Journal of Informatics and Mathematical Sciences
Vol. 13, No. 2, pp. 105–117, 2021
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

DOI: 10.26713/jims.v13i2.1651

Research Article

Fuzzy Soft Positive Implicative Hyper BCK-Ideals

Tolesa Dekeba Bekele*

Department of Mathematics, College of Natural and Computational Science, Dire Dawa University, Dire Dawa, Ethiopia

Received: June 23, 2021 **Accepted:** August 3, 2021 **Published:** August 10, 2021

Abstract. In this paper, the author contributed the concepts of fuzzy soft positive implicative hyper BCK-ideal of types $(\ll,\subseteq,\subseteq)$, $(\ll,\ll,\subseteq)$ and $(\subseteq,\ll,\subseteq)$ are introduced some related properties are considered. Relations between double-framed soft hyper BCK-ideal and double-framed soft strong hyper BCK-ideal are discussed. Additionally, the author demonstrate that the level set of fuzzy soft positive implicative hyper BCK-ideal of types $(\ll,\subseteq,\subseteq)$, $(\ll,\ll,\subseteq)$ and $(\subseteq,\ll,\subseteq)$ are positive implicative hyper BCK ideal of types $(\ll,\subseteq,\subseteq)$, $(\ll,\ll,\subseteq)$ and $(\subseteq,\ll,\subseteq)$, respectively. The conditions for a fuzzy soft set to be a fuzzy soft positive implicative hyper BCK-ideal of types $(\ll,\subseteq,\subseteq)$, $(\ll,\ll,\subseteq)$ and $(\subseteq,\ll,\subseteq)$, are initiated respectively, and the circumstances for a fuzzy soft set to be a fuzzy soft weak hyper BCK-ideal are also considered.

Keywords. Hyper BCK-algebra; Fuzzy soft hyper BCK-ideal; Fuzzy soft positive implicative hyper BCK-ideal of types $(\ll,\subseteq,\subseteq)$, $(\ll,\ll,\subseteq)$ and $(\subseteq,\ll,\subseteq)$

Mathematics Subject Classification (2020). 06F35; 03G25; 06D72

1. Introduction

An algebraic hyper structure characterize a natural extension of classical algebraic structures were first introduced in 1934 by [17] during they introduced the notion of hypergroups. Currently, hyper structures have a lot of applications in several branches of mathematics(geometry, hypergraphs, binary relations, combinatorics, codes, cryptography), probability and computer sciences (see [1, 2, 8, 16, 18, 19, 21]). In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyper structure, the composition of two elements

*Email: toletoobii2020@gmail.com

is a set. The author [3] applied the concept of hyper structures to BCK-algebras, and introduced the idea of a hyper BCK-algebra which is a generalization of a BCK-algebra. In [12] introduced the notions of neutrosophic (strong, weak, s-weak) hyper BCK-ideal and reflexive neutrosophic hyper BCK-ideal and some of the applicable properties and their relations are indicated. The author refers readers to ([1, 2, 4, 9, 13, 14, 22]).

Dealing through uncertainties is a major problematic in several areas such as economics, engineering, environmental science, medical science and social science etc. These problems cannot be disseminated with by classical methods, because classical methods have essential difficulties. At that time to overcome these difficulties [19] predicted an innovative method, which was called soft set theory, for modeling uncertainty. Then [13], [14] applied the notions of soft sets to the theory of BCK/BCI-algebras, and considered ideal theory of BCK/BCI algebras based on soft set theory. In [18] presented the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision-making problem (see [3, 5–7, 9, 10]).

In [20] the concepts of bipolar fuzzy closed, bipolar fuzzy positive implicative, bipolar fuzzy implicative ideals of BCK-algebras were introduced and some related properties were studied. In [11], the authors applied the concepts of fuzzy soft sets demonstrated by [18] to the theory of hyper BCK-algebras. They presented the concept of fuzzy soft positive implicative hyper BCK-ideal, and considered several properties. They discussed the relation between fuzzy soft positive implicative hyper BCK-ideal and fuzzy soft hyper BCK-ideal, and provided characterizations of fuzzy soft positive implicative hyper BCK-ideal. Using the notion of positive implicative hyper BCK-ideal, they established a fuzzy soft weak (strong) hyper BCK-ideal.

In [10] the notations of fuzzy soft positive implicative hyper BCK-ideal of different types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$ were introduced and the relations of fuzzy soft strong hyper BCK-ideal and fuzzy soft positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$ and $(\ll, \ll, \subseteq)$ were introduced. Then, in this paper the author would like to contributes some notations on fuzzy soft positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$. The author demonstrates that the level set of fuzzy soft positive implicative hyper BCK-ideal of the types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$ are positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$, respectively. In addition, the author investigates the circumstances for a fuzzy soft set to be a fuzzy soft positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$, respectively. Finally,the conditions for a fuzzy soft set to be a fuzzy soft weak hyper BCK-ideal is considered.

