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Weak Integer Additive Set-Indexers of Certain Graph Products

Research Article

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Abstract. Let $\mathbb{N}_0$ be the set of all non-negative integers and $\mathscr{P}(\mathbb{N}_0)$ be its power set. An integer additive set-indexer (IASI) of a graph is an injective function $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective, where $f(u)+f(v)$ is the sum set of $f(u)$ and $f(v)$. An IASI f is said to be a weak IASI if $|f^+(uv)| = \max(|f(u)|,|f(v)|)$ $\forall\ uv \in E(G)$. In this paper, we study the admissibility of weak IASI by different products of two weak IASI graphs.

Keywords. Integer additive set-indexers; Mono-indexed elements of a graph; Weak integer additive set-indexers; Sparing number of a graph

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1. Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [12] and [11]. Unless mentioned otherwise, all graphs we consider here are simple, finite and have no isolated vertices.

Let $\mathbb{N}_0$ denotes the set of all non-negative integers and $\mathscr{P}(\mathbb{N}_0)$ be its power set. The *sum set* of two sets A and B is denoted by $A+B$ and is defined by $A+B = \{a+b : a \in A, b \in B\}$. If either A or B is countably infinite, then their sum set is also countably infinite. Hence, only finite sets are taken for this study. We denote the cardinality of a set A by $|A|$.

We call a set B an integral multiple of another set A if every element of B is an integral multiple of the corresponding element of A. That is, for an integer $k > 1$, $B = kA \implies B = \{ka : a \in A\}$.

An *integer additive set-indexer* (IASI) of a graph G is defined in ([7]) as an injective function $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective. A graph that admits an IASI is called an integer additive set-indexed graph (IASI-graph).

Since the set-label of every edge of an IASI-graph G is the sum set of the set-labels of its end vertices, no vertex of G can have the empty set as its set-label. Hence, all sets we consider here are non-empty finite sets of non-negative integers.

The cardinality of an element of a graph G is said to be the *set-indexing number* of that element. An IASI is said to be a k-uniform IASI if $f^+(e) = k$ for all edges e of G. The following result provides the bounds for the set-indexing number of an edge in terms of the set-indexing numbers its end vertices.

Lemma 1.1 ([8]). *If f is an IASI on a graph G, then* $\max(|f(u)|, |f(v)|) \le f^+(uv) \le |f(u)||f(v)|$, $\forall\, uv \in E(G)$.

An IASI f of a given graph G is called, in ([8]), a *weak IASI* of G if $|f^+(uv)| = \max(|f(u)|, |f(v)|)$ for all $u, v \in V(G)$. A weak IASI f is said to be *weakly uniform IASI* if $|f^+(uv)| = k$, for all $u, v \in V(G)$ and for some positive integer k. A graph which admits a weak IASI may be called a *weak IASI graph*. The following lemma provides a necessary and sufficient condition for a given graph to admit a weak IASI.

Lemma 1.2 ([8]). *An IASI f of a given graph G is a weak IASI of G if and only if at least one end vertex of every edge of G has a singleton set-label with respect to f.*

An element (a vertex or an edge) of graph which has the set-indexing number 1 is called a *mono-indexed element* of that graph. The *sparing number* of a graph G is defined as the minimum number of mono-indexed edges required for G to admit a weak IASI. The sparing number of a graph G is denoted by $\varphi(G)$.

The hereditary property of the existence of a weak IASI have been established in the following theorem.

Theorem 1.3 ([15]). *Every subgraph of a weak IASI graph is also a weak IASI graph.*

The following result provides a necessary and sufficient condition for a given graph G to admit an IASI.

Theorem 1.4 ([15]). *A graph G admits a weak IASI if and only if G is bipartite or it has at least one mono-indexed edge.*

In view of the above theorem, it can be noted that the sparing number of a bipartite graph is 0.

The sparing number of certain standard graphs have been estimated as follows.

Theorem 1.5 ([15]). *An odd cycle C_n has a weak IASI if and only if it has at least one mono-indexed edge. That is, the sparing number of an odd cycle C_n is* 1.

