr{K}^*_h$

$$c(x) = \begin{cases} {}_1\mathscr{I}^*_1 \cdots {}_1\mathscr{I}^*_q\, {}_1\mathscr{I}_1 \cdots {}_1\mathscr{I}_n \phi(x), & 0 < x < x_0, \\ {}_1\mathscr{K}^*_1 \cdots {}_1\mathscr{K}^*_{p+1}\, {}_1\mathscr{K}_1 \cdots {}_1\mathscr{K}_m \psi(x), & x_0 < x < x_1. \end{cases} \tag{3.12}$$

Also by applying the contracted forms of the Erd lyi-Kober operators in the following forms again

$$ {}_2\mathscr{I}_j(c(x)) = {}_2\mathscr{I}\left((c_j - e_j), \frac{e_j}{A_j}; \frac{1}{A_j} : c(x)\right), \quad j = 1, \cdots, n \tag{3.13} $$

$$ {}_2\mathscr{I}_k^*(c(x)) = {}_2\mathscr{I}^*\left((f_{m+k} - d_{m+k}), \frac{d_{m+k}}{B_{m+k}} - 1; \frac{1}{B_{m+k}} : c(x)\right), \quad k = 1, \cdots, q \tag{3.14} $$

$$ {}_2\mathscr{K}_l(\xi(x)) = {}_2\mathscr{K}\left((f_l - b_l), \frac{b_l}{B_l}; \frac{1}{B_l} : \xi(x)\right), \quad l = 1, \cdots, m \tag{3.15} $$

$$ {}_2\mathscr{K}_h^*(\xi(x)) = {}_2\mathscr{K}^*\left((a_{n+h} - e_{n+h}), \frac{e_{n+h}}{A_{n+h}} - 1; \frac{1}{A_{n+h}} : \xi(x)\right), \quad h = 1, \cdots, p+1 \tag{3.16} $$

we reduce the equations (3.10), (3.11) into the following equation with the same kernel

$$ \frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{m}\Gamma(b_k + B_k s)\prod_{j=1}^{n}\Gamma(e_j - A_j s)}{\prod_{k=1}^{q}\Gamma(f_{k+m} - B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j} + A_{n+j}s)} F(s)x^{-s}\,ds = s(x) $$

where

$$ s(x) = \begin{cases} {}_2\mathscr{I}_1^*\cdots{}_2\mathscr{I}_q^*\,{}_2\mathscr{I}_1\cdots{}_2\mathscr{I}_n\,{}_1\mathscr{I}_1^*\cdots{}_1\mathscr{I}_q^*\,{}_1\mathscr{I}_1\cdots{}_1\mathscr{I}_n\phi(x) & 0 < x < x_0, \\ {}_2\mathscr{I}_1^*\cdots{}_2\mathscr{I}_q^*\,{}_2\mathscr{I}_1\cdots{}_2\mathscr{I}_n\,{}_1\mathscr{K}_1^*\cdots{}_1\mathscr{K}_{p+1}^*\,{}_1\mathscr{K}_1\cdots{}_1\mathscr{K}_m\psi(x) & x_0 < x < x_1, \\ {}_2\mathscr{K}_1^*\cdots{}_2\mathscr{K}_{p+1}^*\,{}_2\mathscr{K}_1\cdots{}_2\mathscr{K}_m\xi(x) & x > x_1. \end{cases} \tag{3.17} $$

Finally the required function $f(x)$ can be obtained by the inversion of the H-transform as

$$ f(x) = \frac{1}{2\pi i}\int_L \frac{\prod_{k=1}^{q}\Gamma(f_{k+m} - B_{k+m}s)\prod_{j=1}^{p+1}\Gamma(a_{n+j} + A_{n+j}s)}{\prod_{k=1}^{m}\Gamma(b_k + B_k s)\prod_{j=1}^{n}\Gamma(e_j - A_j s)} S(s)x^{-s}\,ds, $$

where $S(s)$ is the Mellin transform of the function $s(x)$.

The formal solution $f(x)$ for the triple integral equations (3.1)-(3.3) may be rewritten in other form by using the Mellin convolution as follows

$$ f(x) = \int_0^\infty H_{n+p+1,m+q}^{q,p}\left[\frac{x}{u}\,\middle|\, \begin{matrix} (a_{n+j}, A_{n+j}), (e_j, A_j) \\ (f_{k+m}, B_{k+m}), (b_k, B_k) \end{matrix}\right] s(u)\frac{du}{u}, $$

and the coefficients α_i, $i = 0, 1, 2, \cdots$ are obtained as

$$ \alpha_i = \frac{1}{i!}\left[\frac{d^i}{dx^i}\int_0^{x_0} H_{n+p+1,m+q}^{q,p}\left(\frac{\ln(\frac{1}{x})}{u}\right) \times {}_2\mathscr{I}_1^*\cdots{}_2\mathscr{I}_q^*\,{}_2\mathscr{I}_1\cdots{}_2\mathscr{I}_n\,{}_1\mathscr{I}_1^*\cdots{}_1\mathscr{I}_q^*\,{}_1\mathscr{I}_1\cdots{}_1\mathscr{I}_n\phi(u)du\right]_{x=0} $$

$$+\frac{1}{i!}\left[\frac{d^i}{dx^i}\int_{x_0}^{x_1} H^{q,p}_{n+p+1,m+q}\left(\frac{\ln(\frac{1}{x})}{u}\right)\right.$$

$$\left.\times\ {}_2\mathscr{I}_1^*\cdots{}_2\mathscr{I}_q^*\,{}_2\mathscr{I}_1\cdots{}_2\mathscr{I}_n\,{}_1\mathscr{K}_1^*\cdots{}_1\mathscr{K}_{p+1}^*\,{}_1\mathscr{K}_1\cdots{}_1\mathscr{K}_m\psi(u)du\right]_{x=0}$$

$$+\frac{1}{i!}\left[\frac{d^i}{dx^i}\int_{x_1}^{\infty} H^{q,p}_{n+p+1,m+q}\left(\frac{\ln(\frac{1}{x})}{u}\right)\right.$$

$$\left.\times\ {}_2\mathscr{K}_1^*\cdots{}_2\mathscr{K}_{p+1}^*\,{}_2\mathscr{K}_1\cdots{}_2\mathscr{K}_m\xi(u)du\right]_{x=0}. \tag{3.18}$$

4. Conclusions

It may be concluded that in this paper finding of the solutions of the two sets of dual and triple series equations involving the Fox H-functions are related to the solutions of dual and triple integral equations reductions involving the Fox H-functions.The Erd lyi-Kober operators are applicable approach to write the formal solutions of these problems.

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