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Some Properties for M-Homomorphism and M-Anti Homomorphism over Q-Fuzzy M-HX Subgroups and its Level

Research Article

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Abstract. In this paper, we define a new algebraic structure of a Q-fuzzy M-HX subgroup and the M-homomorphism and M-anti homomorphism of a Q-fuzzy M-HX subgroups. We discussed some properties in this subject such that the image and pre image of a Q-fuzzy M-HX subgroup also characterizations of level subset of a Q-fuzzy M-HX subgroups of a M-HX group are given.

Keywords. Fuzzy set; Fuzzy group; Q-fuzzy set; Q-fuzzy group; Q-fuzzy M-group; HX group; Q-fuzzy M-HX group; Level subset HX group; M-Homomorphism; M-anti homomorphism

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1. Introduction

Zadeh's classical paper [12] of 1965 introduced the concepts of fuzzy sets. The study of the fuzzy algebraic structures started with the introduction of the concepts of fuzzy subgroups and fuzzy (right, left) ideals in the pioneering paper of Rosenfeld [6]. Choudhury *et al*. [1] defined a fuzzy subgroups and fuzzy homomorphism. Solairaju and Nagarajan [5, 7, 8, 9] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. Subramanian *et al*. [11] defined and discussed some properties about M-fuzzy groups. On the other hand Solairaju *et al*. [10] introduced the new structures of Q-fuzzy M-subgroups of near rings. Li [2] introduced the concept of HX group and the authors Luochengzhong *et al*. [3] introduced the concept of fuzzy HX group. Muthuraj *et al*. [4] discuss the concept of anti Q-fuzzy HX group. In this paper we define a new algebraic structure of a Q-fuzzy M-HX subgroup and level subset of a Q-fuzzy M-HX subgroups and discussed some of its properties.

2. Preliminaries

Definition 2.1 ([2])**.** In $2^G - \{\phi\}$, a non empty-set $\upsilon \subset 2^G - \{\phi\}$ is called a HX group on G, if υ is a group with respect to algebraic operation defined by $AB = \{ab; a \in A$ and $b \in B\}$, which its unit element is denoted by E.

Definition 2.2 ([12])**.** Let X is a non empty-set. A fuzzy subset λ of X is a function $\lambda : X \to [0,1]$.

Definition 2.3 ([3, 4])**.** A fuzzy set λ is called fuzzy HX subgroup of a HX group υ if for $A, B \in \upsilon$.

(i) $\lambda(AB) \geq \min\{\lambda(A), \lambda(B)\}$.

(ii) $\lambda(A^{-1}) = \lambda(A)$.

Definition 2.4 ([4, 9])**.** *Let Q and υ be any two sets. A mapping $\lambda : \upsilon \times Q \to [0,1]$ is called a Q-fuzzy set in υ.*

Definition 2.5 ([4, 9])**.** A Q-fuzzy set λ is called a Q-fuzzy HX subgroup of a HX group υ if for $A, B \in \upsilon$ and $q \in Q$.

(i) $\lambda(AB, q) \geq \min\{\lambda(A, q), \lambda(B, q)\}$.

(ii) $\lambda(A^{-1}, q) = \lambda(A, q)$.

Definition 2.6 ([4])**.** Let υ be a HX group. A Q-fuzzy HX subgroup λ of υ is said to be normal if for all $A, B \in \upsilon$ and $q \in Q$, $\lambda(ABA^{-1}, q) = \lambda(B, q)$ or $\lambda(AB, q) = \lambda(BA, q)$.

Definition 2.7. A HX group with operators is an algebraic system consisting of a HX group υ, a set M and a function defined in the product set $M \times \upsilon$ and having values in υ such that, if mAB denotes the element in υ determined by element AB of υ and the element m of M, then υ is called M-HX group with operators.

A subgroup U of a M-HX group υ is said to be an M-HX subgroup if $mA \in H$ for all $m \in M$ and $A \in H$.

Definition 2.8. Let G be a M-group, a fuzzy subset λ of G is said to be fuzzy M-subgroup of G if its satisfies the following axioms.

(i) $\lambda(mxy) \geq \min\{\lambda(mx), \lambda(my)\}$.

(ii) $\lambda(mx^{-1}) = \lambda(mx)$, for all $x, y \in G$ and $m \in M$.