2. Preliminaries

Let H be a nonempty set endowed with a hyper operation "$\circ$", that is, "$\circ$" is a function from $H \times H$ to $\mathscr{P}^*(H) = \mathscr{P}(H)/\{\emptyset\}$. For any two subsets A and B of H, the set denoted by $A \circ B$ is defined as:

$$A \circ B = \bigcup \{a \circ b : a \in A, b \in B\}.$$

Definition 2.1 ([3])**.** Let H be a nonempty set, and "$\circ$" be hyper operation. Then, H is called hyper BCK-algebra if satisfying the following axioms:

(a) $(a\circ c)\circ(b\circ c)\ll a\circ b$;

(b) $(a\circ b)\circ c= (a\circ c)\circ b$;

(c) $a\circ H\ll\{a\}$;

(d) If $a\ll b$ and $b\ll a$, then $a=b$, for all $a,b,c\in H$,

where $a\ll b$ and $0\in a\circ b$ and for every $A,B\subseteq H$, $A\ll B$ is defined by for all $a\in A$, $b\in B$ such that $a\ll b$.

Let H be a hyper BCK-algebra, then condition $a\circ H\ll\{a\}$ is equivalent to $a\circ b\ll\{a\}$.

Definition 2.2 ([4])**.** In any hyper BCK-algebra H, the following hold:

(a) $a\circ 0\ll(at)$, $0\circ a\ll\{0\}$, and $0\circ 0\ll\{0\}$;

(b) $(A\circ B)\circ C=(A\circ C)B$, $A\circ B\ll A$, $0\circ A\ll\{0\}$;

(c) $0\circ 0=\{0\}$, $0\circ a=\{0\}$, $0\circ A=\{0\}$;

(d) $0\ll a$, $a\ll a$, $A\ll A$;

(e) $A\subseteq B$ implies $A\ll B$;

(f) $A\ll\{0\}$ implies $A=\{0\}$;

(g) $a\in a\circ 0$, $a\circ 0=\{a\}$;

(h) $A\circ 0=A$, for all $a\in H$ and every nonempty subset $A,B,C\subseteq H$.

Definition 2.3 ([2])**.** A subset I of a hyper BCK-algebra H is said to be a hyper BCK-ideal of H, iff $0\in I$, and $a\circ b\ll I$, $b\in I\Longrightarrow a\in I$, for all $a,b\in H$.
Note that a subset I of a hyper BCK-algebra H, is said to be a strong hyper BCK-ideal of H if $0\in I$ and $(a\circ b)\cap I\neq\emptyset$, $b\in I\Longrightarrow a\in I$, for all $a,b\in H$ (see [5]).

Again, every strong hyper BCK-ideal is a hyper BCK-ideal, but the converse might not be true (see [5]).
A subset I of a hyper BCK-algebra H is said to be a weak hyper BCK-ideal of H (see [3,4]) if $0\in I$ and $(a\circ b)\subseteq I$, $b\in I\Longrightarrow a\in I$, for all $a,b\in H$.

Lemma 2.4 ([3])**.** *Every hyper BCK-ideal is a weak hyper BCK-ideal. But, the converse of this lemma may not be true.*

A subset I of a hyper BCK-algebra H is said to be

- Reflexive if $(a\circ a)\in I$ for all $a\in H$,
- Closed if the following statement is valid.

$$(\forall\ a\in H)\,(\forall\ b\in I)\ \ a\ll b\Longrightarrow a\in I).$$

Given a subset I of H and $a,b,c\in H$, we consider the following conditions:

$$(a\circ b)\circ c\subseteq I,\ b\circ c\subseteq I\Longrightarrow a\circ c\in I$$

$$(a \circ b) \circ c \subseteq I,\ b \circ c \ll I \Longrightarrow a \circ c \in I$$
$$(a \circ b) \circ c \ll I,\ b \circ c \subseteq I \Longrightarrow a \circ c \in I$$
$$(a \circ b) \circ c \ll I,\ b \circ c \ll I \Longrightarrow a \circ c \in I$$

Definition 2.5 ([5]). Let I be a nonempty subset of a hyper BCK-algebra H and $0 \in I$. Then, then I is said to be positive implicative hyper BCK-ideal of:

(a) Type $(\subseteq,\subseteq,\subseteq)$, if $(a \circ b) \circ c \subseteq I$, $b \circ c \subseteq I \Longrightarrow a \circ c \in I$,

(b) Type $(\subseteq,\ll,\subseteq)$, if $(a \circ b) \circ c \subseteq I$, $b \circ c \ll I \Longrightarrow a \circ c \in I$,

(c) Type $(\ll,\subseteq,\subseteq)$, if $(a \circ b) \circ c \ll I$, $b \circ c \subseteq I \Longrightarrow a \circ c \in I$,

(d) Type $(\ll,\ll,\subseteq)$, if $(a \circ b) \circ c \ll I$, $b \circ c \subseteq I \Longrightarrow a \circ c \in I$, for all $a,b,c \in H$ (see [3, 10, 14, 15]).

Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denote the power set of U and $A \subseteq E$. Molodtsov [15] defined the soft set in the following way:

Definition 2.6 ([19]). A pair (λ, A) is called a soft set over U, where λ is a mapping given by

$$\lambda : A \longrightarrow \mathscr{P}(U).$$

In other arguments, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.7 ([18]). Let $\mathscr{F}(U)$ denote the set of all fuzzy sets in U. Then a pair $(\check{\lambda}, A)$ is called a fuzzy softset over U where $A \subseteq E$ and $\check{\lambda}$, is a mapping given by:

$$\check{\lambda} : A \longrightarrow \mathscr{F}(U).$$

In general, for any parameter μ in A, the set $\check{\lambda}[\mu]$ is a fuzzy set in U and it is called fuzzy value set of parameters μ. For a given fuzzy set μ in a hyper BCK-algebra H and a subset T of H, the classes of $\mu_*(T)$ and $\mu^*(T)$ are defined as:

$$\mu_*(T) = \inf_{a \in T} \mu(a) \quad \text{and} \quad \mu^*(T) = \sup_{a \in T} \mu(a).$$

Definition 2.8 ([4]). A fuzzy soft set $(\check{\lambda}, A)$ over a hyper BCK-algebra H is called a fuzzy soft hyper BCK-ideal based on a parameter $\mu \in A$ over H, if the fuzzy value set $\check{\lambda}[\mu] : H \longrightarrow [0,1]$ of μ satisfies the following conditions:

$$(\forall\ a,b \in H)\ (a \ll b \Longrightarrow \check{\lambda}[\mu](a) \geq \check{\lambda}[\mu](b))$$

and

$$(\forall\ a,b \in H)\ (a \ll b \Longrightarrow \check{\lambda}[\mu](a) \geq \min\{\check{\lambda}[\mu]_*(a \circ b), \check{\lambda}[\mu](b)\}).$$

Definition 2.9 ([4]). A fuzzy soft set $(\check{\lambda}, A)$ over a hyper BCK-algebra H is called a fuzzy soft weak hyper BCK-ideal based on a parameter $\mu \in A$ over H, if the fuzzy value set $\check{\lambda}[\mu] : H \longrightarrow [0,1]$ of μ satisfies the following conditions:

$$(\forall\ a \in H)\ (\check{\lambda}[\mu](0) \geq \check{\lambda}[\mu](a))$$

and

$$(\forall\ a,b \in H)\ (a \ll b \Longrightarrow \check{\lambda}[\mu](a) \geq \min\{\check{\lambda}[\mu]_*(a \circ b), \check{\lambda}[\mu](b)).$$

Definition 2.10 ([4])**.** A fuzzy soft set $(\check{\lambda}, A)$ over a hyper BCK-algebra H is called a fuzzy soft strong hyper BCK-ideal over H based on a parameter$\mu \in A$, if and only if the fuzzy value set $\check{\lambda}[\mu] : H \longrightarrow [0,1]$ of a parameter μ satisfies the following conditions:

$$(\forall\ a \in H)\ (\{\check{\lambda}[\mu]_*(a \circ a), \check{\lambda}[\mu](a)\})$$

$$(\forall\ a, b \in H)\ (\check{\lambda}[\mu](a) \geq \min\{\check{\lambda}[\mu]_*(a \circ b), \check{\lambda}[\mu](b)\}).$$

3. Fuzzy Soft Positive Implicative Hyper BCK-Ideals

In this section some important definitions, and significant ideals on the class of fuzzy soft positive implicative hyper BCK- ideals are demonstrated.

Definition 3.1. Let $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. Based on a parameter $\mu \in A \subseteq H$, if $\check{\lambda}[\mu] : H \longrightarrow [0,1]$ of μ satisfies the condition:

$$(\forall\ a, b \in H)\ (a \ll b \Longrightarrow \check{\lambda}[\mu](a) \geq \check{\lambda}[\mu](b)). \tag{3.1}$$

Then $(\widetilde{\lambda}, A)$ is called a fuzzy soft positive implicative hyper BCK-ideal of:

(a) Type $(\subseteq, \subseteq, \subseteq)$, if the following condition satisfy:

$$(\forall\ a, b, c \in H)\ (\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]_*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\}). \tag{3.2}$$

(b) Type $(\subseteq, \ll, \subseteq)$, if the following condition satisfy:

$$(\forall\ a, b, c \in H)\ (\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]_*((a \circ b) \circ c), \check{\lambda}[\mu]^*(b \circ c)\}). \tag{3.3}$$

(c) Type $(\ll, \subseteq, \subseteq)$ if the following condition satisfy:

$$(\forall\ a, b, c \in H)\ (\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]^*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\}). \tag{3.4}$$

(d) Type $(\ll, \ll, \subseteq)$, if the following condition satisfy:

$$(\forall\ a, b, c \in H)\ (\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]^*((a \circ b) \circ c), \check{\lambda}[\mu]^*(b \circ c)\}). \tag{3.5}$$

Theorem 3.2. *Let $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. If $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$ or $((\subseteq, \ll, \subseteq))$, then $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \subseteq, \subseteq)$.*

Proof. Suppose that $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-idealof type $(\ll, \subseteq, \subseteq)$ or $((\subseteq, \ll, \subseteq))$. Then, we have

$$\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]^*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\} \geq \min\{\check{\lambda}[\mu]_*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\}$$

or

$$\check{\lambda}[\mu]_*(a \circ c) \geq \min\{\check{\lambda}[\mu]_*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\} \geq \min\{\check{\lambda}[\mu]_*((a \circ b) \circ c), \check{\lambda}[\mu]_*(b \circ c)\}.$$

