The Hierarchical Planning of Production in Agro Alimentary Environment: Case of Interdependent Products

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Abstract. In this paper, we are interested to the problem of planning inside the agro-alimentary industry. In this context, we have take into account many specific constraints such us products interdependence and the existence of many changeable production procedures. We propose a hierarchical model that respects the real capacities of the workshop and the product interdependence. Indeed, we have established a mathematical model according to different production levels. While taking into account real capacities of the shop and the interdependencies between the products, the results of our formulation are satisfactory in terms of quality of solution and time requirements. It's shown that our model is able to reach all optimal solutions for all treated models and for all system levels.

1. Introduction

The agroalimentary industries constitute the link between the agriculture and the consumers. In the planning context, the problem consists of determining a production plan which means knowing the demands among a given horizon, determining the quantities of products which have to be produced in each period, in order to minimize the costs such us production cost, transformation cost, and stocking cost. Furthermore, the planning task in this domain is subjected to several specificities concerning the production planning data because of the existence of high degree of incertitude and imprecision in the data level, perishable finished and semi-finished products and product interdependencies [1]. In fact, the planning task remains an important piloting tool since we have the future demand which refines over the time [4]. Moreover, the environment of these industries is very dynamic due to the raw materials and finished products that have often a very short life cycle [5]. The classics models of planning were not well adapted in the context of agro alimentary environment because they did not offer a homogeneous field at the level of the management of production resources.

In the first section of this article, we propose a hierarchical model for planning that take into account real constraints of *Agro-Alimentary Environment* (AAE). One of the reasons of the choice of the hierarchical approach is that it permits to simplify the global process of decision [7]. Indeed, the decisions transmitted at the lower level are considered as constraints to be satisfied or as objectives to be reached at a superior level.

In the second section, we present the problematic of the planning in the Agro Alimentary Environment. Then, in the third section, we describe our new formulation. To focus in the forth section on the case of a charcuterie to validate our model .finally, the main conclusions and perspectives are presented in section five.

2. **Problematic of Planning in the Agro Alimentary Environment (*AAE*)**

In the context of the AAE, many specific constraints are to be considered in the planning process. In agroalimentary, the costs of the raw materials are not fixed and can depend on some parameters such as the climate. Some products are seasonal and in this time we note an increase in the production level which can leads to an increasing demand for more workers and more supplementary working hours [2].

In the AAE, the process of production is not stationary. A raw material can give a finished product x, two products x and y or even more [6]. So, a resource can be used in several tasks at the same time. In addition, the finished products are sometimes interdependent. For example, we cannot finish producing x without producing another finished product y.

Therefore, even if the demand concerns one variety of products, we must produce a range. In this case, the company will be constrained to stock finished or half-finished products knowing that the stocked products sometimes have very short limit delays of consumption. So, these companies have no choice but to make some sacrifices on their selling prices to liquidate their stock.

The cost of stoking can go up, if the product remains a long time in stock since the weight and the price of some articles sometimes decreases following a congealment. The agroalimentary products are often characterized by a *consumption delay* (*CD*) [10]. The firms must consider not only this deadline, but also they must take into account a *selling delay* (*SD*) for these products. Generally, $SD = \frac{1}{3}CD$, this delay gives a short time for the firm which may be one or two days.

In the AAE, and particularly in the case of the specialized companies in charcuterie, some products can be considered as finished products intended directly to be sold on the market or as half finished products being able to be transformed and/or decomposed into some of other by-products. For example, the thighs are sold directly, or decomposed into top of thigh and pestle. On the other

hand, the production of a product A triggers the production of a product B or of its derivatives without even having a demand of that product from the market.

Generally, in the agoralimentary industries, we distinguish three kinds of products: finished products not decomposable, semi-finished products which cannot be sold directly and finished decomposable products which can be destined to selling or/and decomposed into other products.
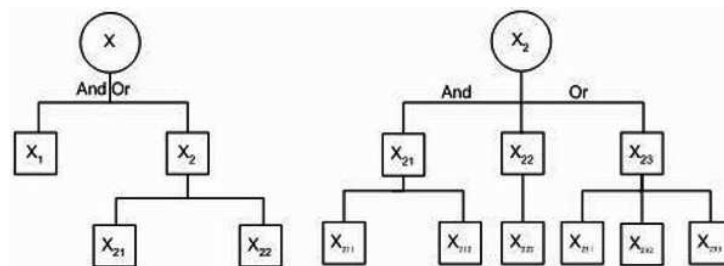

Figure 1. Hierarchical nomenclature "products interdependencies".

