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Analysis of the Form Factors Present in the Description of Heavy Vector Meson Decay

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Abstract In recently published work by the author, in which the decay rates of all known $1S$ vector meson states have been explained via the Gluon Emission Model (GEM), form factors, f_j , come into play as associated with the descriptions of the decays of the $\Psi(1S)$ and the $\Upsilon(1S)$, where, in the case of the $\Psi(1S)$, $f_1 = 1 - q_s^2$ (q_s represents the charge of the strange (s) quark in units of the electron charge), and, in the case of the $\Upsilon(1S)$, $f_2 = 1$. Specifically, f_j represents the fraction of relevant given vector meson states which make a point-like transition to a quark/anti-quark structure of the next lesser mass ...charm/anti-charm (cc^*) to strange/anti-strange (ss^*) in the case of the $\Psi(1S)$ and bottom/anti-bottom (bb^*) to cc^* in the case of the $\Upsilon(1S)$ ...the latter respective structures either forming the major portion of the decay scheme (the $\Psi(1S)$), or its entirety (the $\Upsilon(1S)$). Investigation of $\Psi(NS)$ and $\Upsilon(NS)$ states, with $N > 1$, has revealed three highly interesting eventualities: (1) f_1 retains its form noted above as associated with all presently known $\Psi(NS)$, while (2) f_2 is seen to be unique to the $\Upsilon(1S)$, as for the known $\Upsilon(NS)$ states, the resulting form factor is seen to be $f_3 = 1 - q_c^2$, where q_c represents the charm quark charge. In addition to the above it appears convincingly that (3) for a respective given "N" such that $N \geq 2$, quark color disengagement from lepton production takes place. In the work which follows we attempt to represent the above-mentioned form factors in a logically consistent way as stemming from what we term as "Reduction Operators". Necessarily, f_2 is of a slightly different form than that of f_1 or f_3; therefore, we posit a logical reason as to the nature of the difference, viz., the $\Upsilon(1S)$ never does find itself as a bb^* construction. Rather, it starts out as and decays as a cc^* construction. In addition, from a detailed look into the situation pertaining to the $\psi(NS)$ decay, we suggest that "quark color disengagement" from the decay of the relevant $N \geq 2$ states is consistent what we denote as "dimensional reduction", which is seen to involve reflection-invariant arrays of entangled di-quark structures within an assumed cubic lattice arrangement of same. Investigation of the analogous situation pertaining to the $\Upsilon(NS)$ decay suggests, on the other hand, that "dimensional reduction" is its own phenomenon. Nevertheless, we attempt to make the case that the two phenomena are intricately tied together.

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1. Introduction

The Gluon Emission Model (GEM), first proposed by F.E. Close[1] in the late 1970s, has been shown to be very successful in describing the widths of all known vector mesons in their ground states[2], as well as in describing the leptonic partial widths of all known vector mesons in their excited states[3]. The principal features of the decay scheme of vector mesons in accord with the GEM involve (1) the absorption of a virtual gluon by the vacuum to create a virtual di-quark pair, (2) a spin-spin interaction, proportional to q_i^2, where q_i represents the charge in units of the electron charge associated with the relevant quark flavor, i, to create a virtual spin one state for said di-quark pair, and (3) the emission of a gluon, which terminates in the hadronization process, which produces various hadrons[4]. In the case of the $\rho(770)$ a sum over the constituent up/anti-up (uu^*) and down/anti-down (dd^*) quark charges is required, leading to the GEM description of the ρ width as proportional to $\{q_u^4 + q_d^4\}$. For the $\varphi(1020)$ the GEM describes the associated kaon partial width as proportional to q_s^4, where "s" is the designation for the "strange" quark flavor. The situation regarding the "heavy" vector mesons, i.e., the $\psi(1S)$ and the $\Upsilon(1S)$, turns out to be much more interesting, as, as can be shown[5], the $\psi(1S)$ decays predominately in proportion to q_s^4, *not* in proportion to q_c^4, where "c" designates the "charm" flavor, as one might naively expect, and the $\Upsilon(1S)$ decays *exclusively* in proportion to q_c^4, *not* in proportion to q_b^4, where "b" designates the "bottom" quark flavor. Regarding the latter two cases, extremely accurate determinations of the widths of the $\psi(1S)$ and $\Upsilon(1S)$ may be obtained via the introduction of what we designate as "form factors" (f_j) as associated with each decay scheme ($j = 1$ for the $\psi(1S)$ scheme, and $j = 2$ for the $\Upsilon(1S)$ scheme). Specifically, f_j represents the fraction of relevant given vector meson states which make a point-like transition to a quark/anti-quark structure of the next lesser mass and the same energy as that of the original structure . . . cc^* to ss^* in the case of the $\Psi(1S)$ and bb^* to cc^* in the case of the $\Upsilon(1S)$ . . . the latter respective structures either forming the major portion of the decay scheme (the $\Psi(1S)$), or its entirety (the $\Upsilon(1S)$), as, empirically, it is found that

$$f_1 = 1 - q_s^2 = 8/9 \tag{1}$$

and

$$f_2 = 1\,. \tag{2}$$

[1] F.E. Close, *An Introduction to Quarks and Partons*, Academic Press, 1979.