Theorem 1.6 ([15]). *Let C_n be a cycle of length n which admits a weak IASI, for a positive integer n. Then, C_n has an odd number of mono-indexed edges when it is an odd cycle and has even number of mono-indexed edges, when it is an even cycle.*

The following result explained the admissibility of a weak IASI by the union of two weak IASI graphs and hence estimated its sparing number.

Theorem 1.7 ([16]). *The graph $G_1 \cup G_2$ admits a weak IASI if and only if both G_1 and G_2 are weak IASI graphs. More over, $\varphi(G_1 \cup G_2) = \varphi(G_1) + \varphi(G_2) - \varphi(G_1 \cap G_2)$.*

The sparing number of a complete graph has been given in the following result.

Theorem 1.8 ([15]). *The complete graph K_n admits a weak IASI and it has at least $\frac{1}{2}(n-1)(n-2)$ mono-indexed edges. That is, $\varphi(K_n) = \frac{1}{2}(n-1)(n-2)$.*

The theorem follows from the fact that at most one vertex of a complete graph can have a non-singleton set-label.

In this paper, we discuss the admissibility of weak IASI by certain products of given weak IASI-graphs.

2. Fundamental Products of Weak IASI Graphs

In different products of given graphs, we need to take several layers or copies of some or all given graphs and have to establish the adjacency between these copies, according to certain rules. Hence, it may not be possible to use the weak IASIs of given graphs to construct an induced weak IASI for a product of the given graphs. We have to define an IASI independently for a graph product.

We say that two copies of a graph are *adjacent* to each other in a graph product if there exist some edges between the vertices of those copies in the graph product.

In the first section, we discuss the admissibility of a weak IASI by the three fundamental products such as Cartesian product, direct product and strong product of two weak IASI graphs.

First, let us recall the definition of the Cartesian product of two graphs.

Definition 2.1 ([11]). Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be the two given graphs. The *Cartesian product* of G_1 and G_2, denoted by $G_1 \Box G_2$, is the graph with vertex set $V_1 \times V_2$, such that two points $u = (u_1,u_2)$ and $v = (v_1,v_2)$ in $V_1 \times V_2$ are adjacent in $G_1 \Box G_2$ whenever [$u_1 = v_1$ and u_2 is adjacent to v_2] or [$u_2 = v_2$ and u_1 is adjacent to v_1].
If $|V_i| = n_i$ and $|E_i| = m_i$ for $i = 1,2$, then $|V(G_1 \Box G_2)| = n_1n_2$ and $i = 1,2$ and $|E(G_1 \Box G_2)| = n_1m_2 + n_2m_1$.

The Cartesian product $G_1 \Box G_2$ of two graphs G_1 and G_2 can be viewed as follows. Make n_2 copies of G_1. Denote these copies by G_{1_i}, which corresponds to the vertex v_i of G_2. Now, join the corresponding vertices of two copies G_{1_i} and G_{1_j} if the corresponding vertices v_i and v_j are adjacent in G_2.

The Cartesian product of two bipartite graphs is also a bipartite graph. Also, the Cartesian products $G_1 \Box G_2$ and $G_2 \Box G_1$ of two graphs G_1 and G_2, are isomorphic graphs.

The existence of a weak IASI by the Cartesian product of two weak IASI-graphs is verified in the following theorem.

Theorem 2.2. *Let G_1 and G_2 be two weak IASI graphs. Then, the Cartesian product $G_1 \Box G_2$ also admits a weak IASI.*

Proof. Let G_1 and G_2 be two weak IASI graphs on n_1 and n_2 vertices respectively. We can view the product $G_1 \Box G_2$ as a union of n_2 copies of G_1 and a finite number of edges connecting the corresponding vertices of two copies G_{1i} and G_{1j} of G_1 according to the adjacency of the corresponding vertices v_i and v_j in G_2, where $1 \le i \ne j \le n_2$. Let us define a set-labeling function $f : V(G_1 \Box G_2) \to \mathscr{P}(\mathbb{N}_0)$, with respect to which, the vertices of $G_1 \Box G_2$ are assigned to distinct set-labels as explained below.