Figure 1 presents the notion of interdependencies between products. Both the products X_1 and X_2 use the same raw material X. So, through X, we can produce only X_1, or only X_2, or both X_1 and X_2. X_1 is a non-decomposable finished product directly intended for selling whereas X_2 is a semi-finished product which cannot be sold directly. The latter may be transformed into X_{21}, X_{22} and X_{23} such that we cannot produce X_{22} without producing X_{21} but it's possible to produce X_{23} only without X_{22} and X_{21}.

In the literature, few works treat the problems of production planning in the *Agro Alimentary Industry* (*AAI*) or also in the case of perishable or interdependent products.

Houba *et al.* [8] described a modelling method that reduces the effort required for the development of decision making system in the AAI, based on the techniques of the satisfaction of constrains that most manufacturers make it by hand. The authors proposed a model based on the returns of the products and applied it on dividing salad factory.

Tadei *et al.* [9] proposed a heuristic composed of two stages to treat the problems of planning and of organization of the production in an agro alimentary enterprise. The authors have considered contradictory objectives as the minimization of the labour force cost and the inventory cost.

In this article, we propose a hierarchical planning method to manage the notion of interdependence between products in agro alimentary environment. In a classic approach, the, decision-makers must focus on an important database whereas the hierarchical approach is based on a hierarchical decomposition of the decisions that adapt well to the structure of the ties between the products as a tree

representation [11]. In this tree, the decisions of a superior level can be considered as constraints in a lower level where the decomposition is finer.

3. **Mathematical Modelling Programming Models**

The models used in different levels were constructed by aggregating successively the entities that will be produced in the direction from low to high level. The decisions of a superior level are considered as a constraint in an inferior level [3].

The subjected entities in the model are:

- *Type*: consists of a set of items having the same basic raw-materials and are related by interdependency constraints. The production of items of type i followed by items of type j may require operations machine settings and cleaning. We cannot find any interdependency between the products of two different types.
- *Item*: corresponds to a sold finished product, decomposable finished product or semi-finished product.

We note that any item belongs to only one family, and likewise, every family is part of only one type.

3.1. *Linear Modelling per Type (LMT)*

In the proposed linear model for production hierarchical planning, we consider a production system consisted of several manufacturing stages with a general tree of N products. The required data are: number of planning periods, the demand per type per period, the availability of production resources during supplementary hours, the unit production period per type, a set of unit costs. The used notations are presented as follows:

The decision variables

X_{it}	:	The quantity of type i to produce on period t.
S_{it}	:	The stock of product of type i at the end of period t.
R_t	:	The number of regular working hours used on period t.
O_t	:	The number of supplementary working hours used on period t.
ST_{it}	:	The number of units of product i subcontracted over the period t.

The costs

Cp_{it}	:	Aggregate production cost of type i over the period t.
Cs_{it}	:	Aggregate inventory cost of type i over the period t.
Chs_t	:	Cost of a supplementary working hour over the period t.
Chn_t	:	Cost of a regular working hour over the period t.
C_{it}^{sT}	:	Outsourcing cost per unit of product i over the period t.

The parameters

D_{it} : Aggregate demand of type i over the period t.

thn_t : The total of available regular working hours over the period t.

nh_i : The number of required hours for producing one unit of type i.

C_t : The capacity of production (Kg/hour) over the period t.

Mhn_t : Maximum number of regular working hours per day.

Mhs_t : Maximum number of supplementary working hours per day.

