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# Optimal Control Policy for State Dependent Queueing Model with Service Interruption, Setup and Vacations

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**Abstract.** Present investigation deals with optimal management policy for Markovian queue with server breakdowns, vacations and setup. The customers arrive in Poisson fashion to get the service. The server may breakdown during the service and goes for repair immediately. By applying probability generating function technique queue length distribution is obtain for different states of the server. Further we determine the probability of empty system, expected number of units in the system, total expected cost function etc. Cost analysis has also been done. To validate analytic results, numerical experiment has been performed.

## 1. Introduction

Due to paramount applications in many areas including manufacturing system, production systems, computer processing, communication network, transportation and distribution systems etc., the performance modeling of queueing model with vacation and setup time is more useful in congestion situations to predict quantitatively various operational characteristics of such system. We study queueing system with single removable and non-reliable sever under N-policy

Two phase queueing system with N-policy was considered by Kim and Park (2003). An $M^X/G/1$ queueing system with two phases of heterogeneous service under N-policy was studied by Choudhary and Paul (2004). $M/G/1$ queueing system was considered by Lee and Kim (2006) where the speed of the server depends on the amount of work. Manufacturing lead time in a production has been analyzed by Lee *et al.* (2007) with threshold policy. Chae and Lim (2008) presented a procedure to obtain the joint transform of the length of busy period at the instant busy period ends for $GI/M/c$ queue under N-policy. Using the generating function technique, the system state evolution was analyzed by Wang *et al.* (2009) to determine the joint optimal value of $N$ at a minimum cost. Ke *et*

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*al.* (2010) studied the operating characteristics of a batch arrival queueing system under N policy.

Queueing model with service breakdown is helpful in predicting the performance of various machining system. A queueing model with multiple types of server breakdowns was discussed by Gray *et al.* (2004). Ke (2007) studied the operating characteristics of a $M^X/G/1$ queueing system under vacation policies with startup/closedown times. Single unreliable server in $M^X/G/1$ queueing system with multiple vacations was considered by Wang *et al.* (2007) by assuming that as soon as the system becomes empty the server leaves the system for a vacation. Liu *et al.* (2009) investigated an $M/G/1$ queue with preemptive resume and feedback where an unreliable server leaves the system for N-policy vacation as soon as the system empties.

The performance of a machine is affected by machine failure. This failure can be adjusted by providing spare parts or by repair provisioning. However in many realistic situations some time is needed to initiate service, which is called startup time or setup time. The optimal control of $M/G/1$ queueing system with server vacations, startup and breakdowns was suggested by Ke (2003). Optimal management policy for heterogeneous arrival queueing system with server breakdowns and vacations was provided by Ke and Pearn (2004). Diaz and Moreno (2009) studied a queueing system where the service station operates under an N-policy with early setup.

In this paper we extend the work of Ke and Pearn (2004) for heterogeneous arrival queueing system with server breakdowns, vacations, setup time and machine repair. Firstly we establish the steady state results to obtain probability distributions of the number of units in the system. Secondly we determined the probabilities of empty system and expected number of units in the system. Thirdly we formulate total expected cost for the system and determine the optimal management policy for such type of queueing system.

## 2. **Model Description and Governing Equations**

Consider a single non-reliable removable server Markovian queueing model with breakdown, repair and setup under N-policy. Let the state $i = 0$ represents the state when server is on vacation; $i = 1$, when server is working; $i = 2$, when server is found to be broken down; $i = 3$, when server is under repair. We assume that customer's arrival follows a Poisson process with rates $\lambda_i$ $(i = 0, 1, 2, 3)$ where $0, 1, 2, 3$ denote the arrival rates of customers during the idle, busy, breakdown and repair period, respectively.

The server may serve only one unit at a time and the service rates are exponentially distributed with mean $1/\mu$. Whenever the system is empty the server goes for vacation, If there are at least $N$ units in the system the server will come back and starts service. The duration of vacations is exponentially distributed with mean $1/\theta$. Duration of each vacation is independent of arrival process, the service time and breakdown times. During the service the server may breakdown at any time with Poisson breakdown rate $\alpha$. The setup time to initiate the repair is exponentially distributed with rate $\nu$. When the server fails, it is immediately repaired with repair rate $\beta$. Once the repair of the server is completed, it immediately starts to provide service.