[2] See, for example, the chapter titled, "The Gluon Emission Model for Vector Meson Decay" (by D. White) in *Quantum Mechanics, Book 2* (2011), published by InTech Publishing.

[3] Ibid. as #2 above.

[4] See, for example, D. White (2008), The Gluon Emission Model for Hadron Production Revisited, *Journal of Interdisciplinary Mathematics* 11 (4), 543–551.

[5] See, for example, #2 above and D. White, GEM and the $\Upsilon(1S)$, *The Journal of Informatics and Mathematical Sciences* 2 (2-3) (2010), 71–93.

To see how f_1 and f_2 fit into the general GEM calculations of the widths of the $\psi(1S)$ and the $\Upsilon(1S)$, we exhibit the basic ansatz below in terms of the general GEM framework. Specifically, the width of the $\psi(1S)$ is given via the GEM as

$$\Gamma_{\psi(1S)}(\text{GEM}) = f_1 \; \{ss^* \text{ decay scheme}\} + (1 - f_1) \; \{cc^* \text{ decay scheme}\}, \tag{3a}$$

or

$$\Gamma_{\psi(1S)}(\text{GEM}) = (8/9) \; \{ss^* \text{ decay scheme}\} + (1/9) \; \{cc^* \text{ decay scheme}\}. \tag{3b}$$

The result obtained by carrying out the GEM calculation is[6]

$$\Gamma_{\psi(1S)}(\text{GEM}) = 93.43 \text{ Kev}, \tag{3c}$$

which represents a match to the Particle Data Group (PDG) 2008 result[7] of (93.2 ± 2.1) Kev.

The width of the $\Upsilon(1S)$ is likewise given via the GEM generally as[8]

$$\Gamma_{\Upsilon(1S)}(\text{GEM}) = f_2\{cc^* \text{ decay scheme}\} + (1 - f_2) \; \{bb^* \text{ decay scheme}\}, \tag{4a}$$

or

$$\Gamma_{\Upsilon(1S)}(\text{GEM}) = \{cc^* \text{ decay scheme}\}, \tag{4b}$$

leading to

$$\Gamma_{\Upsilon(1S)}(\text{GEM}) = 54.02 \text{ Kev}, \tag{4c}$$

an exact match to the PDG result[9] of 2008.

It turns out to be possible to determine[10] the electron/positron (ee^*) partial widths of the excited states of the $\psi(1S)$ and $\Upsilon(1S)$, viz., the $\psi(NS)$ and $\Upsilon(NS)$ states, respectively, with $N = 2$. The evidence is very strong that for the $\psi(NS)$ states, the relevant form factors required[11] are each equal to $f_1 = (1 - q_s^2) = (8/9)$, and that for the $\Upsilon(NS)$ states, the relevant form factors[12] are each equal to $f_3 = (1 - q_c^2) = (5/9)$. In addition it is discovered[13] that, apparently, fewer than three quark colors are operative in the ee^* decays associated with all $\psi(NS)$ decays with $N \geq 2$ and all $\Upsilon(NS)$ decays with $N = 3$. In the following section we will attempt to develop a logical framework for representing the various form factors encountered in the GEM schemata, accounting for their mathematical forms by

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[6] See #2 and #5 above and D. White (2009), GEM and the Leptonic Width of the $J(3097)$, *Journal of Applied Global Research* 2 (4), 1–5.

[7] PDG (2008), pdg.lbl.gov, "Meson Table".

[8] Ibid. as #6 above.

[9] Ibid. as #7 above.

[10] D. White (2010), Evidence for Color-by-Color Disengagement from the Process of Lepton Production Associated with the ψ-Series and Υ-Series Mesons, *Communications in Mathematics and Applications* 1 (3), 183–193.

[11] Form Factor Analysis Derived from the Gluon Emission Model Applied to the $\psi(2S)$ and the $\Upsilon(2S)$, *Communications in Mathematics and Applications* 1 (3) (2010), 165–181.

[12] Ibid. as #11 above.

[13] Ibid. as # 10 above.

means of looking carefully at the associated physics. Further on we will attempt to represent quark color disengagement in the process of ee^* decay as a phenomenon involving what we may call, "access arrays".

2. f_j in Terms of Reduction Operators

Defining $\boldsymbol{\Psi}_k$ as representing the relevant virtual di-quark state of a vector meson prior to its decay, where $k = 1 - 6$ indicates quark flavor involved ($k = 1$ indicates "up"; $k = 2$ indicates "down"; $k = 3$ indicates "strange" ; $k = 4$ indicates "charm"; $k = 5$ indicates "bottom"; $k = 6$ indicates "top"), we may define what we term the "Reduction Operator", $\boldsymbol{R}$, associated with the partially point-like transitions described above via the following:

$$\boldsymbol{R}\boldsymbol{\Psi}_k = (1 - q_{k-1}^2)\boldsymbol{\Psi}_{k-1} + q_{k-1}^2\boldsymbol{\Psi}_k, \tag{5}$$

where $\boldsymbol{\Psi}_{k-1}$ is identical to $\boldsymbol{\Psi}_k$ in every respect, except as regards the charge associated with its di-quark structure. The reduction operator thus describes the transition from the original di-quark state associated with any of the $\Psi(NS)$ states ($N = 1$) or any of the $\Upsilon(NS)$ states such that $N = 2$ to a di-quark state associated with quarks of the next lowest mass relative to the quarks making up the original di-quark state. Associated with the $\Upsilon(1S)$ would be a unique reduction operator, $\boldsymbol{R}$, such that