Then, the proposed linear mathematical model, denoted LMT, is presented as follows:

$$(3.1) \quad \text{Min} f = \text{Min} \sum_{t=1}^{T} \left(\sum_{i=1}^{N} (Cp_{it}X_{it} + Cs_{it}S_{it} + C_{it}^{sT}ST_{it}) + Chn_t R_t + Chs_t O_t \right)$$

subject to:

$$(3.2) \quad S_{it-1} + X_{it} - D_{it} = S_{it} \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.3) \quad S_{i0} = 0 \quad \forall \, i = 1, 2, \ldots, N$$

$$(3.4) \quad \sum_{i=1}^{T} nh_i X_{it} \leq R_t + O_t \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.5) \quad \sum_{i=1}^{N} X_{it} \leq C_t \quad \forall \, t = 1, 2, \ldots, T$$

$$(3.6) \quad 0 \leq R_t \leq Mhn_t \quad \forall \, t = 1, 2, \ldots, T$$

$$(3.7) \quad 0 \leq O_t \leq Mhs_t \quad \forall \, t = 1, 2, \ldots, T$$

$$(3.8) \quad S_{it} + D_{it} = S_{i,t-1} + X_{it} + ST_{it} \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.9) \quad X_{it} \geq 0 \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.10) \quad S_{it} \geq 0 \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.11) \quad ST_{it} \geq 0 \quad \forall \, i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

The objective function (3.1) is the sum of production costs, inventory costs, cost of total regular and supplementary working hours for all products over the horizon period. The constraints (3.2) are balancing stocks equations. They state that the stock at the end of period t is equal to the stocks at the period $(t-1)$ plus the produced quantity at the period t minus the demand of the period t. The constraints (3.3) ensure that the initial stock is equal to 0 at $(t = 1)$. The constraints (3.4)ensure that the planned production cannot exceed the available capacity over regular and supplementary working hours over the period t. The production capacity is restricted to an upper bound at each period t (constraints (3.5)). The constraints (3.6) and (3.7) require that the number of working hours cannot exceed the maximum number of regular and supplementary working hours respectively. The constraints (3.8) show that, in the overload case, the firm can use outsourcing. The total quantity of product i produced over given period is

calculated as the sum of its production for different products. The inequalities (3.9), (3.10) and (3.11) are the non-negativity conditions.

The proposed model aims to determine the produced quantities of each type of item. The output of this model is considered as specific constraints to the second proposed model presented in the next section.

3.2. *Linear Modelling per item (LMI)*

This modelling consists in sharing the available capacity of an item from a level i into each obtained item processed from it to a level $i+1$. This is done subject to satisfaction demand constraints with the objective to minimize production costs and inventory costs. The used notations are presented as follows:

Cp_{ikt} : The production cost of product ik over the period t.

Cs_{ikt} : The inventory cost of product ik over the period t.

q_{ikt} : Minimum starting quantity of product ik over the period t.

X_{ikt} : The quantity of product ik to produce over the period t.

S_{ikt} : The stocked quantity of product ik over the period t.

X_{it}^* : The obtained quantity of the product X_i from the MLT.

X_i is a decomposable product into n_i items X_{ik} $(k = 1, \ldots, n_i)$.

The structure of the LMI can be formulated as follows:

$$(3.12) \quad \text{Min} g(S, X) = \text{Min} \sum_{t=1}^{T} \sum_{k=1}^{n_i} Cs_{ikt} S_{ikt} + Cp_{ikt} X_{ikt}$$

Subject to:

$$(3.13) \quad S_{ikt-1} + X_{ikt} - S_{ikt} = D_{ikt} \quad \forall\, K = 1, 2 \ldots, n_i; i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.14) \quad \sum_{K=1}^{n_i} X_{ikt} \leq X_{it}^* \quad \forall\, K = 1, 2 \ldots, n_i; i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.15) \quad X_{ikt} \geq q_{ikt} \quad \forall\, K = 1, 2 \ldots, n_i; i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.16) \quad X_{ikt} \geq 0 \quad \forall\, K = 1, 2 \ldots, n_i; i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

$$(3.17) \quad S_{ikt} \geq 0 \quad \forall\, K = 1, 2 \ldots, n_i; i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

The objective function (3.12) consists of both the production costs, the inventory costs for all products over all periods. The constraints (3.13) represent the state equations of the inventory level. They indicate that the stock at the end of period t is equal to the stock at the period $(t-1)$ plus the produced quantity over the period t minus the demand of the period t. The constraints (3.14) insure that the sum of produced quantities of sub products of X_i cannot exceed X_i^*.

