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Research Article

Improved Binary Tree Coding for Image Compression using Modified Singular Value Decomposition

Naveen Kumar R.[1,*], B.N. Jagadale[1] and J.S. Bhat[2]

[1] *Department of Electronics, Kuvempu University, Shimoga, Karnataka, India*
[2] *Department of Physics, Karnatak University, Dharwad, Karnataka, India*
*__Corresponding authors:__ nkr.hsd@gmail.com

Abstract. Reducing the transmission cost while maintaining the quality of image data is the most challenging part in data transmission. In this paper, we report the possibility of improving the quality of image reconstruction by using modified singular value decomposition (SVD) and binary tree coding with adaptive scanning order (BTCA) for grayscale image compression. This method uses modified rank one updated SVD as a pre-processing step for binary tree coding to increase the quality of the reconstructed image. The high energy compaction in SVD process offers high image quality with less compression and is requires more number of bits for reconstruction. BTCA compression, also gives high image quality by coding more significant coefficients using adaptive scanning order from bottom to top with high compression rate. The proposed method uses both SVD and BTC for image compression and is tested with several test images and results are compared with those of SPIHT, JPEG, JPEG2000 and BTCA. The results show significant improvement in PSNR at high bitrates as compared to other methods.

Keywords. Image compression; Modified singular value decomposition; Binary tree coding

MSC. 68P30

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1. Introduction

Digital storage and transmission bandwidth of large-sized image data as obtained in remote sensing and multimedia applications are extremely reliant on image compression techniques [17]. The aim is to represent large size image data with the reduced number of bits in order to speed up the communication. The image compression is classified into lossy and lossless compression, based on the quality of a reconstructed image. If the reconstructed image resembles the original image then it is called as lossless, otherwise lossy compression [22, 24]. The lossy compression schemes have become popular by achieving high compression ratio and transmission rate, by neglecting the subjective redundancy in still images. Hence, it achieves a tradeoff between compression ratio and image quality [11, 12]. Several lossy image compression techniques follow transform based compression method, because they transform the signals into a few highly de-correlated expansions of coefficients, which will reduce the redundancy in image representation, and also increases the compression ratio and quality of reconstructed image [8]. Most popular compression algorithms like JPEG and JPEG2000 introduced by joint photographic experts group uses discrete cosine transform (DCT) and wavelet transforms for image compression [22]. DCT based compression techniques suffer due to the blocking of an artifact, but multiresolution and overlapping nature of wavelet alleviates the blocking artifact and creates superior energy compaction [16]. Lossy compression is popular in many multimedia and remote sensing applications. A lossy image compression approaches, like Embedded coding of an image using zero blocks of wavelet coefficients (EZBC) [9], set partition hierarchical tree (SPIHT) [21], spherical representation (SPHE) [3], hierarchical classification (HIC) [2], use wavelets.

The SVD is a powerful numerical tool widely used in image compression and data hiding. The SVD factorizes a matrix into three component matrices, called left singular vectors, singular values in diagonal, and right singular vectors [5, 6, 15]. During refactorization, some of the singular values are neglected for reconstruction and it results in the compression of the image. Many attempts have been made to hybridize this SVD with many lossy compression methods and yielded significant improvement in image quality [13, 18, 26]. A hybrid compression algorithm proposed by A.M. Rufai [20], using SVD and wavelet difference reduction (WDR) shows some improvement in image quality.

This paper is organized as follows: The proposed lossy compression algorithm is presented in Section 2. The experimental results and comparison with other methods are tabulated and discussed in Section 3, followed by conclusion in Section 4.

2. Proposed Compression Method

2.1 Modified Rank One Updated SVD

An image is a two-dimensional matrix of $m \times n$ pixels, each pixel represents its intensity value. The SVD is applied to the matrix representing the image to get $U\Sigma V^T$, where U and V are the orthonormal matrices of $m \times m$ and $n \times n$, respectively, Σ is a diagonal nonnegative matrix of $m \times n$. The non-zero diagonal elements of Σ determine the rank of the original matrix. Selection of less number of ranks to approximate the original image is required for good compression. Blocks based process for the complete image reduces the ranks and which helps to reconstruct high-quality image [7].