$$\boldsymbol{R}_0\Upsilon(1S) = \Psi(1S)^*, \tag{6}$$

where $\Psi(1S)^*$ signifies an excited state of the $\Psi(1S)$ whose mass equals that of the $\Upsilon(1S)$ and which decays via a cc^* mechanism as per the GEM. As the general form for $\boldsymbol{R}_0$ would be given by

$$\boldsymbol{R}_0\boldsymbol{\Psi}_k = \boldsymbol{\Psi}_{k-1}, \tag{7}$$

note from Eq. (5) and Eq. (7) that the expectation value of $\boldsymbol{R}$, denoted by $\langle R\rangle$, is given by

$$\langle R\rangle = \int \boldsymbol{\Psi}_k * \boldsymbol{R}\boldsymbol{\Psi}_k d^4x = q_{k-1}^2 \tag{8}$$

and that that of $\boldsymbol{R}_0$, denoted by $\langle R_0\rangle$, is given by

$$\langle R_0\rangle = \int \boldsymbol{\Psi}_k * \boldsymbol{R}_0\boldsymbol{\Psi}_k d^4x = \int \boldsymbol{\Psi}_k * \boldsymbol{\Psi}_{k-1} = 0\,, \tag{9}$$

assuming, of course, that $\boldsymbol{\Psi}_k$ is orthogonal to $\boldsymbol{\Psi}_{k-1}$. In cases involving any of the $\Psi(NS)$ or any of the $\Upsilon(NS)$ states, therefore, it is always such that

$$f_{1,3} = 1 - \langle R\rangle, \tag{10a}$$

and

$$f_2 = 1 - \langle R_0\rangle\,. \tag{10b}$$

Hence, we see that there is a very simple relationship between the form factors which are empirically deduced as part of the GEM formalism and the associated reduction operators as defined above. Now, since it is actually $\boldsymbol{R}\,\boldsymbol{\Psi}_k$ (or $\boldsymbol{R}_0\,\boldsymbol{\Psi}_k$) that decays via a spin-spin interaction of the resulting di-quark pair, it is seen from above that the $\Upsilon(1S)$ is in actuality a "ghost state", in that it never really materializes at all: rather, what materializes out of the vacuum at the $\Upsilon(1S)$ mass is an excited cc^* state, not a bb^* state. To emphasize the point, a bb^* construction *plays no role* in the decay of the $\Upsilon(1S)$.

3. Color Disengagement from Lepton Production

In what follows it will be convenient to make the following definitional/notational significations:

Let $\boldsymbol{R}_1$ be such that

$$\boldsymbol{R}_1\psi(NS) = (1-q_s^2)\phi(NS)^* + q_c^2\psi(NS) = (8/9)\phi(NS)^* + (1/9)\psi(NS), \tag{11a}$$

where $\phi(NS)^*$ is an excited state of the ϕ-meson, which is identical with the relevant $\psi(NS)$ state in every respect, except that it comprises an ss^* virtual state, rather than a cc^* virtual state. Hence,

$$\langle R_1 \rangle = 1/9, \tag{11b}$$

which, again, corresponds to $f_1 = 8/9$. Let $\boldsymbol{R}_2 = \boldsymbol{R}_0$, so that $\boldsymbol{R}_2\Upsilon(1S) = \Psi(1S)^*$ (see Eq. (6)) and $\langle R_2 \rangle = 0$, leading to, again, $f_2 = 1$.

Finally, let $\boldsymbol{R}_3$ be such that

$$\boldsymbol{R}_3\Upsilon(NS) = (1-q_c^2)\psi(NS)^* + q_b^2\Upsilon(NS) = (5/9)\psi(NS)^* + (4/9)\Upsilon(NS), \tag{12a}$$

where $N \geq 2$ and $\psi(NS)^*$ is an excited state of the ψ-meson, which is identical in every respect with the relevant $\Upsilon(NS)$, except that it comprises a cc^* virtual state, rather than a bb^* virtual state. Hence,

$$\langle R_3 \rangle = 4/9, \tag{12b}$$

which, again, corresponds to $f_3 = 5/9$.

3.1. *Color Disengagement Associated with the $\boldsymbol{\psi}(NS)$ Array*

That the effect due to f_1 upon the width calculation of the entire $\psi(NS)$ array be on the order of $\langle R_1 \rangle = q_s^2 = 1/9 \sim 11\%$ is most surprising …shocking, really: Strictly electromagnetic effects in Quantum Electrodynamics always go as the square of the relevant charge multiplied by the fine structure constant, $\alpha = 1/137.036$, not merely as the square of the relevant charge. So …how to make sense of the empirical result from Reference [4] that $f_1 = 1 - \langle R_1 \rangle = 8/9$? We believe we have the answer to such question: We believe that the reduction operator, $\boldsymbol{R}_1$, not only reforms the relevant initial wave function ($\psi(NS)$), but also reforms the neighboring *space*. Specifically, assuming a cubic quark lattice as a constituent of the vacuum, we postulate that, concomitant with Eq. (11a), $\boldsymbol{R}_1$ also transforms the cc^* lattice point describing the initial resonant state of the $\psi(NS)$ into a planar square, which contains a cc^* state at its center and eight surrounding ss^* states according to the following diagram:

ss^*	ss^*	ss^*
ss^*	cc^*	ss^*
ss^*	ss^*	ss^*

Figure 1. The result of $\boldsymbol{R}_1$ operating on the original cc^* state.