The demand of each type is obtained by aggregating the demands of finished items belonging to such type. In order to determine this aggregate demand we multiply the sum of demands of all finished items by a coefficient λ. So $D_{it} = \lambda_{it} \sum_{j=1}^{n_i} d_{ij}$ will be minimized. As shown in Figure 2, first, we reinitialize the value of λ and we solve the proposed hirerachical model. If the system provide
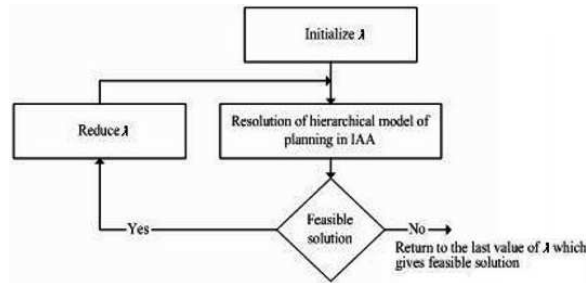
Figure 2. The steps of the resolution procedure.

a feasible solution, then we substract λ and we restart the resolution. These steps will be repeated until reaching a value of λ where the system cannot provide a feasible solution.

4. Case Study: Butcher's Shop Firm

In order to validate our proposed approach, we consider a real case study in a butcher's shop firm. Its activities consist in raising, slaughtering and selling of poultries. In our modelling, several aggregate products are considered which consist of two families: chicken and turkey. The compositions of these two families are limited to items representing similarities in their ranges in order to modelling the interdependencies between the products. The trees of turkey and chicken items are represented respectively in Figure 3 and 4.

The planning horizon (T) is set to a week subdivided into 7 periods. The predictions of the demand of each finished item to sell for the chicken family, over a week, are given in Table 1.
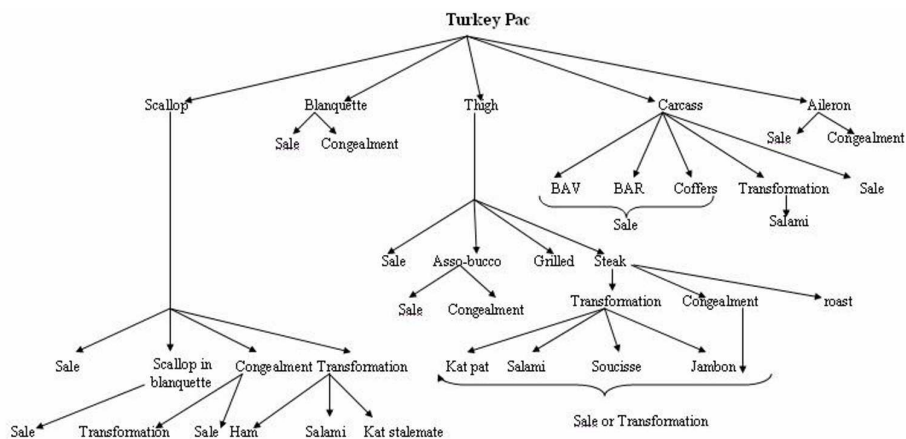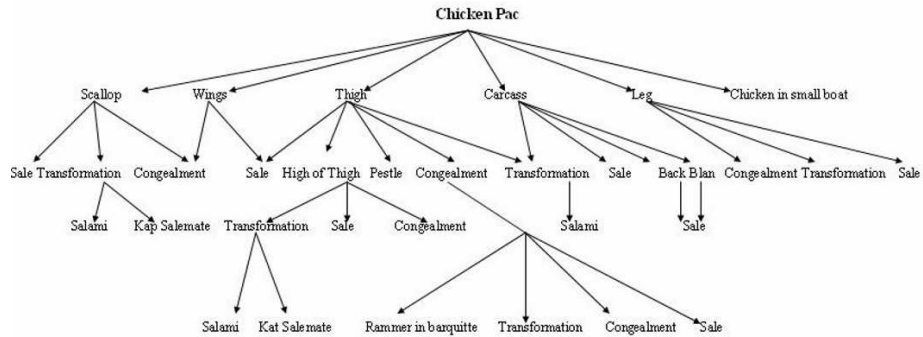

Figure 3. The items of Turkey Pac product.