Therefore $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \subseteq, \subseteq)$. Hence the result. □

Theorem 3.3. *Suppose that $(\widetilde{\lambda}, A)$ is a fuzzy soft set over H. Let $(\widetilde{\lambda}, A)$ be a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \ll, \subseteq)$, then $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$ and $(\subseteq, \ll, \subseteq)$.*

Proof. Suppose that $(\widetilde{\lambda},A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\ll,\subseteq)$. Then, the following conditions satisfied:

$$\check{\lambda}[\mu]_*(a\circ c)\geq \min\{\check{\lambda}[\mu]^*((a\circ b)\circ c),\check{\lambda}[\mu]^*(b\circ c)\}\geq \min\{\check{\lambda}[\mu]^*((a\circ b)\circ c),\check{\lambda}[\mu]_*(b\circ c)\},\ \text{and}$$

$$\check{\lambda}[\mu]_*(a\circ c)\geq \min\{\check{\lambda}[\mu]^*((a\circ b)\circ c),\check{\lambda}[\mu]^*(b\circ c)\}\geq \min\{\check{\lambda}[\mu]_*((a\circ b)\circ c),\check{\lambda}[\mu]^*(b\circ c)\}.$$

Therefore, $(\widetilde{\lambda},A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\subseteq,\subseteq)$ and $(\subseteq,\ll,\subseteq)$ consequently. □

Corollary 3.4. *Let $(\widetilde{\lambda},A)$ be a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\ll,\subseteq)$, then $(\widetilde{\lambda},A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq,\subseteq,\subseteq)$.*

Lemma 3.5 ([11])**.** *Every fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq,\subseteq,\subseteq)$ is a fuzzy soft hyper BCK-ideal. The converse is not true.*

Corollary 3.6. *Every fuzzy soft positive implicative hyper BCK-ideal $(\widetilde{\lambda},A)$ of types $(\ll,\subseteq,\subseteq)$, $(\subseteq,\ll,\subseteq)$ or $(\ll,\ll,\subseteq)$ is a fuzzy soft hyper BCK-ideal.*

It is clear that any fuzzy soft hyper BCK-ideal may not be a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq,\ll,\subseteq)$. Now, we deliberate relation between a fuzzy soft positive implicative hyper BCK-ideal ofany type and a fuzzy soft strong hyper BCK-ideal.

Theorem 3.7. *Every fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\subseteq,\subseteq)$ is a fuzzy soft strong hyper BCK-ideal of H.*

Proof. Let $(\widetilde{\lambda},A)$ be a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\subseteq,\subseteq)$ and let μ be any parameter in A. Since $a\circ a\ll a$ for all $a\in H$, then $\check{\lambda}[\mu]^*(a\circ a)\geq \check{\lambda}[\mu]_*(a)=\check{\lambda}[\mu](a)$. Then by putting $c=0$ in equation (3.4) implies that

$$\check{\lambda}[\mu](a)=\check{\lambda}[\mu]_*(a\circ 0)\geq \min\{\check{\lambda}[\mu]^*((a\circ b)\circ 0),\check{\lambda}[\mu]_*(b\circ 0)\}=\min\{\check{\lambda}[\mu]^*(a\circ b),\check{\lambda}[\mu]_*(b)\}.$$

Therefore $(\widetilde{\lambda},A)$ is a fuzzy soft strong hyper BCK-ideal of H. Hence the result. □

Corollary 3.8. *Every fuzzy soft positive implicative hyper BCK-ideal of type $(\ll,\ll,\subseteq)$ is a fuzzy soft strong hyper BCK-ideal of H. Any fuzzy soft positive implicative hyper BCK-ideal of types $(\subseteq,\subseteq,\subseteq)$ and $(\subseteq,\ll,\subseteq)$ is not a fuzzy soft strong hyper BCK-ideal of H. Given a fuzzy soft set $(\widetilde{\lambda},A)$ over H and $t\in[0,1]$, we consider the following set:*

$$U(\widetilde{\lambda}[\mu];t):=\{a\in H:\check{\lambda}[\mu](a)\geq t\},\tag{3.6}$$

where μ is a parameter in A, which is called level set of $(\widetilde{\lambda},A)$.

Lemma 3.9. *Let $(\widetilde{\lambda},A)$ be a fuzzy soft set over H, and satisfies the condition (3.1), then $0\in U(\widetilde{\lambda}[\mu];t)$ for all $t\in[0,1]$ and any parameter μ in A with $U(\widetilde{\lambda}[\mu];t)\neq\emptyset$.*

Proof. Let $(\widetilde{\lambda},A)$ be a fuzzy soft set over H which satisfies the condition (3.1). For any $t\in[0,1]$ and any parameter μ in A. Suppose that $U(\widetilde{\lambda}[\mu];t)\neq\emptyset$. Since $0\ll a$, $\forall\ a\in H$, it follows from equation (3.1) that $\check{\lambda}[\mu](0)\geq\check{\lambda}[\mu](a)$, $\forall\ a\in H$. Hence $\check{\lambda}[\mu](0)\geq\check{\lambda}[\mu](a)$ for all $a\in U(\widetilde{\lambda}[\mu];t)$ and accordingly $\check{\lambda}[\mu](0)\geq t$.
Therefore $0\in U(\widetilde{\lambda}[\mu];t)$. Hence the result. □