Associated with a colliding beams experiment, defining the beams' directional axis at the site of collision as the x-axis, the y-axis in the horizontal plane and perpendicular to the x-axis, and the z-axis, vertical, orthogonal to the xy horizontal plane, the lattice structure depicted above may form in either the xy plane, the xz plane, or the yz plane. Note that there are eight viable ss^* structures involved in the decay process and only one cc^* structure, each assumed to be accessible with equal probability, commensurate with $f_1 = 8/9$.

At the present juncture we reproduce the table from ref. [7] which illustrates the empirical evidence associated with color disengagement from lepton production associated with the $\psi(NS)$ array (all widths in Kev):

Table 1. Color Participation in Lepton Production in the $\boldsymbol{\Psi}$-Series

Meson	Mass (Mev)	Γ_{ee}(GEM)	$\Gamma_{ee}(PDG)$	# of Colors Operative
$\boldsymbol{\Psi}(1S)$	3097	5.72	5.55 ± 0.16	3
$\boldsymbol{\Psi}(2S)$	3686	2.26	2.36 ± 0.04	2
$\boldsymbol{\Psi}(3S)$	4039	0.86	0.86 ± 0.07	1
$\boldsymbol{\Psi}(4S)$	4153	0.79	0.83 ± 0.07	1
$\boldsymbol{\Psi}(5S)$	4421	0.65	0.58 ± 0.07	1

The ee^* partial width of the $\psi(1S)$ calculated via the GEM is a near match to experiment, as seen from viewing Table 1 above, assuming three quark colors are operative in the decay. In terms of Figure 1 above, the near match is gained by assuming that all nine entities pictured are "on", i.e., the decay is due to a set of viable di-quark resonances comprising eight ss^* states to one cc^* state, each accessible with equal probability. Since the cc^* state decays proportional to $q_c^4 = 16/81$, and each ss^* state decays proportional to $q_s^4 = 1/81$, the $\psi(2S)$ decay may be seen to decay via *only* the cc^* structure, the eight ss^* structures "turned off", leading to two-thirds the "everything on" effect . . . or . . . two colors operating out of three. The "one color operating" associated with the $\psi(3S)$, $\psi(4S)$, and $\psi(5S)$ may be seen as due to the "turning on" of all eight ss^* states, with the cc* state "turned off". Quark color may well be, then, tied intimately to "quark flavor entanglement" associated with an inherent vacuum lattice structure.

3.2. *Color Disengagement Associated with the $\Upsilon(NS)$ Array*

Color disengagement is a bit more complicated as associated with the $\Upsilon(NS)$ array as compared to that associated with the $\psi(NS)$ array. At the present juncture it would be best to take a look at the chart from ref. [7] which illustrates the details as to color disengagement associated with the $\Upsilon(NS)$ array (all widths in Kev):

Table 2. Color Participation in Lepton Production in the Υ-Series

Meson	Mass (Mev)	Γ_{ee}(GEM)	$\Gamma_{ee}(PDG)$	# of Colors Operative
$\Upsilon(2S)$	10023	0.624	0.612 ± 0.11	3
$\Upsilon(3S)$	10355	0.471	0.443 ± 0.008	21/2
$\Upsilon(4S)$	10579	0.266	0.272 ± 0.029	11/2
$\Upsilon(5S)$	10860	0.33	0.31 ± 0.07	2
$\Upsilon(6S)$	11019	0.157	0.13 ± 0.03	1

Recall that in $\Upsilon(1S)$ decay the associated form factor, f_2, is one. Thus, $\boldsymbol{R}_2$ converts the bb^* virtual state instantaneously to a cc^* state. The form factor associated with each item in Table 2, however, is $f_3 = 5/9$, consistent with $\langle R_3 \rangle = 4/9$. Therefore, $\boldsymbol{R}_3$ may be thought of as transforming the original relevant bb^* virtual state at the collision site into, again, a planar array in the quark lattice as follows:

$$\begin{matrix} cc^* & bb^* & cc^* \\ bb^* & cc^* & bb^* \\ cc^* & bb^* & cc^* \end{matrix}$$

Figure 2. The result of $\boldsymbol{R}_3$ operating on the original bb^* state.

As associated with the $\Upsilon(2S)$, we assume that each entity in Figure 2 is accessible for decay with equal probability, as all three colors participate therein. Focusing next upon the $\Upsilon(5S)$, which is listed as having only two colors participate in its decay, in analogous manner to the above regarding the $\psi(2S)$, we would now search for a subsection of the array pictured in Figure 2 such that its being "on", i.e., accessible for decay, represents two thirds of the decay rate associated with all entities in the full array participating. Given that $q_b^4 = 1/81$, unfortunately, no such sub-array exists. The closest that we can come to a parity-conserving representation of "two-thirds what's possible" in the full array is the sub-array where any two opposite-corner cc^* states are "off" . . . the remaining entities "on". Such a sub-array would be $(52/84) = 61.90\%$ "on" relative to the full array's capability. If one now recalculates the GEM-theoretical width of the $\Upsilon(5S)$ on such basis, one obtains an exact match with experiment, i.e., a result of 0.31 Kev for the width of the $\Upsilon(5S)$.