Figure 4. The items of chiken Pac product.

Table 1. The prediction of the demand of some finished items for chicken's case.

Demand	designation	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Chicken	d_{1t}	1800	1500	1000	1300	1800	1100	1700
Turkey	d_{2t}	1000	1200	2300	1800	1300	2100	1600
Cut out chicken	d_{11t}	1000	1200	500	900	1050	850	1100
Chicken in the small boot	d_{12t}	450	375	400	275	350	400	420
Scallop selling	d_{1111t}	400	350	250	400	550	220	350
Scallop congealment	d_{1113t}	350	360	250	270	200	200	250
Wings sale	d_{1121t}	150	100	90	130	165	100	110
Wings congealment	d_{1122t}	130	100	110	110	100	60	100
thigh sale	d_{1141t}	250	220	200	220	250	140	250
Thigh congealment	d_{1143t}	190	140	200	180	180	115	130
Carcass sale	d_{1132t}	85	75	45	60	70	35	50
Transformation of scallop	d_{1112t}	350	300	250	270	300	210	250

Starting from the demands of finished products, we will determine the aggregate demands of each semi-finished product. Starting from the lowest level of the tree of each type and rising from one level to another by aggregating the demand of lower level. The forecasts of demands of semi-finished products are obtained by aggregation. Aggregate demand of a semi finished product may be the sum of requests for these derivatives or the weighted sum by coefficients provided by the company. Concerning the aggregate demand of a type, it cannot be equal to the sum of its derivatives demands because of the existence of the interdependency constraints between the products and other specificities. The results of the hierarchical model for different values of λ are summarized in Table 2.

By using the WinQSB program, it's shown that, for $\lambda = 1.8$, all the demands of finished products intended directly for sale are satisfied and the holding inventory costs is very high. So, it's recommended to reduce the value of λ. By reducing

Table 2. Results of hierarchical model for $\lambda = 1.8$; $\lambda = 1.5$; $\lambda = 1.4$.

Finished item	$t=1$ S	$t=1$ Q	$t=2$ S	$t=2$ Q	$t=3$ S	$t=3$ Q	$t=4$ S	$t=4$ Q	$t=5$ S	$t=5$ Q	$t=6$ S	$t=6$ Q	$t=7$ S	$t=7$ Q
Chicken	0	1800	50	1500	35	1000	12	1300	26	1800	12	1100	21	1700
	32	1600	32	1350	24	950	0	1210	29	1650	0	1050	26	1570
	0	1200	35	1195	30	800	11	1040	30	1480	10	900	30	1300
Turkey	0	100	14	1200	31	2300	14	1800	34	1300	15	2100	45	1600
	0	900	20	1075	37	2100	16	1700	50	1157	17	1900	47	1400
	33	730	27	800	41	1970	18	1500	42	1010	10	1700	50	1100
Chicken in the small boot	42	402	68	350	72	320	80	240	63	316	15	280	40	300
	34	318	54	300	68	303	69	215	0	300	62	210	33	280
	30	250	48	265	57	242	59	200	8	278	58	187	27	241
Scallop selling	33	380	62	300	71	240	35	300	80	489	32	189	90	280
	17	310	54	257	66	208	31	280	60	400	27	166	63	210
	13	280	37	218	59	175	27	235	54	307	24	112	59	184
Total	99	3582	194	3350	209	3860	141	3640	203	3905	134	3369	196	3880
	83	3128	170	2982	195	3561	116	3405	139	3507	106	3326	169	3460
	76	2460	147	2478	187	3187	115	297	134	3075	102	2899	166	2825

this value, we observe that the demands remain satisfied but stocks are still generated then it's possible to decrease λ. We have solved the hierarchical model by setting λ to several values $\lambda = \{1.8; 1.5; 1.4; 1.3; 1.29; 1.28; 1.27\}$. It's shown that more reducing the value of λ, more the total quantity of stocks decreases. For $\lambda = 1.27$,the model has not provided a feasible solution. So, we have returned to the last value which corresponds to a feasible solution $\lambda = 1.28$.In general, it's observed that our model is able to reach optimal solutions for all system levels with minimum cost.

