Journal of Informatics and Mathematical Sciences
Vol. 8, No. 2, pp. 123–127, 2016
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

A Note on McPherson Number of Graphs

Research Article

C. Susanth, Sunny Joseph Kalayathankal and N.K. Sudev*

[1] Department of Mathematics, Research & Development Centre, Bharathiar University, Coimbatore 641046, Tamilnadu, India
[2] Department of Mathematics, Kuriakose Elias College, Mannanam, Kottayam 686561, Kerala
[3] Department of Mathematics, Vidya Academy of Science and Technology, Thrissur 680501, Kerala, India
***Corresponding author:** sudevnk@gmail.com

Abstract. By the explosion of a vertex v in a graph G, we mean drawing edges from v to all other vertices in G that are not already adjacent to it. The recursive concept of vertex explosions is called the McPherson recursion. The McPherson number of a graph is the minimum number of vertex explosions required in G so that the resultant graph becomes a complete graph. In this paper, we determine the McPherson number of a given graph.

Keywords. Vertex explosion; McPherson number; McPherson recursion

MSC. 05C75; 05C40

Received: March 13, 2016 **Accepted:** April 19, 2016

1. Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [1, 4, 7]. Unless mentioned otherwise, all graphs considered here are simple, finite and connected.

By the term *explosion* of a vertex v in a graph G, we mean drawing edges from v to all other vertices in G that are not adjacent to it (see [6]). The concept of the *McPherson number* of a simple connected graph G on n vertices, denoted by $\Upsilon(G)$, is introduced in [6] as the minimum number of vertex explosions required in G so that the resultant graph becomes a complete graph.

The recursive concept of vertex explosions is called the *McPherson recursion*. That is, a McPherson recursion is a series of vertex explosions such that on the first iteration a vertex $u \in V(G)$ explodes to arc (directed edges) to all vertices $v \in V(G)$ for which the edge $uv \notin E(G)$, to obtain the mixed graph G'_1. Now, G'_1 is considered on the second iteration and a vertex $w \in V(G'_1) = V(G)$ may explode to arc to all vertices $z \in V(G'_1)$ if edge $wz \notin E(G)$ and arc (w,z) or $(z,w) \notin E(G'_1)$.

The *McPherson number* of a simple connected graph G is the minimum number of iterative vertex explosions say l, to obtain the mixed graph G'_l such that the underlying graph of G'_l denoted G^*_l has $G^*_l \cong K_n$.

An initial study on McPherson number of certain fundamental graph classes has been done in [6]. In particular, this parameter has been determined for empty graphs, paths, cycles, bipartite graphs, n-partite graphs etc. Motivated by this study, in this paper, we determine the McPherson number of an arbitrary graph.

2. McPherson Number of Graphs

In [6], the McPherson number of a particular type of Jaco graph $J_n(1)$ (see [5] for the definition linear Jaco graph) has been determined. This result remained as a counter example for many general expressions for finding the McPherson number of different graph classes and as a result, the problem of finding a closed formula for the McPherson number for a finite and connected arbitrary graph remained open.

However, it transpired that the well-defined vertex labeling imbedded in the definition of linear Jaco graphs could assist in finding a closed formula. Noting that a linear Jaco graph of order n has the property that it has the maximum number say, k of vertices v_i such that for $j > i$, $i \neq n$ vertex v_i is adjacent to v_j. Henceforth, let all graphs G of order n have vertex labeling $v_1, v_2, v_3, \ldots, v_n$ such that maximum number k of such vertices v_i are labeled. The latter is always possible. The following theorem determines the McPherson number of any finite connected graph.

Theorem 2.1. *The McPherson number of any connected graph G on n vertices is $n-k-1$, where k is the maximum cardinality of the vertex set $\{v_i \in V(G) : v_i \sim v_j \ \forall \ j > i\}$, '$\sim$' denotes the adjacency between vertices.*

Proof. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertex set of a graph G such that the vertices of G are labeled in such a way that the vertex v_{n-1} is adjacent to v_n. Let S be the subset of $V(G)$, defined by $S = \{v_i \in V(G) : v_i \sim v_j \ \forall \ j > i\}$ over all vertex labelings} and $k = |S|$. Clearly, S is non-empty as $v_{n-1} \in S$. Since, the graph is loop-free, $v_n \notin S$.

Let $G' = G - S$. Let $u_1, u_2, \ldots, u_r = v_n$ be the vertices of G', where $r = n-k$. Note that all vertices in G' must be exploded. Explode vertices of G' in a sequential manner starting from u_1.

It can be observed that the number of edges generated in the i^{th} explosion is always greater than or equal to the number of new edges generated during the $(i+1)^{\text{th}}$ explosion. Also note that every vertex in G', after its explosion becomes adjacent to all other vertices in G. If the vertex u_j is adjacent to a vertex $w_i \in S$, after the explosion of the vertex u_j, all the vertices $u_1, u_2, \ldots, u_j$ and the vertex w_i attain the degree $n-1$ and we need not explode the vertex w_i. Therefore, we can see that no vertex in S requires vertex explosion in the McPherson recursion.