Lemma 3.10 ([3]). *A fuzzy soft set $(\widetilde{\lambda}, A)$ over H is a fuzzy soft hyper BCK-ideal of H iff the set $U(\widetilde{\lambda}[\mu];t)$ is a hyper BCK-ideal of H for all $t \in [0,1]$, a parameter μ in A with $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$.*

Theorem 3.11. *Suppose $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. Let $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$, then the set $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$, $\forall\ t \in [0,1]$ and any parameter μ in A with $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$.*

Proof. Suppose that a fuzzy soft set $(\widetilde{\lambda}, A)$ over H is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$. Then $0 \in U(\widetilde{\lambda}[\mu];t)$. Let $a, b, c \in H$ such that $(a \circ b) \circ c \subseteq U(\widetilde{\lambda}[\mu];t)$ and $b \circ c \in U(\widetilde{\lambda}[\mu];t)$, then;

$$\widetilde{\lambda}[\mu](x) \geq t, \quad \text{for all } x \in (a \circ b) \circ c \tag{3.7}$$

and

$$(\forall\ y \in b \circ c)\ (\exists\ z \in U(\widetilde{\lambda}[\mu];t))(y \ll z). \tag{3.8}$$

Then $\widetilde{\lambda}[\mu](x) \geq t$, $\forall\ x \in (a \circ b) \circ c \Longrightarrow \check{\lambda}[\mu]_*((a \circ b) \circ c) \geq t$ and

$$(\forall\ y \in b \circ c)\ (\exists\ z \in U(\widetilde{\lambda}[\mu];t))(x \ll y)$$

$$\Longrightarrow \quad \widetilde{\lambda}[\mu](y) \geq \widetilde{\lambda}[\mu](z) \geq t, \quad \forall\ y \in (b \circ c).$$

Let $x \in (a \circ c)$, by using equation (3.3), we have:

$$\widetilde{\lambda}[\mu](x) \geq \widetilde{\lambda}[\mu]_*(a \circ c) \geq \min\{\widetilde{\lambda}[\mu]_*(a \circ b) \circ c), \widetilde{\lambda}[\mu]^*(b \circ c)\} \geq t.$$

Therefore, $x \in U(\widetilde{\lambda}[\mu];t)$, and accordingly $a \circ c \in U(\widetilde{\lambda}[\mu];t)$.
Thus $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$. Hence the result. □

Lemma 3.12 ([5]). *Every positive implicative hyper BCK-ideal of type $(\subseteq, \subseteq, \subseteq)$ isa weak hyper BCK-ideal of H.*

Lemma 3.13 ([2]). *Let I be a reflexive hyper BCK-ideal of H. Then*

$$(\forall\ a, b \in H)((a \circ b) \cap I \neq \emptyset \Longrightarrow a \circ b \ll I). \tag{3.9}$$

Lemma 3.14. *Let I be a closed subset of a set H, and satisfies $(a \circ b) \subseteq I$, $b \in I \Longrightarrow a \in I$, then the condition $a \circ b \ll I$, $b \in I \Longrightarrow a \in I$, is valid for all $a, b \in H$.*

Proof. Suppose that $a \circ b \in I$ and $b \in I$ for all $a, b \in H$. Let $x \in (a \circ b)$. Then there exists $y \in I$ such that $x \ll y$. Now, since I is closed, we have $x \in I$ and thus $(a \circ b) \subseteq I$. Therefore, we can conclude that $a \in I$. Hence the result. □

Theorem 3.15. *Suppose that A is a fuzzy soft set over H. Let $(a \ll b \Longrightarrow \check{\lambda}[\mu](a) \geq \check{\lambda}[\mu](b))$, for all $a, b \in H$ and*

$$(\forall\ T \in \mathscr{P}(H))(\exists\ x \in T)(\widetilde{\lambda}[\mu](x) = \widetilde{\lambda}[\mu]^*(T)). \tag{3.10}$$

If the set $Ut(\widetilde{\lambda}[\mu];t)$ is a reflexive positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$ for all $t \in [0,1]$ and any parameter μ in A with $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$ then $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$.