In modified rank one updated SVD, the ranks are reduced by subtracting the median value of original image before performing the SVD and then added after reconstruction. Additional ranks are further reduced by dividing the image into sub-block to make use of the irregular complication of the original image. Appropriate ranks have been selected adaptively for each sub-block by specifying the percentage of the sum of singular values instead of a fixed value [4]. For image I_o, modified rank one updated SVD process is expressed as:

For original Image:

$$I_o - \text{median}(I_o) = U\Sigma V^T, \tag{1}$$

where U is m by n, V is n by n, and $\Sigma = \text{diag}(\tau_1, \tau_2, \dots \tau_k, 0, \dots, 0)$.

$$\text{Specified percent age} = \frac{(\tau_1 + \tau_2 + \tau_3 + \dots + \tau_{k_1})}{(\tau_1 + \tau_2 + \tau_3 + \dots + \tau_k)}, \tag{2}$$

where k_1 is rank for each sub-blocks of image.

Reconstructed Image:

$$I_o^1 = U\Sigma_1 V^T + \text{median}(I_o), \tag{3}$$

where U is m by k_1, V is k_1 by n, and $\Sigma_1 = \text{diag}(\tau_1, \tau_2, \tau_3, \dots, \tau_{k_1})$.

This process compacts the distribution of singular values. When a sub-block contains the complex image information, its singular values are scattered out. Table 1, shows that how the average ranks and the percentage of ranks are used for the 8×8 block of peppers image using the median based rank-one update from 25 to 85 percent of singular value. Perceived values from 25 to 70 percentages of sum of singular values use only one rank for all sub-blocks. This indicates that high compression is achieved without affecting the psycho-visual quality as shown in Figure 1.

Table 1. Percentage of ranks obtained for specified singular values to reconstruct pepper image (8×8 block)

% sum of singular values	Average ranks	Average % of ranks
85	2.238	0.279
70	1.022	0.127
55	1.000	0.125
40	1.000	0.125
25	1.000	0.125