Following probabilities are used through out the paper for formulating the model mathematically:

- $P_0(n)$ The probability of being $n$ customers in the system and server is on vacation.
- $P_1(n)$ The probability of being $n$ customers in the system when server is working
- $P_2(n)$ The probability of being $n$ customers in the system when server is found to be broken down.
- $P_3(n)$ The probability of being $n$ customers in the system when server is under repair.
- $H_i(z)$ The probability generating function of $P_i(n)$, $i = 0, 1, 2, 3$
- $E(N_i)$ Expected number of costumers in the system when the server is in the state $i$, $i = 0, 1, 2, 3$.

Steady state equations governing the model are given as follows:

$$\lambda_0 P_0(0) = \mu P_0(1), \tag{2.1}$$

$$\lambda_0 P_0(n) = \lambda_0 P_0(n-1), \quad 1 \leq n \leq N-1, \tag{2.2}$$

$$(\lambda_0 + \theta) P_0(n) = \lambda_0 P_0(n-1), \quad n \geq N, \tag{2.3}$$

$$(\lambda_1 + \mu + \alpha) P_1(1) = \mu P_1(2) + \beta P_3(1), \tag{2.4}$$

$$(\lambda_1 + \mu + \alpha) P_1(n) = \mu P_1(n+1) + \beta P_3(n) + \lambda_1 P_1(n-1), \quad 2 \leq n \leq N-1, \tag{2.5}$$

$$(\lambda_1 + \mu + \alpha) P_1(n) = \mu P_1(n+1) + \beta P_3(n) + \lambda_1 P_1(n-1) + \theta P_0(0), \quad n \geq N, \tag{2.6}$$

$$(\lambda_2 + \vartheta) P_2(1) = \alpha P_1(1), \tag{2.7}$$

$$(\lambda_2 + \vartheta) P_2(n) = \alpha P_1(n) + \lambda_2 P_2(n-1), \quad n \geq 2, \tag{2.8}$$

$$(\lambda_3 + \beta) P_3(1) = \vartheta P_2(1), \tag{2.9}$$

$$(\lambda_3 + \beta) P_3(n) = \vartheta P_2(n) + \lambda_3 P_3(n-1), \quad n \geq 2. \tag{2.10}$$

## 3. **Probaility Generating Functions**

We define the generating function corresponding the probabilities $P_i(n)$, $i = 0, 1, 2, 3$ as follows:

$$H_0(z) = \sum_{n=0}^{\infty} z^n P_0(n),$$

$$H_i(z) = \sum_{n=1}^{\infty} z^n P_i(n), \quad i = 1,2,3\,.$$

Multiplying equation (2.1) by $z$, equations (2.2) and (2.3) by $z^n\,(n \geq 1)$, and then adding for all possible values of $n$, we get:

$$(3.1) \qquad H_0(z) = \left[\frac{1-z^n}{1-z} + \frac{\lambda_0 z^n}{\lambda_0+\theta+\lambda_0 z}\right] P_0(0)\,.$$

Now multiplying (2.4)-(2.6) by $z^{n+1}(n \geq 1)$ and adding for all possible value of $n$, we obtain:

$$(3.2) \quad (\lambda_1 z^2 - (\lambda_1+\mu+\alpha)z+\mu)H_1(z)+\beta(z)H_3(z) = \lambda_0\left(z - \frac{\theta z^{N+1}}{\lambda_0+\theta-\lambda_0 z}\right)P_0(0)\,.$$

Multiplying (2.7) and (2.8) by $z^n(n \geq 1)$ and then adding, we have:

$$(3.3) \qquad (\lambda_2 z - \lambda_2 - \vartheta)H_2(z) + \alpha H_1(z) = 0\,.$$

Similarly from (2.9) and (2.10), we obtain:

$$(3.4) \qquad (\lambda_3 z - \lambda_3 - \beta)H_3(z) + \vartheta H_2(z) = 0\,.$$

Solving the equations (3.2)-(3.4) for $H_1(z)$, $H_2(z)$ and $H_3(z)$, we find

$$(3.5) \qquad H_1(z) = \frac{\lambda_0(\lambda_2 z-\lambda_2-\vartheta)(\lambda_3 z-\lambda_3-\beta)(z\lambda_0+z\theta-\lambda_0 z^2-\theta z^{N+1})P_0(0)}{\begin{bmatrix}[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_3-\beta)(\lambda_2 z-\lambda_2-\vartheta)+\alpha\beta z\vartheta] \\ \times(\lambda_0+\theta-\lambda_0 z)\end{bmatrix}},$$