Proof. For any $a,b,c \in H$, let us consider

$$t := \min\{\widetilde{\lambda}[\mu]_*((a \circ b) \circ c), \widetilde{\lambda}[\mu]^*(b \circ c)\}.$$

Then, we have $\widetilde{\lambda}[\mu]_*((a \circ b) \circ c) \geq t$.
Hereafter $\widetilde{\lambda}[\mu](x) \geq t, \quad \forall\ x \in ((a \circ b) \circ c)$ and

$$\widetilde{\lambda}[\mu]^*(b \circ c) \geq t \Longrightarrow \widetilde{\lambda}[\mu](a_0) = \widetilde{\lambda}[\mu]^*(b \circ c) \geq t, \quad \text{for some } y \in (b \circ c).$$

Hence $y \in U(\widetilde{\lambda}[\mu];t)$, and so $U(\widetilde{\lambda}[\mu];t) \cap (b \circ c) \neq \emptyset$. Since $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$ and hence of type $(\subseteq, \subseteq, \subseteq)$, $U(\widetilde{\lambda}[\mu];t)$ is a weak hyper BCK-ideal of H by Lemma 3.12.
Let $a \in H$ be such that $a \ll b$. If $b \in U(\widetilde{\lambda}[\mu];t)$, then $\widetilde{\lambda}[\mu](a) \geq \widetilde{\lambda}[\mu](b) \geq t$ by equation (3.1). Therefore, $a \in U(\widetilde{\lambda}[\mu];t) \Longrightarrow b \in U(\widetilde{\lambda}[\mu];t)$ is closed. Hence $U(\widetilde{\lambda}[\mu];t)$ is a hyper BCK-ideal of H following Lemma 3.14. Since $U(\widetilde{\lambda}[\mu];t)$ is reflexive,then following Lemma 3.13, we have that $b \circ c \ll U(\widetilde{\lambda}[\mu];t)$. Hence $a \circ c \in U(\widetilde{\lambda}[\mu];t)$.
Now, since $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$. Hence

$$\widetilde{\lambda}[\mu](x) \geq t = \min\{\widetilde{\lambda}[\mu]_*((a \circ b) \circ c), \widetilde{\lambda}[\mu]^*(b \circ c)\}, \quad \text{for all } x \in (a \circ c).$$

Therefore, we have the following equation:

$$\widetilde{\lambda}[\mu]_*(a \circ c) \geq \min\{\widetilde{\lambda}[\mu]_*((a \circ b) \circ c), \widetilde{\lambda}[\mu]^*(b \circ c)\}, \quad \forall\ a,b,c \in H.$$

Thus $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-idealof type $(\subseteq, \ll, \subseteq)$. Hence the result. □

Corollary 3.16. *Let A be a fuzzy soft set over H. If $a \ll b \Rightarrow \check{\lambda}[\mu](a) \geq \check{\lambda}[\mu](b)$, $\forall\ a,b \in H$, $\widetilde{\lambda}[\mu](x) = \widetilde{\lambda}[\mu]^*(T)$ for some $x \in T \in \mathscr{P}(H)$. If $t \in [0,1]$ and μ be any parameter in A, and consider that $U(\widetilde{\lambda}[\mu];t)$ is nonempty and reflexive.*
Then, also $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$, if and only if $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$.

Theorem 3.17. *Let $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. Then, if $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$, the set $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$, $\forall\ t \in [0,1]$ and any parameter μ in A with $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$.*

Proof. Suppose that $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$. Then, $0 \in U(\widetilde{\lambda}[\mu];t)$. Let $a,b,c \in H$ such that $((a \circ b) \circ c) \ll U(\widetilde{\lambda}[\mu];t)$ and $b \circ c \subseteq U(\widetilde{\lambda}[\mu];t)$.
Then, we have the following equation:

$$(\forall\ x \in (a \circ b) \circ c)(\exists\ y \in U(\widetilde{\lambda}[\mu];))(x \ll y). \tag{3.11}$$

Again, following equation (3.1), we have that $\check{\lambda}[\mu](x) \geq \check{\lambda}[\mu](y)$, for all $x \in (a \circ b) \circ c$.
Now, since $b \circ c \in U(\widetilde{\lambda}[\mu];t)$, we have the following condition:

$$\check{\lambda}[\mu](x) \geq t, \quad \text{for all } x \in b \circ c. \tag{3.12}$$

Let $z \in a \circ c$, then

$$\widetilde{\lambda}[\mu](z) \geq \widetilde{\lambda}[\mu]_*(a \circ c) \geq \min\{\widetilde{\lambda}[\mu]^*(a \circ b) \circ c), \widetilde{\lambda}[\mu]_*(b \circ c)\} \geq t, \quad \forall\ a,b,c \in H.$$

Thus, $z \in U(\widetilde{\lambda}[\mu];t)$. Therefore, $a \circ c \subseteq U(\widetilde{\lambda}[\mu];t)$. Thus $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\subseteq, \ll, \subseteq)$. Note that the converse of Theorem 3.17 is not true. □

Lemma 3.18. *Every reflexive hyper BCK-ideal I of H satisfies the following implication:*

$$(\forall\ a,b)\ (a \circ b) \cap I \neq \emptyset \Longrightarrow a \circ b \subseteq I.$$

Lemma 3.19. *Every positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$ is a hyper BCK-ideal.We deliver the conditions for a fuzzy soft set to be a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$.*

Theorem 3.20. *Suppose that A is a fuzzy soft set over H. Let $\widetilde{\lambda}[\mu](x) = \widetilde{\lambda}[\mu]^*(T)$, $x \in T \in \mathscr{P}(H)$. If the set $U(\widetilde{\lambda}[\mu];t)$ is a reflexive positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$ for all $t \in [0,1]$ and any parameter μ in A with $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$, then $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$.*