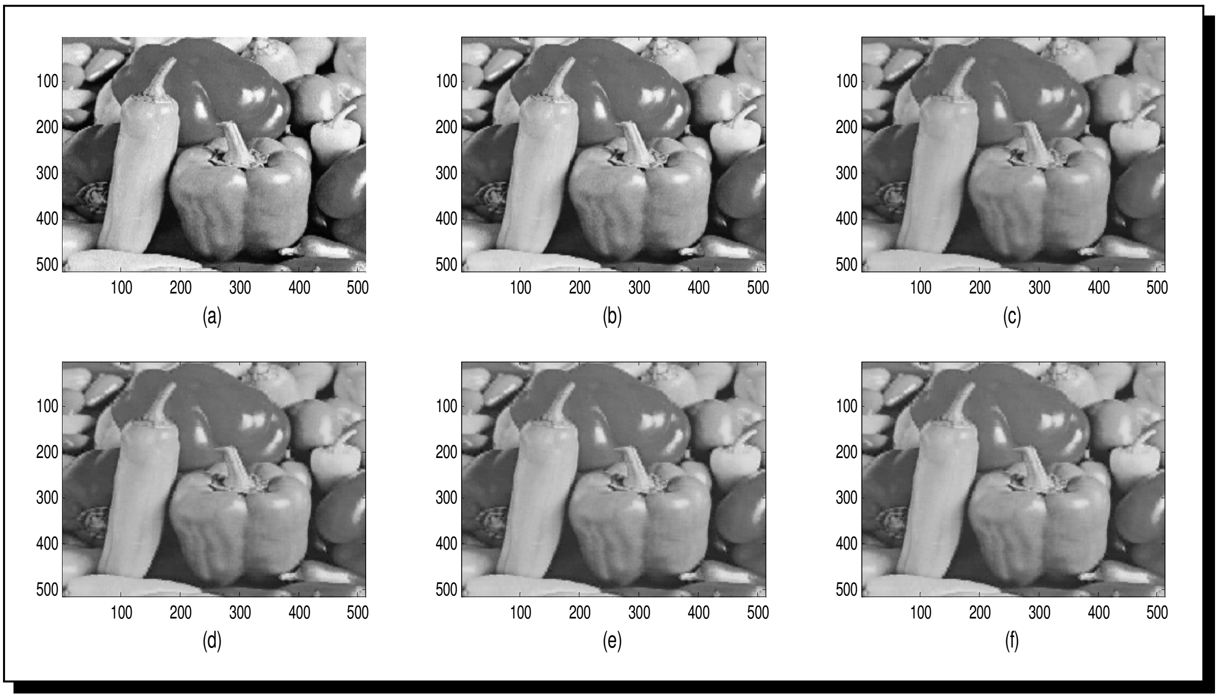

Figure 1. (a) original 'peppers' image of size 512×512 reconstructed with percentages of ranks (b) 85% of ranks, (c) 70% of ranks, (d) 55% of ranks, (e) 40 % of ranks, (f) 25% of ranks

2.2 Binary Tree Coding with Adaptive Scanning Order

The wavelet decomposition distributes the energy of subband into clusters, hence coding of wavelet information becomes more important in image compression. In binary tree coding, the wavelet coefficients are divided into significant and insignificant sub-blocks (i.e. code block) based on the threshold and assign them a binary bit for representations.

In binary tree coding algorithm, we consider a code block 'S' of wavelet image with the size of $2^N \times 2^N$ and is converted into a one-dimensional indexed array using Morton scanning order [27]. Then the binary tree is constructed from bottom to top with node $\lambda(k)$, where $1 \le k \le 2 \times S$. The bottom level of the binary tree consists all wavelet coefficients of Morton scanning order. Upper levels of the tree are defined as follows:

$$\lambda(k) = \max\{\lambda(2k), \lambda(2k+1)\}, \quad \text{for } 1 \le k \le S,$$

where $\lambda(2k)$ and $\lambda(2k+1)$ are the offspring of $\lambda(k)$ and tree depth is $P = N + N + 1$. After construction of binary tree for each code blocks, span the tree by depth, from top to bottom of the sub tree in a bit plane. If tree node is insignificant it is coded with "0" otherwise with "1", and the process is repeated for its two offspring. If the process reaches the bottom level and then corresponding coefficient becomes significant, then its sign is coded. It allows us to concentrate on areas of high energy and also codes the 'zero pixels' compactly. The wavelet coefficients of edges are the treasure of significant coefficients with high magnitude, but they gradually change in natural images. Hence adaptive scanning of this significant coefficient along with it's neighbor are effectively encodes edges and improves the image quality [10]. In this proposed algorithm entropy coding for encoded bit stream is avoided to speed up the execution process.

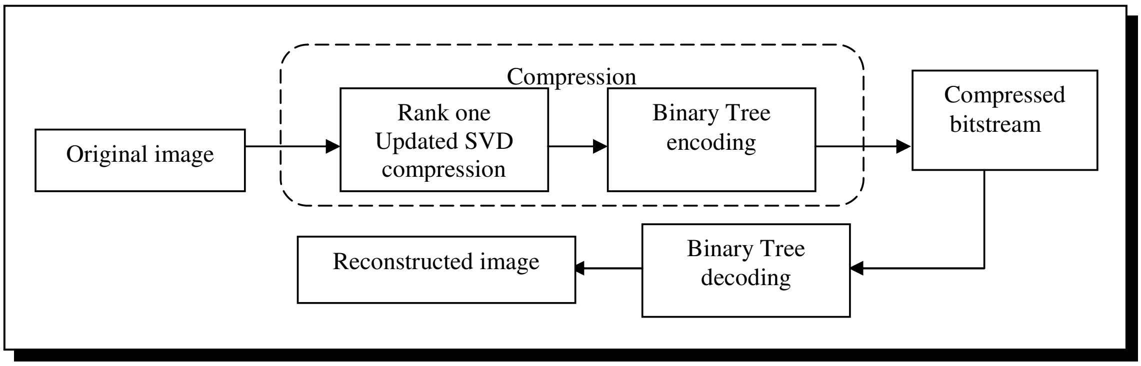

Figure 2. Pipelined structure of proposed image compression method

The detailed steps of the proposed method (Figure 2) described in two parts with functions *Span_MSVD*, *Span_depth* and *Span_level*, where I is 8-bit grayscale image matrix. Ending percentage (EP) is the Specified percentage of ranks used for reconstruction.