$$(3.6) \qquad H_2(z) = \frac{(-\alpha)(\lambda_3 z-\lambda_3-\beta)(z\lambda_0+z\theta-\lambda_0 z^2-\theta z^{N+1})\lambda_0 P_0(0)}{\begin{bmatrix}[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_3-\beta)(\lambda_2 z-\lambda_2-\vartheta)+\alpha\beta z\vartheta] \\ \times(\lambda_0+\theta-\lambda_0 z)\end{bmatrix}},$$

$$(3.7) \qquad H_3(z) = \frac{\lambda_0\alpha\vartheta(z\lambda_0+z\theta-\lambda_0 z^2-\theta z^{N+1})P_0(0)}{\begin{bmatrix}[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_3-\beta)(\lambda_2 z-\lambda_2-\vartheta)+\alpha\beta z\vartheta] \\ \times(\lambda_0+\theta-\lambda_0 z)\end{bmatrix}}.$$

Now $H(z)$ which represents the p.g.f. of the total number of customers in the system, is obtained as

$$(3.8) \qquad H(z) = \sum_{i=0}^{3} H_i(z)\,.$$

We evaluate $H_0(1)$, $H_1(1)$, $H_2(1)$ and $H_3(1)$ by applying L'Hospital's rule in equations (3.1), (3.5), (3.6) and (3.7), as the numerator and denominator are both 0 in these equations. Thus, we obtain

$$(3.9) \qquad H_0(1) = \left[N + \frac{\lambda_0}{\theta}\right] P_0(0)\,,$$

$$H_1(1) = \frac{\beta\vartheta\lambda_0(\lambda_0+\theta N)P_0(0)}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}, \tag{3.10}$$

$$H_2(1) = \frac{\alpha\beta\lambda_0(\lambda_0+\theta N)P_0(0)}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}, \tag{3.11}$$

$$H_3(1) = \frac{\lambda_0\alpha\vartheta(\lambda_0+\theta N)P_0(0)}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}. \tag{3.12}$$

To establish $P_0(0)$, we use the normalizing condition and obtain

$$P_0(0) = \left[N+\frac{\lambda_0}{\theta}+\frac{\lambda_0(\lambda_0+\theta N)(\beta\vartheta+\alpha\beta+\alpha\vartheta)}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}\right]^{-1}. \tag{3.13}$$

## 4. The Expected Queue Length

To determine $E\ (N_i)$, $i = 0,1,2,3$ we use the probability generating function $H_i(Z)$ given in equations (3.9)-(3.12) and obtain

$$E(N_0) = \left\{\frac{N(N-1)}{2}+\frac{\lambda_0(\lambda_0+N\theta)}{\theta^2}\right\}P_0(0), \tag{4.1}$$

$$\begin{aligned} E(N_1) = \Bigg[ & \frac{\lambda_0\beta\vartheta(2\lambda_0+N\theta(N+1))-2(\lambda_0+\theta N)}{2\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \\ & +\frac{\vartheta\beta\lambda_0^2(\lambda_0+N\theta)}{\theta^2(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \\ & -\frac{\begin{pmatrix}\lambda_0\vartheta\beta(\lambda_0+N\theta)\{(\lambda_1-\mu-\alpha)(\lambda_3\vartheta+\beta\lambda_2)\\ +\alpha\lambda_2\lambda_3-\lambda_1\beta\vartheta\}\end{pmatrix}}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)^2}\Bigg]P_0(0), \end{aligned} \tag{4.2}$$

$$\begin{aligned} E(N_2) = \Bigg[ & \frac{\alpha\beta\lambda_0(2\lambda_0+N\theta(N+1))-2\alpha\lambda_0\lambda_3(\lambda_0+N\theta)}{2\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \\ & +\frac{\alpha\beta\lambda_0^2(\lambda_0+N\theta)}{\theta^2(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \\ & -\frac{\begin{pmatrix}\lambda_0\alpha\beta(\lambda_0+N\theta)\{(\lambda_1-\mu-\alpha)(\lambda_3\vartheta+\beta\lambda_2)\\ +\alpha\lambda_2\lambda_3-\lambda_1\beta\vartheta\}\end{pmatrix}}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)^2}\Bigg]P_0(0), \end{aligned} \tag{4.3}$$

$$\begin{aligned} E(N_3) = \Bigg[ & \frac{\alpha\vartheta\lambda_0(2\lambda_0+N\theta(N+1))}{2\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \\ & +\frac{\alpha\vartheta\lambda_0^2(\lambda_0+N\theta)}{\theta^2(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)} \end{aligned} \tag{4.4}$$

$$- \frac{\begin{pmatrix} \lambda_0\alpha\vartheta(\lambda_0+N\theta)\{(\lambda_1-\mu-\alpha)(\lambda_3\vartheta+\beta\lambda_2) \\ +\alpha\lambda_2\lambda_3-\lambda_1\beta\vartheta\} \end{pmatrix}}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)^2}\Bigg] P_0(0).$$