Proof. Suppose that $U(\widetilde{\lambda}[\mu];t) \neq \emptyset$, $\forall\ t \in [0,1]$ and parameter μ in A. Suppose that $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$. Then $U(\widetilde{\lambda}[\mu];t)$ is a hyper BCK-ideal of H by Lemma 3.19. Then $(\widetilde{\lambda}, A)$ is a fuzzy soft hyper BCK-ideal of H. Therefore, the condition of equation (3.1) is valid. Now, let us consider;

$$t = \min\{\widetilde{\lambda}[\mu]^*((a \circ b) \circ c), \widetilde{\lambda}[\mu]_*(b \circ c)\}, \quad \forall\ a,b,c \in H.$$

Since $(\widetilde{\lambda}, A)$ satisfies the condition of equation (3.10), there is $x \in ((a \circ b) \circ c)$ such that:

$$\widetilde{\lambda}[\mu](x) = \widetilde{\lambda}[\mu]^*((a \circ b) \circ c) \geq t.$$

Therefore $x \in U(\widetilde{\lambda}[\mu];t)$, and hence $((a \circ b) \circ c) \cap U(\widetilde{\lambda}[\mu];t) \neq \emptyset \Longrightarrow ((a \circ b) \circ c) \ll U(\widetilde{\lambda}[\mu];t)$.
Furthermore, $\widetilde{\lambda}[\mu](y) \geq \widetilde{\lambda}[\mu]_*(b \circ c) \geq t$, $\forall\ y \in (b \circ c)$.
Therefore $y \in U(\widetilde{\lambda}[\mu];t) \Longrightarrow (b \circ c) \subseteq U(\widetilde{\lambda}[\mu];t)$. Since $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$. Consequently, $(a \circ c) \subseteq U(\widetilde{\lambda}[\mu];t)$. Accordingly, $\widetilde{\lambda}[\mu](x) \geq t$, $\forall\ x \in (a \circ c)$. Hence, we have the following equation;

$$\widetilde{\lambda}[\mu]_*(a \circ c) \geq t = \min\{\widetilde{\lambda}[\mu]^*((a \circ b) \circ c), \widetilde{\lambda}[\mu]_*(b \circ c)\}.$$

Therefore, $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$. □

Corollary 3.21. *Suppose A be a fuzzy soft set over H. Let $\widetilde{\lambda}[\mu](x) = \widetilde{\lambda}[\mu]^*(T)$, $(\forall\ T \in \mathscr{P}(H)$ and $\exists\ x \in T$. For any $t \in [0,1]$ and parameter μ in A, consider nonempty and reflexive set $U(\widetilde{\lambda}[\mu];t)$. Then $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$ if and only if $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of type $(\ll, \subseteq, \subseteq)$.*
By using a positive implicative hyper BCK-ideal of type $(\subseteq, \subseteq, \subseteq)$ (resp. $(\subseteq, \ll, \subseteq)$, $(\ll, \subseteq, \subseteq)$ and $(\ll, \ll, \subseteq)$, we establish a fuzzy soft weak hyper BCK-ideal.

Theorem 3.22. *Suppose I be a positive implicative hyper BCK-ideal of type $(\subseteq, \subseteq, \subseteq)$ (resp. $(\subseteq, \ll, \subseteq)$, $(\ll, \subseteq, \subseteq)$ and $(\ll, \ll, \subseteq)$). Let $a \in H$, for a fuzzy soft set $(\widetilde{\lambda}, A)$ over H and any parameter μ in A, if we define the fuzzy value set $\widetilde{\lambda}[\mu]$ by*

$$\widetilde{\lambda}[\mu] : H \longrightarrow [0,1], \quad A \longmapsto \begin{cases} t & \text{if } a \in I_c \\ s & \text{otherwise} \end{cases} \tag{3.13}$$

where $s < t \in [0,1]$ and $I_c := \{b \in H \mid b \circ c \subseteq I\}$, then $(\widetilde{\lambda}, A)$ is a μ-fuzzy soft weak hyper BCK-ideal of H.

Proof. Obviously $\widetilde{\lambda}[\mu](0) \geq \widetilde{\lambda}[\mu](a)$, $\forall\ a \in H$. Let $a,b \in H$, and if $b \notin I_c$, then $\widetilde{\lambda}[\mu](b) = s$. Therefore, we have the following condition:

$$\widetilde{\lambda}[\mu](a) \geq s = \min\{\widetilde{\lambda}[\mu](b), \widetilde{\lambda}[\mu]_*(a \circ b)\}. \tag{3.14}$$

If $(a \circ b) \nsubseteq I_c$, then there exists $x \in (a \circ b) - I_c$ and thus $\widetilde{\lambda}[\mu](b) = s$. Hence

$$\min\{\widetilde{\lambda}[\mu](b), \widetilde{\lambda}[\mu]_*(a \circ b)\} = s \leq \widetilde{\lambda}[\mu](a). \tag{3.15}$$

Assume that $(a \circ b) \subseteq I_c$ and $b \in I_c$. Then

$$(a \circ b) \circ c \subseteq I \text{ and } (b \circ c) \subseteq I. \tag{3.16}$$

If I is of type $(\subseteq,\subseteq,\subseteq)$, then $(a \circ c) \subseteq I$, i.e., $a \in I_c$. Thus, we have:

$$\widetilde{\lambda}[\mu](a) = t \geq \min\{\widetilde{\lambda}[\mu](b), \widetilde{\lambda}[\mu]_*(a \circ b)\}. \tag{3.17}$$

Then, by equation (3.16) implies that $(a \circ b) \circ c \ll I$ and $(b \circ c) \ll I$.