Let u_{ij} be the i-th vertex of G_{1j}, the j-the copy of G_1. For odd values of j, label the vertices of G_{1j} in such a way that the corresponding vertices of G_{1j} have the same type of set-labels as that of G_1. That is, for odd j, let u_{ij} has a singleton set-label or non-singleton set-label according to whether the corresponding vertex u_i of G_1 has a singleton set-label or non-singleton set-label.

If u_i be not an end vertex of a mono-indexed edge in G_1, then for even values of j, label the corresponding vertex u_{ij} in such a way that u_{ij} has a singleton set-label (or non-singleton set-label) according as the vertex u_i of G_1 has non-singleton set-label (or singleton set-label). Also, label the vertices of G_{1j} which are corresponding to the adjacent mono-indexed vertices of G_1, by singleton sets.

If $u_i u_j$ is a mono-indexed edge of G_2, then label the vertices of G_{1i} and G_{1j} such that no two corresponding vertices of G_{1i} and G_{1j} simultaneously have singleton set-labels or non-singleton set-labels.

We can see that no two adjacent vertices in $G_1 \Box G_2$ have non-singleton set-labels with respect to this labeling f. Therefore, f is a weak IASI for the graph $G_1 \Box G_2$. □

If a graph G is the Cartesian product of two graphs G_1 and G_2, then G_1 and G_2 are called the *factors* of G. A graph is said to be *prime* with respect to a given graph product if it is non-trivial and can not be represented as the product of two non trivial graphs.

The following result establishes the existence of an induced weak IASI for every factor of a non-prime graph that admits a weak IASI.

Theorem 2.3. *Let G is a non-prime graph which admits a weak IASI f. Then, every factor of G also admits a weak IASI that is induced by f.*

Proof. Let G be a non-prime weak IASI graph with a weak IASI f defined on it. If G_1 is a factor of G, then there is a subgraph G_{1i} in G which is isomorphic to G_1 such that v_i is the vertex of G_1 corresponding to a vertex v of G_1. Define a function $g : V(G_1) \to \mathscr{P}(\mathbb{N}_0)$ defined by $g(v) = f'(v_i)$ where $f' = f|_{G_{1i}}$, the restriction of f to the subgraph G_{1i}. By Theorem 1.3, f' is a weak IASI of G_{1i} and hence g is a weak IASI of G_1. □

Next, recall the definition of another graph product called the direct product of two graphs.

Definition 2.4 ([11])**.** The *direct product* of two graphs G_1 and G_2, is the graph whose vertex set is $V(G_1) \times V(G_2)$ and for which the vertices (u,v) and (u',v') are adjacent if $uu' \in E(G_1)$ and $vv' \in E(G_2)$. The direct product of G_1 and G_2 is denoted by $G_1 \times G_2$.

Note that the direct product of two connected graphs can be a disconnected graph also. The direct product is also known as *tensor product* or *cardinal product* or *cross product* or *categorical product* or *Kronecker product*. The following theorem establishes the admissibility of weak IASI by the direct product of two weak IASI graphs.

Theorem 2.5. *The direct product of two weak IASI graphs is also a weak IASI-graph.*

Proof. Let G_1 and G_2 be two weak IASI graphs on n_1 and n_2 vertices, m_1 and m_2 edges respectively. Let $V(G_1) = \{u_1, u_2, u_3, \ldots u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, v_3, \ldots v_{n_2}\}$. Make n_2 copies of $V(G_1)$ and denote the j-th copy of V by V_j, where $1 \le j \le n_2$. Let $V_j = \{u_{ij} : 1 \le i \le n_1, 1 \le j \le n_2\}$. Since the vertex u_{ij} is adjacent to a vertex u_{rs} if u_i and u_r are adjacent G_1 and u_j and u_s are adjacent in G_2, no two vertices in the copy V_j can be adjacent to each other in $G_1 \times G_2$. Hence, define a function f_j on the vertex set V_j such that it assigns the set-labels to the vertices of V_j, which are integral multiples of the set-labels of the corresponding vertices of G_1. Clearly, no two adjacent edges in $G_1 \times G_2$ have non-singleton set-labels. Therefore, this labeling is a weak IASI on $G_1 \times G_2$. □

Next, let us consider the following definition of the strong product of two graphs.