The original image is pre-processed by modified rank one updated SVD.

$FunctionCode = span\_MSVD(L, Ep, Block, size)$

- Calculate the median m of I and $I_m = I - m$.
- Divide the I_m into sub-blocks for defined block size 8×8.
- Apply SVD for each block and calculate the average percentage of ranks (PR).
  - if $PR \leq EP$
    - Ranks used for reconstruction.
  - else
    - Neglect the ranks.
- Recombine the blocks into I_m.
- $I_m = I + m$.

After MSVD process reconstructed image is subjected to wavelet decomposition by Cohen-Daubechies-Feauveau (CDF 9/7) tap wavelet filter. The wavelet coefficients are under Morton scanning order gives the indexed array for the binary tree, where k is the index of the node of a binary tree, and T_b is the threshold, $T_0 = 2^{[\log_2 T(1)]}$ and $T_b = \frac{T_0}{2^b}$.

1: $FunctionCode = span\_depth(\lambda, k, T_b)$

- if $\lambda(k)$ coded with significant with the large threshold value, $\lambda(k) \geq T_{k-1}$
  - if $k \leq S$
    - $Jl = Span\_depth(\lambda, 2k, T_b)$
    - $Jr = Span\_depth(\lambda, 2k+1, T_b)$
    - $code = Jl \cup Jr$
  - else
    - $code = \{sign(V(k-s))\}$

- *Else if* $\lambda(k)$ has a significant parent and the neighbors of $\lambda(k)$ has just been coded with insignificant, namely, $k \rhd 1$ and $t \bmod 2 = 1$ $\lambda(k-1) \lhd T_b$,
  - if $k < S$,
    - ◇ $Jl = Span\_depth(\lambda, 2k, T_b)$
    - ◇ $Jr = Span\_depth(\lambda, 2k+1, T_b)$
    - ◇ $code = Jl \cup Jr$
  - *else*
    - ◇ $code = \{sign(V(k-s))\}$
- *Else if* $\lambda(k) \geq T_b$
  - if $k < S$,
    - ◇ $Jl = Span\_depth(\lambda, 2k, T_b)$
    - ◇ $Jr = Span\_depth(\lambda, 2k+1, T_b)$
    - ◇ $code = \{1\} \cup Jl \cup Jr$
  - *else*
    - ◇ $code = \{1\} \cup \{sign(v(k-S)\}$
- *Else*
  - $code = \{0\}$.

2: For adaptive scanning, after spanning the tree by depth with $Span\_depth(\lambda, 1, T_0)$ the function we obtain the previously scanned significant nodes with a threshold $\{T_Z | Z \geq 0\}$. From bottom to top of the tree, find the brother of previously significant nodes. For depth $n = N$, repeat up to $n > 1$

- $Functioncode = Span\_level(T_Z)$

  $For\ K = \sum_{i=1}^{n-1} 2^i + 1\ to\ \sum_{i=0}^{n} 2^i$
- $ck = \{\cdot\}\ if\ \lambda(k) \geq T_{k-1}$,
  - $if\ k \bmod 2 = 0\ and\ \lambda(k+1) < T_{Z-1}$,

    $then\ ck = Span\_depth(\lambda, k+1, T_Z)$;

    $else\ if\ k \bmod 2 = 1\ and\ \lambda(k-1) < T_{Z-1}$,

    $then\ ck = Span\_depth(\lambda, k-1, T_{Z-1})$;
- $code = \{code, ck\}$
- $n = n - 1$.