The expected number of customers is given by

$$(4.5)\qquad E(N) = \sum_{i=0}^{3} E(N_i)$$

$$= \Bigg[\frac{N(N-1)}{2} + \frac{\lambda_0(\lambda_0+N\theta)}{\theta^2}$$

$$+ \frac{\begin{pmatrix} \lambda_0(2\lambda_0+N\theta(N+1))(\beta\vartheta+\alpha\beta+\alpha\vartheta) \\ -2\lambda_0(\lambda_0+N\theta)(\lambda_3\vartheta+\lambda_2\beta+\lambda_3\alpha) \end{pmatrix}}{2\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}$$

$$+ \frac{\lambda_0^2(\lambda_0+N\theta)(\alpha\beta+\alpha\vartheta+\beta\vartheta)}{\theta^2(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)}$$

$$- \frac{\begin{pmatrix} \lambda_0(\lambda_0+N\theta)(\alpha\beta+\alpha\vartheta+\beta\vartheta) \\ \times\{(\lambda_1-\mu-\alpha)(\lambda_3\vartheta+\beta\lambda_2)+\alpha\lambda_2\lambda_3-\lambda_1\beta\vartheta\} \end{pmatrix}}{\theta(\mu\beta\vartheta-\lambda_1\beta\vartheta-\alpha\vartheta\lambda_3-\alpha\beta\lambda_2)^2}\Bigg] P_0(0).$$

## 5. **Optimal N-policy and Cost Analysis**

In order to design optimal policy we shall derive some performance indices for different system states, which are defined as follows:

(a) *Idle period* $(I)$: This is the length of time per cycle when server is turned off.

(b) *Busy period* $(B)$: This is the length of time per cycle when server is turned on and is operational.

(c) *Down period* $(D)$: This is the length of time per cycle when server is turned on and found to be broken down.

(d) *Repair period* $(R)$: The duration for which server is found to be under repair is known as repair period.

Let the expected lengths of the idle period, the busy period, the down period and the repair period are denoted by $E[I]$, $E[B]$, $E[D]$ and $E[R]$, respectively. The length of idle period is the sum of $N$ exponential random variables each having mean $1/\lambda$. Thus, the expected length of idle period is given by

$$E[I] = N/\lambda_0 .$$

The expected length of completion period $E[H]$ and the expected length of cycle $E[C]$ is given by

$$E[H] = E[B] + E[D] + E[R],$$
$$E[C] = E[I] + E[B] + E[D] + E[R].$$

From equations (3.9)-(3.12), we obtain the long run fraction of time, for which server is idle, busy, broken down and under repair, respectively as follows:

$$\frac{E[I]}{E[C]} = H_0(1) = \left[N + \frac{\lambda_0}{\theta}\right] P_0(0), \tag{5.1}$$

$$\frac{E[B]}{E[C]} = H_1(1) = \frac{\beta\vartheta\lambda_0(\lambda_0 + \theta N)P_0(0)}{\theta(\mu\beta\vartheta - \lambda_1\beta\vartheta - \alpha\vartheta\lambda_3 - \alpha\beta\lambda_2)}, \tag{5.2}$$

$$\frac{E[D]}{E[C]} = H_2(1) = \frac{\alpha\beta\lambda_0(\lambda_0 + \theta N)P_0(0)}{\theta(\mu\beta\vartheta - \lambda_1\beta\vartheta - \alpha\vartheta\lambda_3 - \alpha\beta\lambda_2)}, \tag{5.3}$$

$$\frac{E[R]}{E[C]} = H_3(1) = \frac{\lambda_0\alpha\vartheta(\lambda_0 + \theta N)P_0(0)}{\theta(\mu\beta\vartheta - \lambda_1\beta\vartheta - \alpha\vartheta\lambda_3 - \alpha\beta\lambda_2)}. \tag{5.4}$$

From equations (5.2) and (5.3) we get $E[C]$ as

$$\frac{1}{E[C]} = \frac{\lambda_0(N\theta + \lambda_0)}{N\theta} P_0(0). \tag{5.5}$$

Now we develop a cost model by considering $N$ as decision variable. Our objective is to determine the optimal value of $N$, say $N^*$ so that the cost function is minimized. Let us denote the cost factors associated with different activities as follows

$C_h$ = holding cost per unit time for each unit present in the system.

