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Research Article

Studies of Rotation Effect on Plane Harmonic Waves in a Micropolar Elastic Medium

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Abstract. In this article, we investigate the effect of rotation on plane harmonic waves in an isotropic homogeneous micropolar elastic solid with rotation having uniform angular velocity. It is observed that two sets of coupled dilation and shear waves are propagating with distinct speeds. Out of these, only two shear waves are propagating without elasticity and without micropolarity and two dilation waves are propagating with elasticity and without micropolarity. All these waves are dispersive in nature and affected by the rotation of the medium. It is interesting to observed that harmonic plane waves are not allowed to propagate in high rotating solids. Numerical example have been performed to discuss the behaviour of the speed of the waves.

Keywords. Micropolar elasticity, Harmonic plane waves, Rotation

Mathematics Subject Classification (2020). 74Jxx, 74Bxx, 76E07

1. Introduction

A nonlinear theory of micro elasticity was developed and modified as a linear theory of micropolar elasticity. Classical theory of elasticity and micropolar elasticity is differ by an independent microrotation vector, so, motion in micropolar elastic solid is characterized by six freedom parameters namely three translation and three rotation. The two parts of micropolar media introduced by a force vector, symmetric force stress tensor and couple stress tensor. Many authors studied the plane wave propagation in micropolar elastic solid. The studies on shear waves having a huge importance for investigators of seismologists,geophysicists for their predictions on seismic behavior at different parts of earth.

With the introduction of Cosserats' theory [7], a new wave of workers started for solution of various classical problems of elasticity. Theory of Cosserat and Cosserat [7] is the fore runner for postulating the theory of micropolar continuum mechanics. Based on this theory Eringen and Suhubi [9, 18] and Eringen [10] have formulated what is now called the micropolar theory of continuous media. In case of micropolar elasticity however along with displacement vector the rotation vector is also needed to determine all the quantities. A complete discussion can be found Eringen and Suhubi [9]. This theory has vide applications where the microstructure is also important.

In the past the number of authors focused on wave propagation studies in materials having voids, among those are Chandrasakharaiah and Srikanth [5], Chandrasakharaiah [3, 4], Wright [19], Puri and Cowin [12], Ciarletta and Sumbatyam [6], Day *et al*. [8]. Some authors investigated surface waves in different elastic materials. Surface wave propagation along the plane surface of an elastic solid was studied by Rayleigh [13]. Rayleigh waves along isothermal and insulated boundaries are examined by Chadwic and Windle [2]. Agarwal [1] studied the surface waves in generalized thermoelasticity. Thermoelastic surface waves with thermal relaxation in a transversely isotropic half space are investigated by Sharma and Singh [14]. Mayer [11] have discussed the thermoelastic attenuation of surface acoustic waves. The effects of rotation and voids on plane waves in thermoelastic solid are investigated by Singh and Tomar [16], while the effect of voids on surface waves in a non-rotating thermoelastic solid was studied by Singh and Pal [15] studied the plane waves in a rotating thermoelastic solid with voids.

Recently, Somaiah and Kumar [17] studied the propagation of plane longitudinal waves in micro-isotropic, elastic solids. Many researchers explained the angular rotation effect on plane waves in different elastic materials, but in this article we study the angular rotation effect on plane harmonic waves in micropolar elastic solid. In this we derive the shear and dilational waves and they are influenced by angular rotation of the medium and also they are dispersive in nature.

2. Formulation of the problem

Let us assume that a linear homogeneous micropolar elastic medium with rotating uniform angular velocity $\vec{\Omega} = \Omega_0 \hat{p}$, where $\hat{p}$ is the unit vector that represents the direction of the axis of the rotation.The macro displacement equation of the motion involves two additional terms in the rotating frame work namely, Centripetal acceleration $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ and the Coriolis acceleration $2\vec{\Omega} \times \dot{\vec{u}}$, where $\vec{u}$ is the displacement vector. The field equations in terms of displacement, microrotation for micropolar elastic solid and angular velocity $\vec{\Omega}$ in the absence of body forces and body couples are given by Eringen [10] as follows:

$$\rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\vec{\Omega}\times\frac{\partial\vec{u}}{\partial t}\right]=(\lambda+\mu)\nabla\nabla\cdot\vec{u}+(\mu+K)\nabla^2\vec{u}+K\nabla\times\vec{\phi}, \tag{1}$$