Definition 2.9. A Q-fuzzy set λ is called a Q-fuzzy M-subgroup of a M-group G if for $x, y \in G$, $q \in Q$ and $m \in M$.

(i) $\lambda(m(xy), q) \geq \min\{\lambda(mx, q), \lambda(my, q)\}$.

(ii) $\lambda(mx^{-1},q) = \lambda(mx,q)$.

Definition 2.10. A Q-fuzzy set λ is called a Q-fuzzy M-HX subgroup of a M-HX group υ if for $A,B \in \upsilon$, $q \in Q$ and $m \in M$.

(i) $\lambda(m(AB),q) \geq \min\{\lambda(mA,q),\lambda(mB,q)\}$.

(ii) $\lambda(mA^{-1},q) = \lambda(mA,q)$.

Definition 2.11. Let υ be a M-HX group. A Q-fuzzy M-HX subgroup λ of υ is said to be normal if for all $A,B \in \upsilon$, $m \in M$ and $q \in Q$, $\lambda(m(ABA^{-1}),q) = \lambda(mB,q)$ or $\lambda(m(AB),q) = \lambda(m(BA),q)$.

3. Q-fuzzy M-HX groups under M-homomorphism and M-anti homomorphism

Definition 3.1. Let υ_1 and υ_2 be any two M-HX groups, then the function $\varphi : \upsilon_1 \to \upsilon_2$ is said to be an M-homomorphism if

(i) $\varphi(AB) = \varphi(A)\cdot\varphi(B)$.

(ii) $\varphi(mA) = m\cdot\varphi(A)$ for all $A,B \in \upsilon_1$ and $m \in M$.

Definition 3.2. Let υ_1 and υ_2 be any two M-HX groups (not necessarily commutative) then the function $\varphi : \upsilon_1 \to \upsilon_2$ is said to be an M-anti homomorphism if

(i) $\varphi(AB) = \varphi(B)\cdot\varphi(A)$.

(ii) $\varphi(mA) = m\cdot\varphi(A)$ for all $A,B \in \upsilon_1$ and $m \in M$.

Theorem 3.3. *Let φ be a M-homomorphism form a M-HX group υ_1 onto a M-HX group υ_2. If λ is an Q-fuzzy M-HX subgroup of υ_1 and λ is φ-invariant, then $\varphi(\lambda)$ "the image of λ under φ" is a Q-fuzzy M-HX subgroup of υ_2*

Proof. Let $\alpha \in$ image $\varphi(\lambda)$ for some $B \in \upsilon_2, \varphi(\lambda)(mB,q) = \sup_{(mA,q)\in\varphi^{-1}(mB,q)} \lambda(mA,q) = \alpha$.

Such that $\alpha \leq \lambda(mE,q)$. Clearly λ_α is an M-HX subgroup of υ_1, if $\alpha = 1$ then $(\varphi(\lambda))_\alpha = \upsilon_2$. If $0 < \alpha < 1$ then $(\varphi(\lambda))_\alpha = \varphi(\lambda_\alpha)$ since $(mC,q) \in (\varphi(\lambda_))_\alpha$ iff

$$\sup_{(mA,q)\in\varphi^{-1}(mC,q)} \lambda(mQ,q) \geq \alpha \quad \varphi(\lambda)(mC,q) \leq q \quad \text{if and only}$$

$(0 < \alpha < 1)$ iff there exist $A \in \upsilon_1$ such that $\varphi(\lambda)(mA,q) = (mC,q)$ and $\lambda(mA,q) \geq \alpha$ iff $(mC,q) \in \varphi(\lambda_\alpha)$ therefore $(\varphi(\lambda))_\alpha = \varphi(\lambda_\alpha)$. Since φ is an M-homomorphism, $\varphi(\lambda_\alpha)$ is a M-HX subgroup of υ_2 hence $(\varphi(\lambda))_\alpha$is a M-HX subgroup of υ_2. Then $\varphi(\lambda)$ is a Q-fuzzy M-HX subgroup of υ_2. □

Corollary 3.4. *Let φ be a M-anti homomorphism form a M-HX group υ_1 onto a M-HX group υ_2. If λ is an Q-fuzzy M-HX subgroup of υ_1 and λ is φ-invariant, then $\varphi(\lambda)$ "the image of λ under φ" is a Q-fuzzy M-HX subgroup of υ_2.*