Definition 2.6 ([11])**.** The *strong product* of two graphs G_1 and G_2 is the graph, denoted by $G_1 \boxtimes G_2$, whose vertex set is $V(G_1) \times V(G_2)$, the vertices (u, v) and (u', v') are adjacent in $G_1 \boxtimes G_2$ if $[uu' \in E(G_1)$ and $v = v']$ or $[u = u'$ and $vv' \in E(G_2)]$ or $[uu' \in E(G_1)$ and $vv' \in E(G_2)]$.

From this definition, we understand that $E(G_1 \boxtimes G_2) = E(G_1 \Box G_2) \cup E(G_1 \times G_2)$. Now, we prove the existence of weak IASI for the strong product of two weak IASI graphs in the following theorem.

Theorem 2.7. *The strong product of two weak IASI graphs also admits a weak IASI.*

Proof. Let G_1 and G_2 be two weak IASI graphs on n_1 and n_2 vertices with the corresponding weak IASIs f_1 and and f_2 respectively. Let $G = G_1 \boxtimes G_2$. Then, G can be viewed as follows.

Take n_2 copies of G_1, denoted by G_{1i}, for $1 \le i \le n_2$. Let u_{ij} be the i-th vertex of the j-th copy of G_1, where $1 \le i \le n_1$, $1 \le j \le n_2$. If a copy G_{1j} is adjacent to another copy G_{1k} in G, then the vertex u_{ij} will be adjacent to the vertices $u_{i,k}, u_{i+1,j}, u_{i-1,j}$, if they exist.

Let $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ be an IASI defined on G, which labels the vertices of G in the following way. Label the corresponding vertices of the first copy G_{11} of G_1 by the same set-labels of the vertices of G_1. Now, by Definition 2.6, a vertex of the copies of G_1 that are adjacent to G_{11} can have a non-singleton set label if and only if the corresponding vertex and its adjacent vertices in G_{11} are mono-indexed. Let G_{1r} be the next copy of G_1 which is not adjacent to G_{11}. Label the vertices of this copy by an integral multiple of the set-labels of the corresponding vertices of G_1 and label the vertices of adjacent copies of G_{1r} such that no vertex of G_{1r} has a non-singleton set-label unless the corresponding vertex and its adjacent vertices in G_{1r} are mono-indexed. Proceed in this way until all the vertices in G are set-labeled. Then, we have a set-labeling in which no two adjacent vertices of G have non-singleton set-labels. Hence, f is a weak IASI on $G = G_1 \boxtimes G_2$. This completes the proof. □

3. Other Products of Weak IASI Graphs

In the previous section, we have discussed the admissibility of weak IASI by three fundamental products of weak IASI graphs. Now, we proceed to discuss the existence of weak IASI for certain other graph products.

Now, recall the definition of lexicographic product of two graphs.

Definition 3.1 ([13]). The *lexicographic product* or *composition* of two graphs G_1 and G_2 is the graph, denoted by $G_1[G_2]$, is the graph whose vertex set $V(G_1) \times V(G_2)$ and for two vertices (u,v) and (u',v') are adjacent if $[uu' \in E(G_1)]$ or $[u = u'$ and $vv' \in E(G_2)]$.

Admissibility of weak IASI by the lexicographic product of two weak IASI graphs is established in the following theorem.

Theorem 3.2. *The lexicographic product of two weak IASI graphs admits a weak IASI.*

Proof. Let G_1 and G_2 be two weak IASI graphs on n_1 and n_2 vertices respectively. The composition of G_1 and G_2 can be viewed as follows. Take n_1 copies of G_2, denoted by G_{2i}; $1 \le i \le n_1$. Every vertex of a copy G_{2i} is adjacent to all vertices of another copy G_{2j} in $G_1 \circ G_2$ if the corresponding vertices v_i and v_j are adjacent in G_1.