$$\rho J\frac{\partial^2\vec{\phi}}{\partial t^2}=\gamma(\nabla^2\vec{\phi})+(\alpha+\beta)\nabla(\nabla\cdot\vec{\phi})+K\nabla\times\vec{u}-2K\vec{\phi}, \tag{2}$$

where λ,μ are Lame's constants, K is the elastic constant, ρ is the density of the medium, J is the moment of micro inertia and α,β,γ are micro polar parameters, while $\vec{u}$ is the macro displacement vector and $\vec{\phi}$ is the micro rotation vector. Equations (1) and (2) rewrite as

$$c_1^2\nabla^2\vec{u}+c_2^2\nabla\nabla\cdot\vec{u}+c_3^2\nabla\times\vec{\phi}=\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\vec{\Omega}\times\frac{\partial\vec{u}}{\partial t}, \tag{3}$$

$$c_4\nabla^2\vec{\phi}+c_5^2\nabla\nabla\cdot\vec{\phi}+c_6^2\nabla\times\vec{u}-c_7^2\vec{\phi}=\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{4}$$

where

$$c_1^2=\frac{\mu+K}{\rho},\ c_2^2=\frac{\lambda+\mu}{\rho},\ c_3^2=\frac{K}{\rho},\ c_4^2=\frac{\gamma}{\rho J},\ c_5^2=\frac{\alpha+\beta}{\rho J},\ c_6^2=\frac{K}{\rho J}=\frac{c_3^2}{J},\ c_7^2=\frac{2K}{\rho J}=\frac{2c_3^2}{J}. \tag{5}$$

3. Derivation of Plane Harmonic Wave

Under the method of plane harmonic solutions, the equations (3) and (4) have the solutions in the following form

$$[\vec{u},\vec{\phi}]=[\vec{A},\vec{B}]\exp[ik(\hat{n}\cdot\vec{r}-vt)] \tag{6}$$

where $\vec{A}$, $\vec{B}$ are vector constants, $\vec{r}$ is the position vector, v is the phase velocity and k is the wave number with $\omega=kv$; ω being the angular frequency, $\hat{n}$ is the unit vector along the direction of propagation. On using the vector calculus results for equation (6) we obtain

$$\left.\begin{aligned}&\nabla^2\vec{u}=-\vec{A}k^2\exp[ik(\hat{n}\cdot\vec{r}-vt)],\quad \nabla\nabla\cdot\vec{u}=-k^2\hat{n}(\hat{n}\cdot\vec{A})\exp[ik(\hat{n}\cdot\vec{r}-vt)],\\&\nabla\times\vec{u}=(\hat{n}\times\vec{A})ik\exp[ik(\hat{n}\cdot\vec{r}-vt)],\quad \nabla^2\vec{\phi}=-\vec{B}k^2\exp[ik(\hat{n}\cdot\vec{r}-vt)];\\&\nabla\nabla\cdot\vec{\phi}=-k^2\hat{n}(\hat{n}\cdot\vec{B})\exp[ik(\hat{n}\cdot\vec{r}-vt)],\quad \nabla\times\vec{\phi}=(\hat{n}\times\vec{B})ik\exp[ik(\hat{n}\cdot\vec{r}-vt)].\end{aligned}\right\} \tag{7}$$

Inserting equations (7) in (3) and (4) we obtain

$$(\omega^2+\vec{\Omega}^2-c_1^2k^2)\vec{A}-c_2^2k^2(\hat{n}\cdot\vec{A})\hat{n}+c_3^2ik(\hat{n}\times\vec{B})-[(\vec{\Omega}\cdot\vec{A})\Omega+2i\omega(\vec{\Omega}\times\vec{A})]=0 \tag{8}$$

and

$$[k^2(c_4^2-v^2)+c_7^2]\vec{B}+c_5^2k^2\hat{n}(\hat{n}\cdot\vec{B})-c_6^2ik(\hat{n}\times\vec{A})=0 \tag{9}$$

Solving equation (9) for $\vec{B}$ we obtain

$$\vec{B}=\frac{c_6^2ik(\hat{n}\times\vec{A})}{[c_4^2+c_5^2-v^2]k^2+c_7^2} \tag{10}$$

substituting equation (10) in equation (8) we obtain

$$\left[\omega^2+\Omega_0^2-c_1^2k^2+\frac{c_3^2c_6^2k^2}{[c_4^2+c_5^2-v^2]k^2+c_7^2}\right]\vec{A}-\left[c_2^2+\frac{c_3^2c_6^2k^2}{[c_4^2+c_5^2-v^2]k^2+c_7^2}\right]k^2(\hat{n}\cdot\vec{A})\hat{n}$$
$$-[(\vec{\Omega}\cdot\vec{A})\vec{\Omega}+2i\omega(\vec{\Omega}\times\vec{A})]=0. \tag{11}$$

$\Omega_0 = |\vec{\Omega}|$ is the magnitude of $\vec{\Omega}$. From equation (11),the shear waves and purely dilational waves are evaluated as follow.