$C_s$ = start up cost per unit time.

$C_b$ = cost per unit time for keeping the server working.

$C_d$ = breakdown cost per unit time for broken server.

$C_r$ = repair cost per unit time.

Using the definition of each cost element listed above, the total expected cost function per unit time is given by

$$T_c(N) = C_n L_s + C_s \frac{1}{E[C]} + C_b \frac{E[B]}{E[C]} + C_d \frac{E[D]}{E[C]} + C_r \frac{E[R]}{E[C]}. \tag{5.6}$$

We can establish the optimal value $N^*$, by differentiating (**??**) with respect to $N$ and setting the result to be zero, which gives the minimum cost function, i.e.

$$\frac{\partial T_c(N)}{\partial N} = 0. \tag{5.7}$$

The value of $N$ may not be integer. The best positive integer value of $N$ is one of the integers surrounding $N^*$ which gives a smaller cost $T_c$.

## 6. Sensitivity Analysis

We carry out extensive computations for various combination of system parameter by setting the following cost elements

$C_h = 40$, $C_s = 4000$, $C_b = 1000$, $C_r = 200$, and $C_d = 800$.

The optimal threshold parameter $N^*$ and the corresponding minimum expected cost $TC(N^*)$ are summarized in Table 1-4 for different sets of $(\lambda_0, \lambda_1)$, $(\lambda_1, \lambda_2)$, $(\lambda_2, \lambda_3)$ and $(\lambda_1, \lambda_3)$, respectively by setting other parameters.

Tables 1-7 depict the expected number of customers in the system $E(N)$ for default parameters we have $(l = 0.5, m = 1.5, a = 0.03, n = 0.01, q = 0.15, N = 5, b = 1.5)$ and by varying arrival rate $(\lambda)$, service rate $(\mu)$, breakdown rate $(\alpha)$, threshold value $(N)$ and repair rate $(\beta)$ respectively for the following sets of arrival rates:

*Set* 1. $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda$
*Set* 2. $\lambda_0 = \lambda$, $\lambda_1 = 1.4\lambda$, $\lambda_2 = 0.9\lambda$, $\lambda_3 = 0.7\lambda$
*Set* 3. $\lambda_0 = \lambda$, $\lambda_1 = 1.2\lambda$, $\lambda_2 = \lambda$, $\lambda_3 = 0.8\lambda$

**Table 1.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\lambda_0, \lambda_1)$

| $(\lambda_0, \lambda_1)$ | (0.2,0.2) | (0.2,0.4) | (0.2,0.6) | (0.3,0.4) | (0.5,0.4) | (0.5,0.6) |
|---|---|---|---|---|---|---|
| $N^*$ | 46 | 44 | 41 | 48 | 59 | 57 |
| $T_c(N^*)$ | 1031.91 | 990 | 950 | 1120.72 | 1436.80 | 1401.21 |

**Table 2.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\lambda_1, \lambda_2)$

| $(\lambda_1, \lambda_2)$ | (0.1,0.2) | (0.2,0.2) | (0.3,0.2) | (0.4,0.3) | (0.4,0.5) | (0.4,0.6) |
|---|---|---|---|---|---|---|
| $N^*$ | 48 | 46 | 45 | 44 | 44 | 44 |
| $T_c(N^*)$ | 1053.24 | 1031.90 | 1010.90 | 991.09 | 989.42 | 988.30 |

**Table 3.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\lambda_2, \lambda_3)$

| $(\lambda_2, \lambda_3)$ | (0.4,0.3) | (0.6,0.3) | (0.8,0.3) | (0.2,0.6) | (0.2,0.4) | (0.2,0.3) |
|---|---|---|---|---|---|---|
| $N^*$ | 45 | 45 | 45 | 45 | 45 | 45 |
| $T_c(N^*)$ | 1010.91 | 1009.10 | 1007.30 | 1011.96 | 1012.23 | 1012.30 |

**Table 4.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\lambda_1, \lambda_3)$

| $(\lambda_1, \lambda_3)$ | (0.3,0.2) | (0.4,0.2) | (0.5,0.2) | (0.6,0.3) | (0.6,0.5) | (0.6,0.7) |
|---|---|---|---|---|---|---|
| $N^*$ | 45 | 44 | 43 | 42 | 42 | 42 |
| $T_c(N^*)$ | 1012.51 | 991.80 | 971.54 | 951.68 | 951.50 | 951.32 |