This process should be repeated until the explosion of all vertices in G' except v_n. Note that no explosion is required for the last vertex v_n, as by the time we reach at v_n, all it preceding vertices have become adjacent to it. Therefore, the number of explosions required to find the McPherson number of G is $|V(G') - \{v_n\}|$. Therefore, $\Upsilon(G) = |V(G')| - 1 = n - k - 1$. □

Illustration 2.2. Consider the Petersen graph with vertex labels as shown in Figure 1.

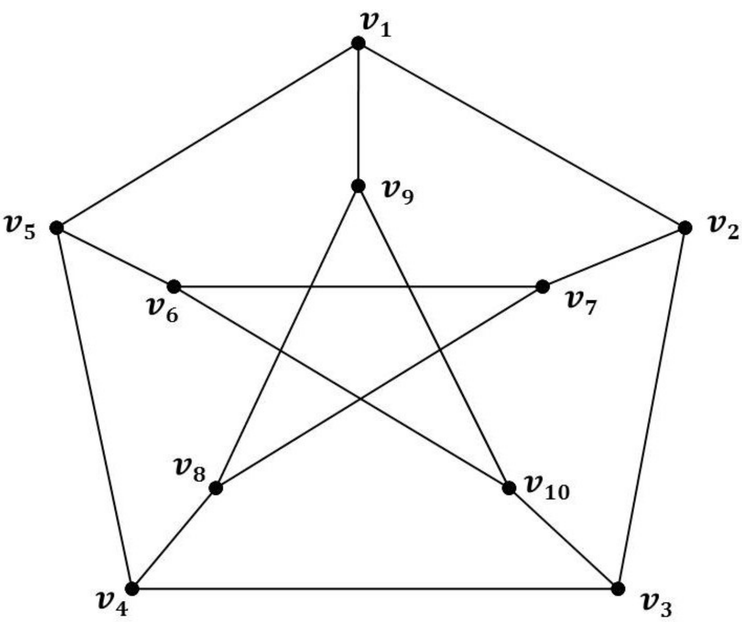

Figure 1

Here, $n = 10$. We also notice that only the vertex v_9 belongs to the set S and hence $k = 1$. Therefore, $n - k - 1 = 8$.

Table 1 represents the vertex explosions made to the above graph in Figure 1. In the table, the (i,j)-th entry represent the iteration number where the vertices v_i and v_j become adjacent. That is, if the (i,j)-th entry is r, where $r < n$ is a positive integer, then the vertices v_i and v_j become adjacent in the r-th explosion. Since the graph K_1 is simple, finite and connected a vertex is inherently adjacent to itself hence, the diagonal elements are all *zero*. Furthermore, because the Peterson graph is 3-regular all columns and rows will have exactly four *zero* entries.

Table 1

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
v_1	0	0	1	1	0	1	1	1	0	1
v_2	0	0	0	2	2	2	0	2	2	2
v_3	1	0	0	0	3	3	3	3	3	0
v_4	1	2	0	0	0	4	4	0	4	4
v_5	0	2	3	0	0	0	5	5	5	5
v_6	1	2	3	4	0	0	0	6	6	0
v_7	1	0	3	4	5	0	0	0	7	7
v_8	1	2	3	0	5	6	0	0	0	8
v_9	0	2	3	4	5	6	7	0	0	0
v_{10}	1	2	0	4	5	0	7	8	0	0

From Table 1, we note that after 8-th vertex explosion, the revised graph becomes a complete graph. Therefore, the McPherson number of the Petersen graph is $8 = n - k - 1$.

3. Conclusion

In this paper, we have determined the McPherson number of a non-empty and non-trivial connected graph. Certain problems in this area are still open.

Describing an efficient algorithm to label the vertices of a graph G of order n such that the maximum number of vertices v_i exist such that v_i is adjacent to all vertices v_j, $j > i$ is a worthy open problem. Such an algorithm will be an alternative to the McPherson algorithm found in [6].

Determining the McPherson number of different graph operations, products and integer powers of graphs and identifying the relation between that of individual graphs, if such a relation exists, promises much for further investigation.

Certain other problems to explore the possibilities of establishing some relations between this parameters and certain other parameters like matching number, chromatic number, independence number etc. are also worthy for future studies.

Acknowledgement

The authors gratefully acknowledge the comments and fruitful advices of Dr. Johan Kok, Director, Licensing Services, Tshwane Metropolitan Police Department, South Africa, who introduced the concept of McPherson number of graphs.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] J.A. Bondy and U.S.R. Murty, *Graph Theory*, Springer, New York (2008).

[2] J. Clark and D.A. Holton, *A First Look at Graph Theory*, Allied Pub., New Delhi (1991).

[3] N. Deo, *Graph Theory with Applications to Engineering and Computer Science*, PHI Learning, Delhi (1974).

[4] F. Harary, *Graph Theory*, Addison Wesley Pub., Reading, Massachusetts (1969).

[5] J. Kok, C. Susanth and S.J. Kalayathankal, A study on linear Jaco graphs, *Journal of Informatics and Mathematical Sciences* **7** (2) (2015), 69–80.

[6] J. Kok and C. Susanth, Introduction to the McPherson number $\Upsilon(G)$ of a simple connected graph, *Pioneer J. Math. Math. Sci.* **13** (2) (2014), 91–102.

[7] D.B. West, *Introduction to Graph Theory*, Pearson Education Asia, New Delhi (2002).