3.1 Shear Waves

To evaluate shear waves, we have

$$\hat{n}\cdot\vec{A} = 0 \tag{12}$$

and equation (11) becomes

$$\left[\omega^2 + \Omega_0^2 - c_1^2k^2 + \frac{c_3^2c_6^2k^2}{(c_4^2+c_5^2-v^2)k^2+c_7^2}\right]\vec{A} - [(\vec{\Omega}\cdot\vec{A})\vec{\Omega} + 2i\omega(\vec{\Omega}\times\vec{A})] = 0. \tag{13}$$

Taking scalar product with $\vec{A}$ of equations (13), we obtain

$$\left[\omega^2 + \Omega_0^2 - c_1^2k^2 + \frac{c_3^2c_6^2k^2}{(c_4^2+c_5^2-v^2)k^2+c_7^2}\right]A^2 - [(\vec{\Omega}\cdot\vec{A})^2 + 2i\omega(\vec{\Omega}\times\vec{A})\cdot\vec{A}] = 0.$$

where $\vec{A}\cdot\vec{A} = |A|^2 = A^2$.

Therefore,

$$a_1v^4 + a_2v^2 - a_3 = 0, \tag{14}$$

where

$$\left.\begin{aligned}&(\vec{\Omega}\times\vec{A})\cdot\vec{A} = \vec{\Omega}\cdot(\vec{A}\times\vec{A}) = 0,\ \ a_1 = (c_7^2-\omega^2)\Gamma,\ \ a_2 = \omega^2[c_1^2 + (c_4^2+c_5^2)\Gamma] - c_1^2c_7^2 + c_3^2c_6^2,\\ &a_3 = c_1^2(c_4^2+c_5^2)\omega^2,\ \ \Gamma = 1 + \tfrac{\Omega_0^2}{\omega^2}\sin^2\theta,\end{aligned}\right\} \tag{15}$$

and the angle between $\vec{u}$ and the directions of $\vec{\Omega}$ is θ and given by

$$\cos^2\theta = \frac{(\vec{\Omega}\cdot\vec{A})^2}{\Omega_0^2A^2},\ \ \Omega_0 = |\vec{\Omega}|,\ \ A^2 = |\vec{A}|^2.$$

By equation (14), the phase velocities of the shear couple waves due to the rotation of the body are given by

$$v_{s_{1,2}}^2 = \frac{-a_2 \pm (a_2^2 + 4a_1a_3)^{\frac{1}{2}}}{2a_1} \tag{16}$$

and these are dispersive in nature.

3.1.1 Case (i)

When $\theta = 0$, the couple shear wave phase velocities in non-rotating medium are given by

$$v_{s_{1,2}}^2 = \frac{c_1^2c_7^2 + c_3^2c_6^2 - \omega^2(c_1^2+c_4^2+c_5^2)}{2(c_7^2-\omega^2)} \pm \frac{\sqrt{(c_1^2c_7^2 - c_3^2c_6^2 - \omega^2(c_1^2+c_4^2+c_5^2))^2 + 4c_1^2\omega^2(c_7^2-\omega^2)(c_4^2+c_5^2)}}{2(c_7^2-\omega^2)}. \tag{17}$$

3.1.2 Case (ii)

When $\Omega_0 \to \infty$ then $v_{s_{1,2}}^2 \to 0$, i.e., the shear waves are not exist in very high speed rotating media.

3.1.3 Case (iii): With Elasticity and Without Micropolarity

Presence of elasticity ($K \neq 0$) and without micropolarity ($\alpha = \beta = \gamma = 0$) leads to $c_4 = 0 = c_5$. The speed of the shear waves v_s are given by

$$v_s^2 = c_1^2 + \frac{c_3^2 c_6^2}{(c_7^2 - \omega^2)\Gamma}, \tag{18}$$

this means at presence of elasticity, without micropolarity, we get two waves.