Similarly, the best we can do as associated with the $\Upsilon(6S)$, which ostensibly has only one color operating, is to test the sub-array of Figure 2 where any two of the opposite corner cc^* states are "on" ... the rest "off", leading to not one third of the decay rate relative to all entities "on", but $(32/84) = 38.10\%$ of them. A subsequent recalculation of the GEM-theoretical width of the $\Upsilon(6S)$ yields 0.18 Kev, which is about 0.02 Kev outside the range of experimental uncertainty. Things work out fairly well, however, when considering the "integer plus a half" color situations. The sub-array that would be associated with the $\Upsilon(3S)$, in which, ostensibly, "2 1/2 colors" are operating, would be the one in which only the central cc^* state is "off". The fractional contribution from such an array would be $(68/84) = 80.95\%$. Here, the associated recalculated GEM-theoretical width is a better match to experiment at 0.458 Kev. The best sub-array associated with the $\Upsilon(4S)$ ("1 1/2 colors operating") is one in which only three cc^* states along a diagonal are "on". Then, the new GEM-theoretical width comes out to be 0.311 Kev ... just outside of the range of experimental uncertainty. Overall, then, the average relative discrepancy between the GEM-theoretical widths associated with the $\Upsilon(NS)$ series $(N = [2-5])$ is augmented only slightly (from 4.2% to 4.9%) by adopting the ansatz associated with the reduction operator, $\boldsymbol{R}_3$, rather than assuming color-by-color disengagement, which reduces the relevant decay widths by integer thirds; for $N = 6$ the reduction operator ansatz moves the GEM-theoretical result for the width of the $\psi(6S)$ just outside the range of experimental uncertainty.

4. Do the Quark Lattice Arrays "Blink"?

Let us now develop some notational symbology associated with the various quark lattice arrays encountered in the previous section. Let $\boldsymbol{A}$ be the general designate of an array such as seen in Figures 1 or 2 above. Specifically, let $\boldsymbol{A}_{\psi 1}$ represent the array as seen in Figure 1 associated with the $\psi(1S)$; further, the arrays associated with the $\psi(2S)$ and the $\psi(3S)$ will have designations in accord with Figures 3 and 4, respectively, immediately below, in which figures the dull grey color associated with a di-quark state signifies that said state is "off".

$$\begin{matrix} ss^* & ss^* & ss^* \\ ss^* & cc^* & ss^* \\ ss^* & ss^* & ss^* \end{matrix}$$

Figure 3. The $\psi(2S)$ Array: $\boldsymbol{A}_{\psi 2}$ (consistent with "2 colors operating")

$$\begin{matrix} ss^* & ss^* & ss^* \\ ss^* & cc^* & ss^* \\ ss^* & ss^* & ss^* \end{matrix}$$

Figure 4. The ψ $(\geq 3S)$ Array: $\boldsymbol{A}_{\psi 3}$ (consistent with "1 color operating")

One thing we immediately notice about $\boldsymbol{A}_{\psi 2}$ and $\boldsymbol{A}_{\psi 3}$ is that they are the complements of each other, i.e., adopting a matrix notation in which all "off" states are represented by zeros, we obviously find:

$$\boldsymbol{A}_{\psi 1} = \boldsymbol{A}_{\psi 2} + \boldsymbol{A}_{\psi 3}\,. \tag{13}$$

At the instant any $\psi(NS)$ state is created, at the center of any of the relevant arrays is a cc^* virtual di-quark state acting as a "transmitter" of the energy contained therein. Each of the off-center points that are "on" in a given array, be it $\boldsymbol{A}_{\psi 1}$, $\boldsymbol{A}_{\psi 2}$, or $\boldsymbol{A}_{\psi 3}$, act as potential receivers of said energy, each with equal probability. In terms of any array, we define "blinking" as transforming back and forth between it and its complement. We may cause $\boldsymbol{A}_{\psi 2}$ and $\boldsymbol{A}_{\psi 3}$ to "blink", therefore, if we can provide for the energy of colliding beams to oscillate back and forth between the threshold for $\psi(2S)$ production and beyond that for $\psi(3S)$ production. So ... yes; quark lattice arrays can "blink". In addition Eq. (13) has a corollary, viz., that the number of colors associated with $\boldsymbol{A}_{\psi 1}$ equals the sum of the number of colors associated with $\boldsymbol{A}_{\psi 2}$ and $\boldsymbol{A}_{\psi 3}$, respectively.