Label the corresponding vertices of the first copy G_{21} of G_2 by the same set-labels of the vertices of G_2. Since every vertex of G_{21} is adjacent to all vertices of its adjacent copies, these vertices must be labeled by distinct singleton sets. Now, label the vertices of the next copy G_{2r} of G_2 which is not adjacent to G_{21} by an integral multiple of the set-labels of the corresponding vertices of G_2 and label the vertices of the adjacent copies of G_{2r} by singleton sets. Proceed in this way until all the vertices in G are set-labeled. This set-labeling is a weak IASI for $G_1[G_2]$. □

The next graph product we are going to discuss now is the corona of two weak IASI graphs.

Definition 3.3 ([12]). The *corona* of two graphs G_1 and G_2, denoted by $G_1 \odot G_2$, is the graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i-th point of G_1 to every point in the i-th copy of G_2. The number of vertices and edges in $G_1 \odot G_2$ are $p_1(1+p_2)$ and $q_1 + p_1q_2 + p_1p_2$ respectively, where p_i and q_i are the number of vertices and edges of the graph $G_i, i = 1,2$.

The following theorem establishes a necessary and sufficient condition for the corona of two weak IASI graphs to admit a weak IASI.

Theorem 3.4. *Let G_1 and G_2 be two weak IASI graphs on m and n vertices respectively. Then,*

(i) *$G_1 \odot G_2$ admits a weak IASI if and only if either G_1 is 1-uniform or it has r copies of G_2 that are 1-uniform, where r is the number of vertices in G_1 that are not mono-indexed.*

(ii) *$G_2 \odot G_1$ admits a weak IASI if and only if either G_2 is 1-uniform or it has l copies of G_2 that are 1-uniform, where l is the number of vertices in G_2 that are not mono-indexed.*

Proof. Consider the corona $G_1 \odot G_2$. Let f be a weak IASI of G_1 and f_i be a weak IASI on the i-th copy G_{2i} of G_2 all whose vertices are connected to the i-th vertex of G_1. Define the function g on $G_1 \odot G_2$ by

$$g(v) = \begin{cases} f(v) & \text{if } v \in G_1 \\ f_i(v) & \text{if } v \in G_{2i},\ 1 \le i \le m \end{cases}$$

Assume that $G_1 \odot G_2$ is a weak IASI graph. If G_1 is 1-uniform, then the proof is complete. If G_1 is not 1-uniform, then the vertex set V of G_1 can be divided into two disjoint sets V_1 and V_2, where V_1 is the set of all mono-indexed vertices in G_1 and V_2 be the set of all vertices that are not mono-indexed in G_1. Then, we observe that any copy G_{2i} of G_2 that are connected to the vertices of V_2 can not have a vertex that is not mono-indexed. That is, r copies of G_2 are 1-uniform, where $r = |V_2|$. Hence, $G_1 \odot G_2$ is a weak IASI graph implies G_1 is 1-uniform or r copies of G_2 are 1-uniform, where r is the number of vertices of G_1 that are not mono-indexed.

Conversely, either G_1 or r copies of G_2 that are 1-uniform, where r is the number of vertices of G_1 that are not mono-indexed. If G_1 is 1-uniform, then the vertices of G_{2i} can be labeled alternately by distinct singleton sets and distinct non-singleton sets under f_i. If G_1 is not 1-uniform, by hypothesis, r copies of G_2 are 1-uniform, where r is the number of vertices of G_1 that are not mono-indexed. Label the vertices of path G_{2i}, which is adjacent to the vertex v_i of G_1 that are not mono-indexed, by distinct singleton sets under f_i. Hence, in both cases, g is a set-indexer and hence a weak IASI for $G_1 \odot G_2$.
The proof for the second part is similar. □

We now proceed to determine the sparing number of the corona of two graphs.