3.1.4 Case (iv): Without Micropolarity and Elasticity

In this case, elastic constant $K = 0$, micropolar parameters $\alpha = \beta = \gamma = 0$ leads to the speed of the shear waves v_s^2 in classical elastic solid and is given by

$$v_s^2 = \frac{\mu}{\rho\Gamma}. \tag{19}$$

3.2 Dilational waves

Purely dilational waves are allowed in the case that $\vec{u}$ and $\hat{n}$ are in the same direction, i.e.,

$$\hat{n}\cdot\vec{A} = A, \quad \text{where } A = |\vec{A}|. \tag{20}$$

On taking the scalar product with $\vec{A}$ of equation (11) and using equation (20) we obtain

$$v^2\Gamma + \frac{c_3^2 c_6^2}{(c_4^2 + c_5^2 - v^2)k^2 + c_7^2} - c_1^2 - c_2^2 - \frac{c_3^2 c_6^2 k^2}{(c_4^2 + c_5^2 - v^2)k^2 + c_7^2} = 0, \tag{21}$$

where Γ is given by equation (15).

On simplication of equation (21), we get the following quadratic equation in v^2,

$$Pv^4 + Qv^2 + R = 0, \tag{22}$$

where

$$\left.\begin{aligned} &P = \Gamma\left(1 - \frac{c_7^2}{\omega^2}\right), \quad Q = \tfrac{1}{\omega^2}[(c_1^2 + c_2^2)c_7^2 - c_3^2 c_6^2] - [\Gamma(c_2^2 + c_4^2) + c_1^2 + c_2^2], \\ &R = (c_1^2 + c_2^2)(c_4^2 + c_5^2) + c_3^2 c_6^2. \end{aligned}\right\} \tag{23}$$

Therefore, the phase velocities of the dilational couple waves due to the rotation of the body are given by

$$v_{d_{1,2}}^2 = \frac{-Q \pm (Q^2 - 4PR)^{\frac{1}{2}}}{2P}. \tag{24}$$

By equation (24), the velocity of the dilational waves are depends on frequency, so they are dispersive in nature.

3.2.1 Case (i)

When $\theta = 0$, the phase velocities of the couple dilational waves in non-rotating body are given by

$$v_{d_{1,2}}^2 = \frac{-Q^* \pm ({Q^*}^2 - 4P^* R)^{\frac{1}{2}}}{2P^*}, \tag{25}$$

where

$$P^* = 1 - \frac{c_7^2}{\omega^2}; \;\; Q^* = \frac{1}{\omega^2}\left[(c_1^2 + c_2^2)c_7^2 - c_3^2 c_6^2\right] - (c_1^2 + 2c_2^2 + c_4^2) \tag{26}$$

and R is given by equation (23).

3.2.2 Case (ii)

When $\Omega_0 \to \infty$; (i.e., $P \to \infty$) then by equation (24),

$$v^2_{d_{1,2}} \to 0,$$

i.e., very high speed rotating media not allowed to propagate dilational waves.

3.2.3 Case (iii): Without Micropolarity and Without Elasticity

In this case elastic constant $K = 0$, micropolar parameters $\alpha = \beta = \gamma = 0$ leads to the velocity of the dilational waves v^2_d in classical case and is given by

$$v^2_d = \frac{1}{\rho}\left[\frac{\lambda + 2\mu}{\Gamma} + (\lambda + \mu)\right]. \tag{27}$$

4. Numerical Example

We investigate the effect of micropolarity and angular rotation on the speed of shear and dilation waves by using the following relevant parameters from Somaiah and Kumar [17] for aluminum epoxy material and this material modelled as an isotropic generalized micropolar materials: $\alpha = 0.036 \times 10^{10}\,\text{N}$; $\beta = 0.037 \times 10^{10}\,\text{N}$; $\gamma = 0.0268 \times 10^{10}\,\text{N}$; $K = 0.0149 \times 10^{10}\,\text{N/m}^2$; $\lambda = 7.59 \times 10^{10}\,\text{N/m}^2$; $\mu = 1.89 \times 10^{10}\,\text{N/m}^2$; $\rho = 2190\,\text{kg/m}^3$; $J = 0.000196\,\text{cm}^2$. Natural angular frequency $\omega = 10\,\text{Hz}$. Magnitude of angular rotation speed Ω_0 taken as $\Omega_0 = 0, 0.2, 0.4, 0.6$. The variation of angle of rotation θ in degrees taken as $0^\circ \le \theta \le 180\circ$. The variation of θ versus the phase speeds of shear waves and dilation waves drawn with the use of MATLAB software.

5. Illustrations

The effect of angular rotation on phase speed of shear waves in micropolar and generalized elastic solids are shown in Figure 1 and Figure 2.