The above function of binary tree coding is the recursive function for adaptive scanning order. Here we have used the non recursive function to accelerate the process for each bit plane.

3. Experiment Results and Discussion

The proposed compression algorithm was tested on 8-bit grayscale (512 × 512) Barbara, Lena, Goldhill, Cameraman, Jet-plane, Peppers images. Tables 2–7, show, the comparison of the proposed method (decomposed at level 5) with SPIHT, BTC (without entropy coding), JPEG and

JPEG2000 in terms of PSNR for the different bit per pixel (BPP) respectively. The PSNR values of SPIHT, JPEG, and JPEG2000 compression were obtained from [1] and same tested images are used for consistency check. For maximum compression in MSVD, we use 70 to 75 percentage ranks for reconstruction which boosts the image quality for binary tree coding. Hence the PSNR values of SVD+BTC in tables are comparatively higher than BTC. Figure 3 shows the Lena image compressed by proposed technique at different bit rates.

Table 2. Comparison of PSNR values for Barbara image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	24.84	19.20	23.69	24.87	28.20
0.250	27.57	25.63	26.42	28.17	28.20
0.500	31.39	30.54	30.53	31.82	32.42
1.000	36.41	36.58	35.60	36.68	37.76
1.250	39.80	36.58	39.03	39.40	41.56

Table 3. Comparison of PSNR values for Lena image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	31.10	26.14	28.45	30.93	29.70
0.250	34.13	29.56	31.90	34.03	33.10
0.500	37.27	35.14	35.51	37.16	38.48
1.000	40.45	40.85	38.78	40.36	43.34
1.250	42.00	42.13	41.45	42.00	43.34

Table 4. Comparison of PSNR values for Goldhill image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	28.47	23.50	27.25	28.48	26.76
0.250	30.55	28.50	29.47	30.58	31.83
0.500	33.12	32.90	32.12	33.27	36.87
1.000	36.54	32.94	35.57	36.81	36.87
1.250	39.60	39.10	40.12	40.45	43.00

Table 5. Comparison of PSNR values for Cameramen image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	25.82	28.23	24.88	25.57	32.34
0.250	29.12	31.16	28.20	29.30	35.27
0.500	33.00	32.89	32.11	33.28	37.71
1.000	37.96	33.82	36.29	38.08	38.82
1.250	39.85	33.82	39.42	39.75	39.72

Table 6. Comparison of PSNR values for Jet plane image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	27.27	27.37	26.05	27.23	32.67
0.250	29.89	30.57	28.83	29.79	36.08
0.500	33.54	33.80	32.47	33.54	39.42
1.000	38.24	37.66	37.11	38.30	44.02
1.250	38.96	37.59	39.02	40.42	44.04

Table 7. Comparison of PSNR values for pepper image at different bit per pixel (BPP)

BPP	SPIHT	BTC	JPEG	JPEG2000	SVD+BTC
0.125	34.24	28.06	29.45	33.83	31.26
0.250	35.44	31.35	31.58	36.03	34.95
0.500	38.86	35.57	35.83	39.96	39.33
1.000	41.45	40.78	38.75	42.36	43.90
1.250	42.15	40.98	39.95	42.87	43.91

Figure 3. (a) Original Lena uncompressed image (b) Compressed at 0.125 BPP; (c) Compressed at 0.250 BPP; (d) Compressed at 0.500 BPP; (e) Compressed at 1.00 BPP; (f) Compressed at 1.250 BPP using proposed method

4. Conclusion

This paper shows, the improvement of image quality in binary tree coding method by using modified singular value decomposition. The proposed method uses optimum percentage sum of singular values for reconstruction in SVD and adaptive scanning of prior significant pixels during reconstruction in BTC process, which leads to improved PSNR in compressed images. However, in some images like Lena, Goldhill, and peppers, the PSNR values at low BPP is lesser than the JPEG2000 because of a large number of edges. A number of edges in the image lead to the use of more number of significant bits and hence PSNR gets tainted. In general, the proposed algorithm shows significant improvement in PSNR of images at higher bit per pixel as compared to other methods.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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