Proof. Straight forward. □

Theorem 3.5. *The M-homomorphic pre image of an Q-fuzzy M-HX subgroup of an M-HX group υ_2 is an Q-fuzzy M-HX subgroup of υ_1.*

Proof. Let $\upsilon_1 \to \upsilon_2$ be a M-homomorphism and the fuzzy set U on υ_2 be an Q-fuzzy M-HX subgroup, we need to prove that any Q-fuzzy set λ on υ_1 is an Q-fuzzy M-HX subgroup where $U = \varphi(\lambda)$.

$$\begin{aligned}\lambda(m(AB),q) &= U(\varphi(m(AB),q)\\ &= U(\varphi(mA,q),\varphi(mB,q))\\ &\geq \min\{U(\varphi(mA,q)),U(\varphi(mB,q))\}\\ &= \min\{\lambda(mA,q),\lambda(mB,q)\}.\end{aligned}$$

Thus $\lambda(m(AB),q) \geq \min\{\lambda(mA,q),\lambda(mB,q)\}$.

$$\begin{aligned}\lambda(mA^{-1},q) &= U(\varphi(mA^{-1},q)\\ &= U(\varphi^{-1}(mA,q)\\ &= U(\varphi(mA,q)\\ &= \lambda(mA,q).\end{aligned}$$

Thus $\lambda(mA^{-1},q) = \lambda(mA,q)$.

Then λ is an Q-fuzzy M-HX subgroup of υ_1. □

Corollary 3.6. *The M-anti homomorphic pre image of an Q-fuzzy M-HX subgroup of an M-HX group υ_2 is an Q-fuzzy M-HX subgroup of υ_1.*

Proof. Straight forward. □

4. Level Subsets of a Q-fuzzy M-HX Groups

Theorem 4.1. *Let λ be a Q-fuzzy subset of a M-HX group υ. If λ is an Q-fuzzy M-HX subgroup of υ then the level subset λ_t, $t \in \mathrm{Im}(\lambda)$ are M-HX subgroup of υ.*

Proof. Let $t \in \mathrm{Im}(\lambda)$ and $A,B \in \lambda_t$ then $\lambda(mA,q) = t$ and $\lambda(mB,q) = t$, since λ is an Q-fuzzy M-HX subgroup of υ thus $\lambda(m(AB),q) \geq \min\{\lambda(mA,q),\lambda(mB,q)\} \geq t$ then $\lambda(m(AB),q) \geq t$ and hence $m(AB) \in \lambda_t$. Also if $mA \in \lambda_t$ then $\lambda(mA^{-1},q) = \lambda(mA,q) \geq t$ thus $mA^{-1} \in \lambda_t$. Therefore λ_t is an M-HX subgroup of υ. □

Theorem 4.2. *Let λ be a Q-fuzzy subset of a M-HX group υ. If λ is an Q-fuzzy M-HX subgroup of υ, if the level subset λ_t, $t \in \mathrm{Im}(\lambda)$ are M-HX subgroup of υ, then λ is an Q-fuzzy M-HX subgroup of υ.*

Proof. Suppose that the level subsets $\lambda_t t \in \text{Im}(\lambda)$ are M-HX subgroup of υ, if there exist $A_0, B_0 \in \upsilon$ such that $\lambda(m(A_0B_0),q) < \min\{\lambda(mA_0,q),\lambda(mB_0,q)\}$. If $\alpha_0 = \frac{1}{2}\{\lambda(m(A_0B_0),q) + \min\{\{\lambda(mA_0,q),(mB_0,q)\}\}$, we have $(m(A_0B_0),q) < \alpha_0 < \min\{\lambda(mA_0,q),\lambda(mB_0,q)\}$. Thus $mA_0, mB_0 \in \lambda_t 0$ but $m(A_0B_0) \notin \lambda_t 0$ which is contradiction. If $A, B \in \upsilon, m \in M$ and $q \in Q$, $\lambda(m(AB),q) \le \min\{\lambda(mA,q),\lambda(mB,q)\}$, let $\alpha_0 = \frac{1}{2}\{\lambda(m(AB),q) + \min\{\{\lambda(mA,q),\lambda(mB,q)\}\}\}$ then $\lambda(m(AB),q) < \alpha_0 < \min\{\lambda(mA,q)\lambda,(mB,q)\}$ that is for $m \in M$ and $AB \in \lambda_t 0$ but $m(AB) \notin \lambda_t 0$ which is contradiction to $\lambda_t 0$ is a M-HX subgroup of υ. □