From these figures we observed that shear waves are propagating with constant speed in non-rotating materials, and the speed of shear waves are increasing with decreasing angular rotations in the given range of θ and they have very low speed at $\theta = 80^\circ$ and high speed at $\theta = 110^\circ$. The effects of rotation on comparative shear waves of micropolar and generalized solids with classical theory for ($K \to 0$) are shown in Figures 3 and 4.

From this figures we observed that shear waves in rotating classical elastic solids are slower than rotating micropolar and rotating generalized (non-micropolar) elastic solids. Classical shear waves are constant in high speed rotating solids, while micropolar shear waves are constant in low speed rotating solids. But shear waves in all rotating generalized elastic solids are constant. The variation of angle of rotation and speed of dilation waves in micropolar and classical elastic solids are shown in Figures 5 and 6, respectively.

From these figures we noticed that dilational waves are proportional to the speed of angular rotation of micropolar solids. Dilational waves also constant in non-rotating solids. Dilational waves are inverse proportional to the angular rotation speed in classical elastic solids.

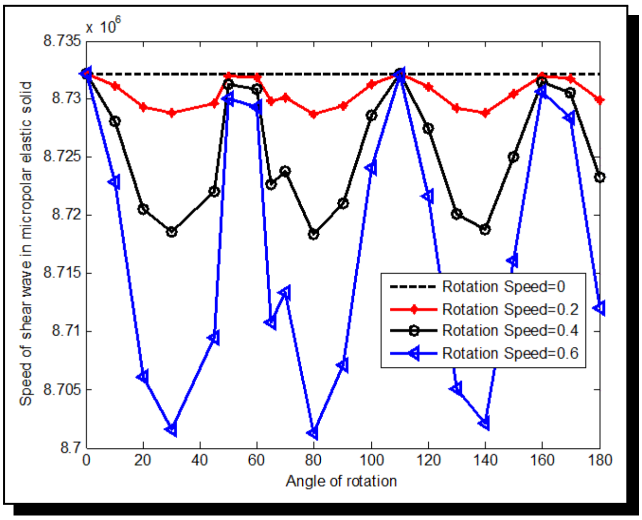

Figure 1. Angle of rotation versus Speed of shear wave in Micropolar elastic solid

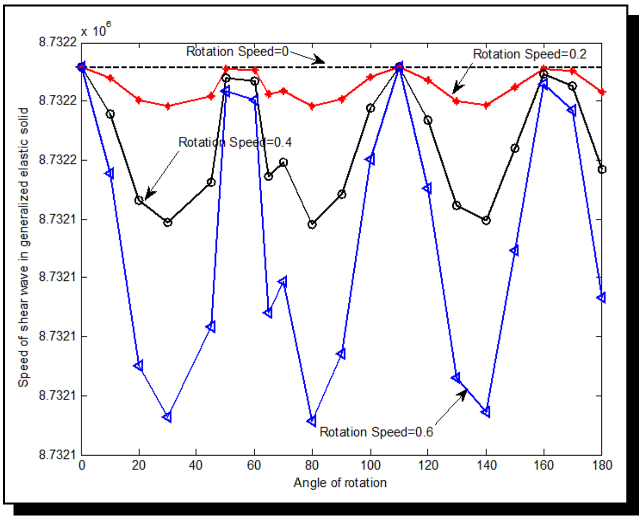

Figure 2. Angle of rotation versus Speed of shear wave in generalized solid

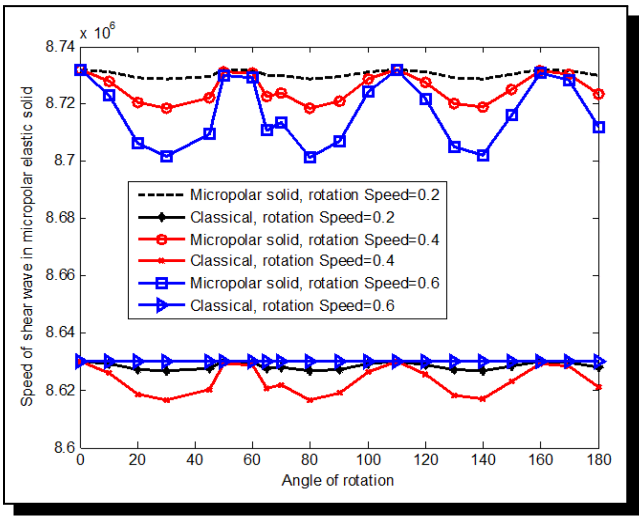

Figure 3. Angle of rotation versus Speed of shear wave in micropolar elastic solid