Let us now designate $\boldsymbol{A}_{\Upsilon 2}$ as the array represented by the array shown in Figure 2, which is consistent with the assumption that all three colors are operative in $\Upsilon(2S)$ decay. Focusing now upon the "2 color decay scheme", that of the $\Upsilon(5S)$, we would represent its associated array as

$$\boldsymbol{A}_{\Upsilon 5} = 1/2\{\boldsymbol{A}^{L}_{\Upsilon 5} + \boldsymbol{A}^{R}_{\Upsilon 5}\},$$

where $\boldsymbol{A}^{L}_{\Upsilon 5}$ and $\boldsymbol{A}^{R}_{\Upsilon 5}$ are as pictured in Figures 5 and 6 immediately below.

cc^*	bb^*	cc^*
bb^*	cc^*	bb^*
cc^*	bb^*	cc^*

Figure 5. $\boldsymbol{A}^{L}_{\Upsilon 5}$ associated with the $\Upsilon(5S)$ (consistent with "1.86 colors operating")

cc^*	bb^*	cc^*
bb^*	cc^*	bb^*
cc^*	bb^*	cc^*

Figure 6. $\boldsymbol{A}^{R}_{\Upsilon 5}$ associated with the $\Upsilon(5S)$ (consistent with "1.86 colors operating")

Again, $\boldsymbol{A}_{\Upsilon 5}$ as the basis for the $\Upsilon(5S)$ decay would yield an associated GEM-theoretical width of $(1.86/2) = 0.93$ times that contained in Table 2, or 0.31 Kev, an exact match to the experimental result, whose range of uncertainty is rather extensive ($\pm 23\%$).

The array associated with the "1 color decay", that of the $\Upsilon(6S)$, designated as $\boldsymbol{A}_{\Upsilon 6}$, is the complement of $\boldsymbol{A}_{\Upsilon 5}$, i.e., $\boldsymbol{A}_{\Upsilon 6} = 1/2\{\boldsymbol{A}^{L}_{\Upsilon 6} + \boldsymbol{A}^{R}_{\Upsilon 6}\}$, where $\boldsymbol{A}^{L}_{\Upsilon 6}$ and $\boldsymbol{A}^{R}_{\Upsilon 6}$ are pictured in Figures 7 and 8 below.

$$\begin{array}{ccc} cc^* & bb^* & cc^* \\ bb^* & cc^* & bb^* \\ cc^* & bb^* & cc^* \end{array}$$

Figure 7. $A^L_{\Upsilon 6}$ associated with the $\Upsilon(6S)$ (consistent with "1.14 colors operating")

$$\begin{array}{ccc} cc^* & bb^* & cc^* \\ bb^* & cc^* & bb^* \\ cc^* & bb^* & cc^* \end{array}$$

Figure 8. $A^R_{\Upsilon 6}$ associated with the $\Upsilon(6S)$ (consistent with "1.14 colors operating")

Again, $\boldsymbol{A}_{\Upsilon 6}$ as the basis for the $\Upsilon(6S)$ decay would yield an associated GEM-theoretical width of 1.14 times that contained in Table 2, or 0.18 Kev, a figure 0.02 Kev outside of the experimental range of (0.13 ± 0.03) Kev. Here, again, the range of experimental uncertainty is rather extensive (again, $\pm 23\%$).

Turning now to the $\Upsilon(3S)$, associated with "2 1/2 colors" operating in its decay, its relevant array, designated by $\boldsymbol{A}_{\Upsilon 3}$, is depicted in Figure 9 below.

$$\begin{array}{ccc} cc^* & bb^* & cc^* \\ bb^* & cc^* & bb^* \\ cc^* & bb^* & cc^* \end{array}$$

Figure 9. $A_{\Upsilon 3}$ associated with the $\Upsilon(3S)$ (consistent with "2.43 colors operating")

Again, $\boldsymbol{A}_{\Upsilon 3}$ as the basis for the $\Upsilon(3S)$ decay would yield an associated GEM-theoretical width of $(2.43/2.50) = 0.972$ times that contained in Table 2, or 0.458 Kev, a figure only 3.4% discrepant from a very well-known experimental result. Relative to assuming "2.5 colors" operating in the decay of the $\Upsilon(3S)$, the reduction operator ansatz represents a significant improvement, the relative discrepancy between theory and experiment dropping by almost a factor of two (from 6.3% to 3.4%). The above array, $\boldsymbol{A}_{\Upsilon 3}$, has a complement, which we may designate as $\boldsymbol{A}^C_{\Upsilon 3}$, the latter being associated with "1/2 color" (actually, "0.57 colors") operation, which is associated with an as yet to be discovered resonance.

The $\Upsilon(4S)$ presents us with a highly interesting situation. Ostensibly, the $\Upsilon(4S)$ decays with "1 1/2 colors" in operation. In the previous section we suggested an associated array, but from the foregoing it is clear that the proper description of the $\Upsilon(4S)$ involves the suggested array *and its complement*, i.e., we define the array associated with the basis for $\Upsilon(4S)$ decay as $\boldsymbol{A}_{\Upsilon 4} = \boldsymbol{A}^A_{\Upsilon 4} + \boldsymbol{A}^B_{\Upsilon 4}$, where $\boldsymbol{A}^A_{\Upsilon 4}$ and $\boldsymbol{A}^B_{\Upsilon 4}$ are complements of each other, as shown below in Figures 10 and 11.