**Table 5.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\theta,\alpha)$

| $(\theta,\alpha)$ | (0.5,0.05) | (1.0,0.05) | (1.5,0.05) | (2.0,0.05) | (2.0,0.2) | (2.0,0.5) |
|---|---|---|---|---|---|---|
| $N^*$ | 39 | 34 | 34 | 40 | 40 | 40 |
| $T_c(N^*)$ | 993.43 | 967.65 | 967.13 | 924.75 | 929.47 | 936.61 |

**Table 6.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\mu,\beta)$

| $(\mu,\beta)$ | (0.8,3.0) | (1.0,3.0) | (1.2,3.0) | (1.0,2.0) | (1.0,4.0) | (1.0,5.0) |
|---|---|---|---|---|---|---|
| $N^*$ | 23 | 26 | 27 | 26 | 26 | 26 |
| $T_c(N^*)$ | 1003.43 | 982.65 | 973.13 | 987.75 | 1064.41 | 1062.46 |

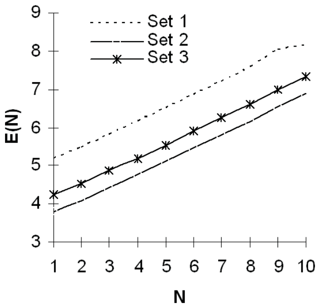
**Table 7.** The optimal threshold parameter $N^*$ and corresponding minimum expected cost for different sets $(\vartheta,\beta)$

| $(\vartheta,\beta)$ | (0.3,1.0) | (0.3,2.0) | (0.3,3.0) | (0.4,2.5) | (0.5,2.5) | (0.6,2.5) |
|---|---|---|---|---|---|---|
| $N^*$ | 28 | 32 39 | 36 | 36 | 37 | |
| $T_c(N^*)$ | 657.11 | 766.61 | 783.42 | 871.20 | 870.65 | 870.89 |

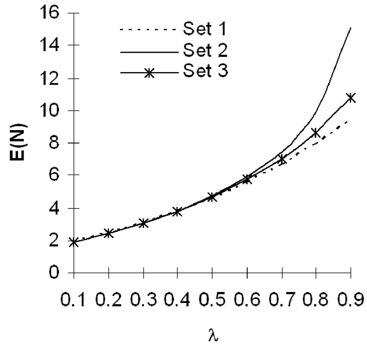
In Figure 1(a) as we increase the threshold level, the queue length increases linearly. Figure 1(b) shows the gradually increment initially and then after there is a sharp increment in $E(N)$ as arrival rate increases. In Figure 1(c) we exhibit the graphs for $E(N)$ and notice that it increases with the increase in $\alpha$. Figure 1(d) displays that as we increase $\mu$, we see that initially average queue length decreases sharply and then becoming almost constant.

## 7. **Conclusion**

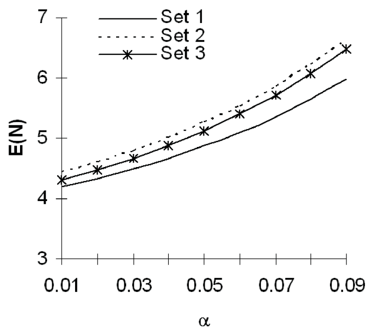
In this paper, we have developed steady state performance indices for N-policy $M/M/1$ queueing system with server breakdowns, vacations and setup time. We have derived the distribution of system size and employed the probability generating function technique to obtain mean queue length. Many existing queueing models are deduced as special cases of our queueing model. Our queueing model accommodates the real world congestion situations more closely in comparison to other similar studies done previously. Sensitivity analysis performed to examine the effect on the average queue length and cost function of different parameters, may be helpful to decision makers and system designers for the choice of optimal control policy.



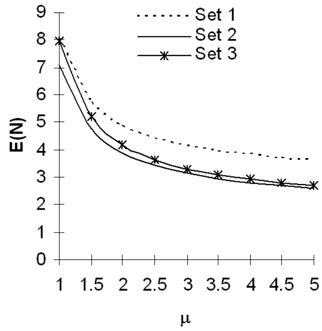

(a) Expected queue length vs. $N$




(b) Expected queue length vs. $\lambda$




(c) Expected queue length vs. $\alpha$




(d) Expected queue length vs. $\mu$

**Figure 1**

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