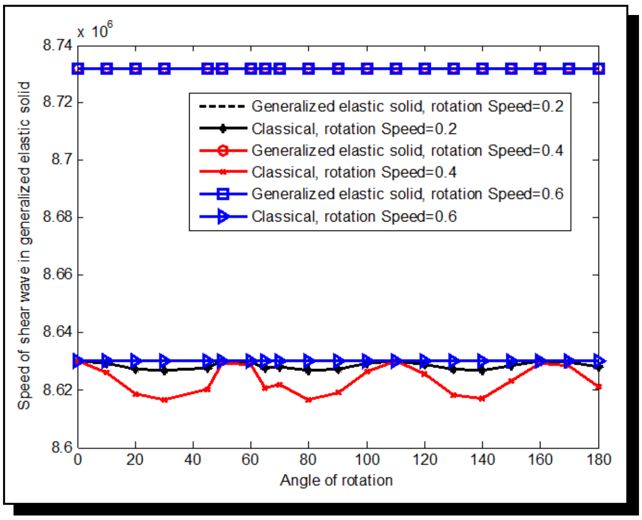

Figure 4. Angle of rotation versus Speed of shear wave in generalized solid

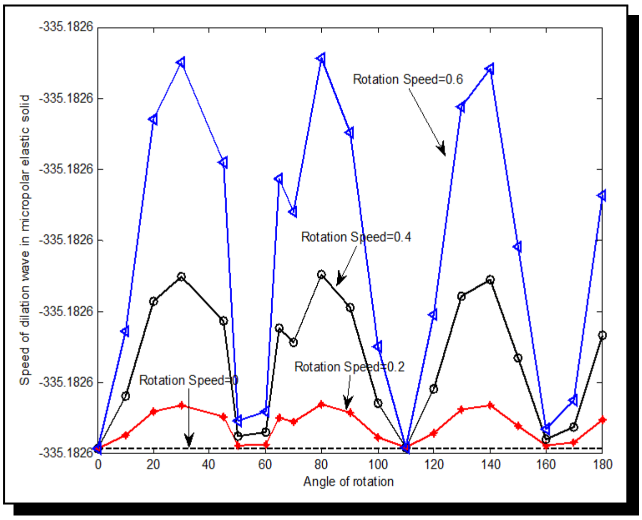

Figure 5. Angle of rotation versus Speed of dilation wave in micropolar elastic solid

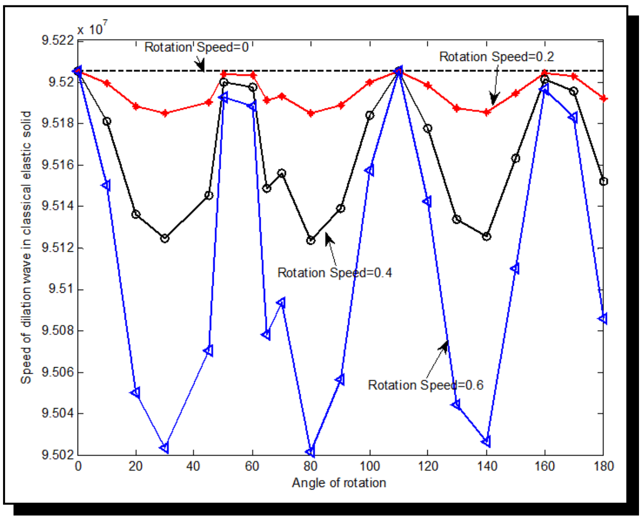

Figure 6. Angle of rotation versus Speed of dilation wave in classical elastic solid

6. Concluding Remarks

We investigate the effect of angular rotation on propagation of plane harmonic waves. From theoritical illustrations and a particular numerical example we conclude that:

(i) Among four harmonic plane waves, we observed that two sets of shear waves and two sets dilation waves are propagating.

(ii) All harmonic plane waves are affected by angular rotation of the solid.

(iii) The plane harmonic waves are not allowed to propagate in the solids with infinite rotation speed.

(iv) All these waves are dispersive in nature.

(v) Plane waves are constant in non-rotating materials.

(vi) Shear waves in micropolar and generalized elastic solids are inverse proportional to the speed of angular rotation of the solids.

(vii) Shear waves in rotating classical elastic solids are slower than rotating generalized elastic solids.

(viii) Dilation waves of classical elastic solids also inverse proportional to the speed of angular rotation.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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