Figure 5 and Figure 6 present the output of the WinQSB software. It's observed that our model is able to reach optimal solutions for all system levels with minimum cost both for LMT and LMI.

5. **Conclusions and perspectives**

In this paper, we have performed the resolution of hierarchical production planning problem in the agroalimentary industries.

We established a mathematical programming relative to the different levels of production, the mathematical program, using the software WinQSB, generated satisfactory results because it gives optimal solutions for all models and levels of the system with a minimal cost. One of the strong points of that program is its flexibility and its adaptation to all new situations. Indeed, decision-maker can use this program in future years.

As future directions, several other problems can be treated in more depth as the problems related to breeding, subcontracting and distribution of finished products.
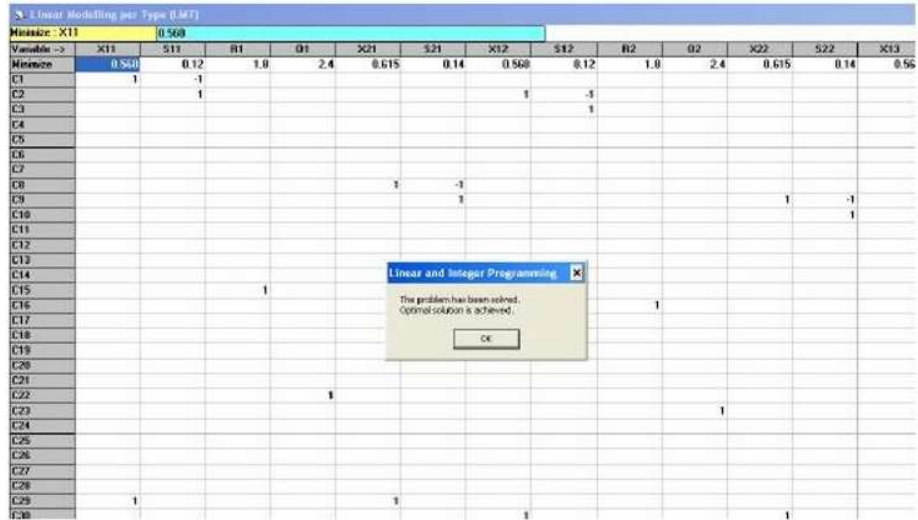
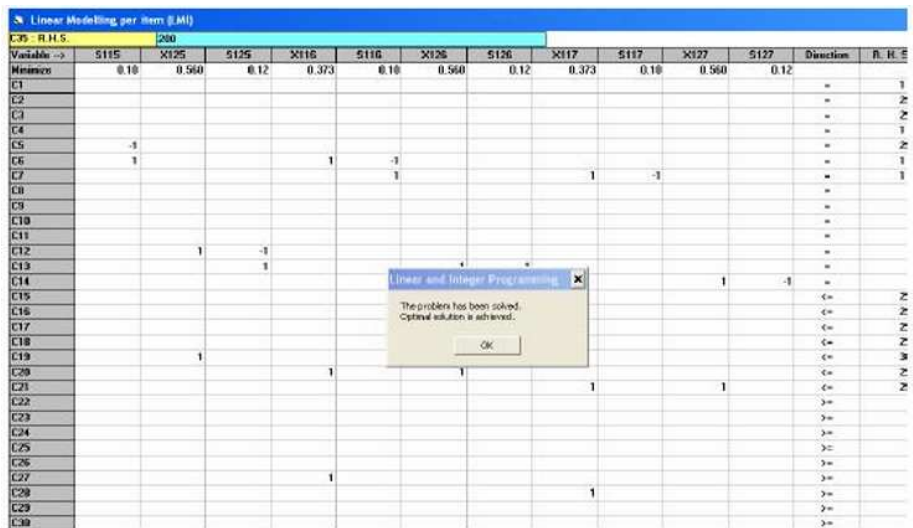**Figure 5.** Results for the LMT with WinQSB interface.

Figure 6. Results for the LMI with WinQSB interface.

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