Theorem 3.5. *Let G_1 be a weak IASI graph on n_1 vertices, m_1 edges and r_1 mono-indexed vertices and G_2 be a weak IASI graph on n_2 vertices, m_2 edges and r_2 mono-indexed vertices. Then, the sparing number of $G_1 \odot G_2$ is $r_1(1+r_2)+(n_1-r_1)m_2$ and the sparing number of $G_2 \odot G_1$ is $r_2(1+r_1)+(n_2-r_2)m_1$.*

Proof. Since G_1 has r_1 mono-indexed vertices, (n_1-r_1) copies of G_2 must be 1-uniform in $G_1 \odot G_2$. In the remaining r_1 copies, label the vertices by the set-labels which are some integral multiples of the set-labels of the corresponding vertices of G_2 (in such a way that no two copies of G_2 have the same set of set-labels). Hence, each of these copies contains the same number of mono-indexed edges as that of G_2. Therefore, the total number of mono-indexed edges in $G_1 \odot G_2$ is $r_1+(n_1-r_1)m_2+r_1r_2 = r_1(1+r_2)+(n_1-r_1)m_2$.
Similarly, we can prove the other part also. □

Another interesting graph product is the rooted product of two graphs. Let us first recall the definition of the rooted product of two given graphs.

Definition 3.6 ([9])**.** The *rooted product* of a graph G_1 on n_1 vertices and rooted graph G_2 on n_2 vertices, denoted by $G_1 \circ G_2$, is defined as the graph obtained by taking n_1 copies of G_2, and for every vertex v_i of G_1, identifying v_i with the root node of the i-th copy of G_2.

The following theorem verifies the admissibility of weak IASI by the rooted product of two graphs.

Theorem 3.7. *The rooted product of two weak IASI graphs is also a weak IASI graph.*

Proof. Let G_1 and G_2 be the given graphs with the weak IASIs f_1 and f_2 defined on them respectively. Also let $V(G_1) = \{u_1,u_2,u_3,\dots,u_{n_1}\}$ be the vertex set of G_1 and let $V(G_2) = \{v_1,v_2,v_3,\dots,v_{n_2}\}$ be the vertex set of G_2. Let $G = G_1 \circ G_2$. Without loss of generality, let v_1 be the root vertex of G_2. Now, make n_1 copies of G_2, denoted by G_{2r}, $1 \le r \le n_1$, with $V(G_{2r} = \{v_{1r},v_{2r},v_{3r},\dots,v_{n_2r}\}$.

Define a function $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ with the following conditions.

(1) For $1 \le i \le n_1$, define a function $f_{2r} : V(G_{2r}) \to \mathscr{P}(\mathbb{N}_0)$ such that $f_{2r}(v_{ir}) = rf_2(v_i)$, where $rf_2(v_i)$ is the set obtained by multiplying the elements of the set-label $f_2(v_i)$ by the integer r.

(2) The vertex u'_r obtained by identifying the vertex u_r of G_1 and the root vertex v_{1r} of the r-th copy G_{2r} of G_2 has the same set-label of u_r unless u_r has a non-singleton set label and v_{1r} is mono-indexed. In this case , let u'_r assumes the same set-label of v_{1r}.

Then, under f, no two adjacent vertices of G have non-singleton set-labels. That is, f is a weak IASI on $G = G_1 \circ G_2$. This completes the proof. □

4. Conclusion

In this paper, we have discussed the admissibility of weak IASI by the certain products of two graphs which admit weak IASIs. Some problems in this area are still open. In our present discussion we have not studied about the sparing number of the graph products, other than corona, of two arbitrary graphs G_1 and G_2. Uncertainty in the adjacency pattern of different graphs makes the study about the sparing number of the products of two arbitrary graphs a little complex. An investigation to determine the sparing number of different products of two arbitrary graphs in terms of their orders, sizes and the vertex degrees in each of them, seem to be promising. The admissibility of weak IASIs by certain other graph products is also worth studying.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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