Table 3. Color Participation in Lepton Production in the Υ-Series

Meson	Mass (Mev)	Γ_{ee}(GEM)	$\Gamma_{ee}(PDG)$	# of Colors Operative
$\Upsilon(2S)$	10023	0.624	0.612 ± 0.11	3
$\Upsilon(3S)$	10355	0.458	0.443 ± 0.008	2.43
$\Upsilon(4S)$	10579	0.266	0.272 ± 0.029	1 1/2
$\Upsilon(5S)$	10860	0.31	0.31 ± 0.07	1.86
$\Upsilon(6S)$	11019	0.18	0.13 ± 0.03	1.14

cc^*	bb^*	cc^*
bb^*	cc^*	bb^*
cc^*	bb^*	cc^*

Figure 10. $A^1_{\Upsilon 4}$ associated with the $\Upsilon(4S)$ (consistent with "1.71 colors operating")

cc^*	bb^*	cc^*
bb^*	cc^*	bb^*
cc^*	bb^*	cc^*

Figure 11. $A^2_{\Upsilon 4}$ associated with the $\Upsilon(4S)$ (consistent with "1.29 colors operating")

Implicit in Figures 10 and 11 is that $A^{1,2}_{\Upsilon 4} = 1/2\{A^{(1,2)L}_{\Upsilon 4} + A^{(1,2)R}_{\Upsilon 4}\}$, where the designations, $\boldsymbol{L}$ and $\boldsymbol{R}$ reflect that the cc^* states that are either all "on" (Figure 10) or all "off" (Figure 11) can be so along either of two diagonals in each array. Hence, assuming that $A_{\Upsilon 4}$ "blinks", i.e., its constituent complements are accessed randomly, each 50% of the time, the GEM-theoretical result from Table 2 as to the $\Upsilon(4S)$ is recovered, i.e., 0.266 Kev, only 2.2% discrepant from another very well-known experimental result.

Below, in Table 3, we reproduce Table 2, except that we insert the GEM-theoretical results for the relevant width calculations as based upon the reduction operator ansatz as presented so far; GEM-theoretical results written in red indicate improvements over the ansatz whereby color is allowed to contribute in integer amounts, while the one entry written in blue represents a setback. Given the rather extensive range in experimental uncertainty associated with the latter, we think that, overall, the reduction operator ansatz (ROA) represents an improvement relative to the way quark color has been priorly considered.

5. Is the ROA a "Color" Phenomenon?

A highly interesting feature of the foregoing is that whereas associated with the $\psi(NS)$ series the number of colors operating decreases monotonically with increasing N (see Table 1), the number of colors operating in the $\Upsilon(NS)$ series decays appears to be somewhat haphazard as they relate to N (see Tables 2 and 3). However, if one looks carefully at the sub-arrays associated with the entities in the $\Upsilon(NS)$ series, the number of colors operating is at least quasi-monotonically tied to a *metric*. Specifically, where $\boldsymbol{S}$ represents the sum of the distances from the center of a given array to the "on" virtual di-quark states, in terms of "d", the distance between neighboring di-quark sites, we obtain the following as associated with the $\Upsilon(NS)$ series:

$$\text{For the } \Upsilon(2S),\ \boldsymbol{S} = 4(1+\sqrt{2})d = 9.66d; \tag{14a}$$

$$\text{for the } \Upsilon(3S),\ \boldsymbol{S} = 4(1+\sqrt{2})d = 9.66d; \tag{14b}$$

$$\text{for the } \Upsilon(4S),\ \boldsymbol{S} = 4(1+\sqrt{2})d = 9.66d; \tag{14c}$$

$$\text{for the } \Upsilon(5S),\ \boldsymbol{S} = 2(2+\sqrt{2})d = 6.83d; \tag{14d}$$

$$\text{for the } \Upsilon(6S),\ \boldsymbol{S} = 2(\sqrt{2})d = 2.83d. \tag{14e}$$

In addition, if the $\Upsilon(7S)$ exists, its $\boldsymbol{S} = 0$. Hence, $\boldsymbol{S}$ either remains the same or decreases with increasing N. Thus, a review of the data shows that, apparently, the color contribution associated with $\psi(NS)$ decay decreases by integer amounts as N increases (see Table 1), while whatever fractional color contribution to $\Upsilon(NS)$ entities depends most strongly, instead, on the value of $\boldsymbol{S}$ as per Eq. (14). Now, according to the GEM, the T meson, a tt^* resonance, where "t" signifies the top quark, will behave exactly as does the ψ, as q_t, the charge of the top quark, equals q_c. So, among the heavy mesons, the Υ is the only one which behaves anomalously in terms of a "color turnoff" that is simply related to N, which, in turn, preserves the notion of "color" as originally constructed in the early days of quark physics theory-building, i.e., quark color is of three distinct types.

The question arises, then, as to whether the ROA is a formalism unrelated to color. Is it simply a coincidence that the $\psi(NS)$ series seems to exhibit color disengagement in its decays with increasing N? Or does the ROA *underlie* the phenomenon of color, in which case color can be "fragmented" as per Tables 2 or 3? To attempt to answer the above pressing questions, let us take a look at the situation involving the first vector meson to be discovered, the ρ-meson: It forms as either a uu^* resonance or a dd^* resonance with equal probability. Given a three dimensional quark lattice structure in the vacuum, it seems quite likely that given a uu^* di-quark state at some given point, there will be one of six likely companion dd^* states as neighbors to the uu^* mentioned which will be sought out by the color field to "balance the load", so to speak ... two on the x-axis, two on the y-axis, and two on the z-axis. For a high energy collision at a given point, if conditions are created such that the dd^* states along the z-axis are precluded from access by said field, then the ρ decay partial width due to the dd^* component would fall to two thirds of its "normal value". Similarly, if the neighboring dd^* sites along the y-axis are in addition precluded from access by the color field, said partial width would fall to only one-third normal. Though inherently different in nature from the reduction operators associated with the ROA, we see that in the above example a form of *spatial reduction* is present in the associated gedanken experiment . . . that of removing from possibility the color field's reaching the z-axis and/or the y-axis neighbors associated with a given uu^*, thus preventing said neighboring dd^* states from participating in the decay process of the ρ-meson, analogous to relevant di-quark states "turned off" as associated with the sub-arrays encountered above. We therefore surmise that, given a three-dimensional quark/anti-quark lattice structure in the vacuum, quark color is a *space* phenomenon, a phenomenon tied intimately to the arrangement of the various flavors of quarks in the lattice within three dimensional space . . . the number of spatial dimensions giving rise to exactly the number of quark colors formulated, i.e., three. Hence, the ROA in our view *does* represent a quark color phenomenon as described.