Definition 4.3. Let λ be an Q-fuzzy M-HX subgroup of an M-HX group υ. Then the M-HX subgroups λ_t for $t \in [0,1]$ and $\lambda(mE,q) \ge t$ are called level M-HX subgroup of λ.

Theorem 4.4. *The M-homomorphic image of a level M-HX subgroup of an Q-fuzzy M-HX subgroup λ of an M-HX group υ_1 is a level M-HX subgroup of an Q-fuzzy M-HX subgroup $\varphi(\lambda)$ of an M-HX group υ_2 where λ is φ-invariant.*

Proof. Let υ_1, υ_2 be any two M-HX groups and $\varphi : \upsilon_1 \to \upsilon_2$ be an M-homomorphism, let λ be an Q-fuzzy M-HX subgroup of υ_1 since $\varphi(\lambda)$ is an Q-fuzzy M-HX subgroup of υ_2, let λ_α be a level M-HX subgroup of an Q-fuzzy M-HX subgroup λ of υ_1 since φ is an M-homomorphism, $\varphi(\lambda_\alpha)$ is an M-HX subgroup $\varphi(\lambda)$ of υ_2 and $\varphi(\lambda_\alpha) = (\varphi(\lambda))_\alpha$. Hence $(\varphi(\lambda))_\alpha$ is a level M-HX subgroup $\varphi(\lambda)$ of υ_2. □

Corollary 4.5. *The M-anti homomorphic image of a level M-HX subgroup of an Q-fuzzy M-HX subgroup λ of an M-HX group υ_1 is a level M-HX subgroup of an Q-fuzzy M-HX subgroup $\varphi(\lambda)$ of an M-HX group υ_2 where λ is φ-invariant.*

Proof. Straight forward. □

Theorem 4.6. *The M-homomorphic pre image of a level M-HX subgroup of an Q-fuzzy M-HX subgroup U of an M-HX group υ_2 is a level M-HX subgroup of an Q-fuzzy M-HX subgroup $\varphi^{-1}(U)$ of an M-HX group υ_1.*

Proof. Let $\varphi : \upsilon_1 \to \upsilon_2$ be an M-homomorphism and U be an Q-fuzzy M-HX subgroup of υ_2. Since $\varphi^{-1}(U)$ is a Q-fuzzy M-HX subgroup of υ_1 let U_t be a level M-HX subgroup of an Q-fuzzy M-HX subgroup U of $\upsilon_2\varphi$, is an M-homomorphism, $\varphi^{-1}(U_t)$ is an M-HX subgroup of $\varphi^{-1}(U)$ of υ_1 and $\varphi^{-1}(U_t) = (\varphi^{-1}(U))_t$ is an M-HX subgroup of a Q-fuzzy M-HX subgroup $\varphi^{-1}(U)$ of υ_1 that is, $(\varphi^{-1}(U))_t$ is a level M-HX subgroup of a Q-fuzzy M-HX subgroup $\varphi^{-1}(U)$ of υ_1. □

Corollary 4.7. *The M-anti homomorphic pre image of a level M-HX subgroup of an Q-fuzzy M-HX subgroup U of an M-HX group υ_2 is a level M-HX subgroup of an Q-fuzzy M-HX subgroup $\varphi^{-1}(U)$ of an M-HX group υ_1.*

Proof. Straight forward. □

5. Conclusion

In this paper, we have given the notion of a Q-fuzzy M-HX subgroup and the M-homomorphism and M-anti homomorphism of a Q-fuzzy M-HX subgroups. The image, the pre image of a Q-fuzzy M-HX subgroup and the level subset of a Q-fuzzy M-HX subgroups are discussed with respect to the M-homomorphism and M-anti homomorphism. We hope that our results can also be extended to other algebraic field.

Competing Interests

Author declares that he has no competing interests.

Authors' Contributions

Author wrote, read and approved the final manuscript.

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