6. Implications as to String Theory

The ROA appears not to be compatible with String Theory (ST). First of all, ST provides for ten spatial dimensions[14] (sic), seven of them "curled around" the "observable" three dimensions of everyday experience. The radii associated with the "curling around" is envisioned to be very small, but as the energy of colliding beams becomes ever higher, the probability of "uncurling" the "hidden dimension(s)" increases. Now, the ROA suggests that quark color (QC) is intimately tied to the three "observable" space dimensions, as noted above. Freeing up the "hidden" dimensions ought, therefore, to reveal yet additional QCs to the known three. However, what the ROA shows is that as the colliding beam energy becomes higher, the number of QCs participating in the leptonic decay of the excited states of a given vector meson *decreases* . . . in exact opposition to what is expected from ST. Indeed, already the news from the LHC is that the "mini black holes" predicted by ST are not present . . . either at the expected energies . . . or at all. Secondly, the lattice array picture developed within the ROA fits perfectly with the empirically-determined mathematical structure of all form-factors associated with the heavy vector meson decays, and while $f_2 = 1$ would be quite compatible with a string changing its vibrational pattern from the bb^* manifestation to a cc^* manifestation at the site of a given collision, the structures associated with f_1 and f_3 are not. Except for the behavior of the $\Upsilon(1S)$, therefore, there is not extant the "economy" one would expect from ST.

7. Concluding Remarks

The quark lattice approach to vector meson decay has been shown to be quite useful as to the understanding of the decay schemes associated with the heavy vector mesons, but we feel that application of a similar approach can be also useful in the understanding of the decay schemes of the light vector mesons. Consider the mystery of the neighbors in energy space, the ρ and the ω: Why does one of them

[14] See, for example, Brian Greene (2004), *The Fabric of the Cosmos*, Knopf.

predominately decay via two pions (the ρ), while the other (the ω) decays via three pions? In terms of the following lattice arrays, each assumed to represent a collision at its center, the total number of pions that could be produced from each array associated with the spin-flip decay of the central di-quark state is five. Perhaps, then, the ρ/ω is *one* entity, an entity associated with the decay of either a dd^* di-quark state at the center of one of the three-by-three arrays below via a spin-flip, or same as regards a central uu^* state.

uu^*		dd^*
	dd^*	
dd^*		uu^*

Figure 12. Possible Array associated with ρ/ω Decay

dd^*		uu^*
	uu^*	
uu^*		dd^*

Figure 13. Possible Array associated with ρ/ω Decay

The GEM has also been successful in describing[15] the width of the $K^*(892)$. By filling in the blank spots in Figures 12 and 13 with ss^* sites, it is quite apparent as to how the decay products of the $K^*(892)$ result from such lattice structure: The $K^*(892)$ is not massive enough to be able to decay via two kaons (sd^*- or su^*-type structures), so, for example, in Figure 14 below a central "d" may join with an "s^*" at one of the ss^* sites, but the central "d^*" remnant must then join with a "u", say, at one of the uu^* sites, resulting in the emission of a pion and a kaon.

uu^*	ss^*	dd^*
ss^*	dd^*	ss^*
dd^*	ss^*	uu^*

Figure 14. Possible Array associated with $K^*(892)$ Decay

dd^*	ss^*	uu^*
ss^*	uu^*	ss^*
uu^*	ss^*	dd^*

Figure 15. Possible Array associated with $K^*(892)$ Decay

Finally, the $\phi(1019)$ decays predominately via two kaons, but it also possesses a small "ω-like" component, i.e., a three-pion decay route. By inspection, one

[15] D. White (2008), GEM and the $K^*(892)$, *Journal of Applied Global Research* 1 (3), 1–4.

realizes that replacing the central di-quark sites in Figures 12 and 13 by ss^* sites would result in two-kaon production as associated with the "s" and "s^*" joining with any "u/u^*" or "d/d^*" quark, leaving enough left in the array for the production of three pions. Given the success demonstrated by the ROA and the applicability of an analogous picture involving quark lattice arrays in the vacuum as associated with the light vector mesons, we suggest that the "quark sea on a lattice structure", so to speak, is a fundamental aspect to all quantum systems where deep inelastic interactions are involved.

$$\begin{array}{ccc} uu^* & & dd^* \\ & ss^* & \\ dd^* & & uu^* \end{array}$$

Figure 16. Possible Array associated with ϕ Decay

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