(a) I is of type $(\ll,\ll,\subseteq)$, then $(a \circ c) \subseteq I \Longrightarrow a \in I_c$. Therefore, we have equation (3.17). From equation (3.16) we have $(a \circ b) \circ c \subseteq I$ and $(b \circ c) \ll I$.

(b) If I is of type $(\subseteq,\ll,\subseteq)$, then $(a \circ c) \subseteq I \Longrightarrow a \in I_c$. Therefore, we have equation (3.17). From the equation (3.16), we have $(a \circ b) \circ c \ll I$ and $(b \circ c) \subseteq I$.

(c) If I is of type $(\ll,\subseteq,\subseteq)$, then $(a \circ c) \subseteq I \Longrightarrow a \in I_c$. Therefore, we have equation (3.17).

Therefore $(\widetilde{\lambda}, A)$ is a μ-fuzzy soft weak hyper BCK-idealof H. Hence the result. □

Theorem 3.23. *Suppose $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. Let $\emptyset \neq U(\widetilde{\lambda}[\mu];t)$ is reflexive for all $t \in [0,1]$. If $(\widetilde{\lambda}, A)$ be a fuzzy soft positive implicative hyper BCK-ideal of H of type $(\ll,\subseteq,\subseteq)$, then the set is a (weak) hyper BCK-ideal of H for all $c \in H$.*

Proof. Let $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of H of type $(\ll,\subseteq,\subseteq)$. Clearly, $0 \in \widetilde{\lambda}[u]_c$. Then $(\widetilde{\lambda}, A)$ is a fuzzy soft hyper BCK-ideal of H, and therefore $U(\widetilde{\lambda}[\mu];t)$ is a hyper BCK-ideal of H.

Let $a,b \in H$ such that $(a \circ b) \subseteq \widetilde{\lambda}[u]_c$ and $b \in \widetilde{\lambda}[u]_c$. Then $(a \circ b) \circ c \subseteq U(\widetilde{\lambda}[\mu];t)$ and $(b \circ c) \subseteq U(\widetilde{\lambda}[\mu];t)$ for all $t \in [0,1]$. Then, we recognize that $(a \circ b) \circ c \ll U(\widetilde{\lambda}[\mu];t)$.

Since $U(\widetilde{\lambda}[\mu];t)$ is a positive implicative hyper BCK-ideal of H of type $(\ll,\subseteq,\subseteq)$, then:

$$(a \circ c) \subseteq U(\widetilde{\lambda}[\mu];t) \Longrightarrow a \in \widetilde{\lambda}[u]_c.$$

This determine that $\widetilde{\lambda}[u]_c$ is a weak hyper BCK-ideal of H.

Let $a,b \in H$ be such that $(a \circ b) \ll \widetilde{\lambda}[u]_c$ and $b \in \widetilde{\lambda}[u]_c$, and let $x \in (a \circ b)$. Then, there exists $y \in \widetilde{\lambda}[u]_c$ such that $x \ll y \Longrightarrow 0 \in x \circ y$. Thus $(x \circ y) \cap U(\widetilde{\lambda}[\mu];t) \neq \emptyset$. Since $U(\widetilde{\lambda}[\mu];t)$ is a reflexive hyper BCK-ideal of H, then $(x \circ c) \circ (y \circ c) \ll (x \circ y) \subseteq U(\widetilde{\lambda}[\mu];t)$.

Therefore $(x \circ c) \subseteq U(\widetilde{\lambda}[\mu];t)$, since $(y \circ c) \subseteq U(\widetilde{\lambda}[\mu];t)$.

Hence $x \in \widetilde{\lambda}[u]_c$ and $a \circ b \subseteq \widetilde{\lambda}[u]_c$.

Now, since $\widetilde{\lambda}[u]_c$ is a weak hyper BCK-ideal of H, we get $a \in \widetilde{\lambda}[u]_c$. Consequently, $\widetilde{\lambda}[u]_c$ is a hyper BCK-ideal of H. Hence the result. □

Corollary 3.24. *Suppose $(\widetilde{\lambda}, A)$ be a fuzzy soft set over H. If a level set $U(\widetilde{\lambda}[u];t)$ of $(\widetilde{\lambda}, A)$ is reflexive $\forall\ t \in [0,1]$. If $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of H of type $(\ll, \ll, \subseteq)$, then the set:*

$$\widetilde{\lambda}[u]_c := \{a \in H\ : a \circ c \subseteq (\widetilde{\lambda}[\mu];t)\} \tag{3.18}$$

is a (weak) hyper BCK-ideal of H for all $c \in H$.

4. Conclusion

In this paper, the author considered some notations on fuzzy soft positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$. The author established that the level set of fuzzy soft positive implicative hyper BCK-ideal of the types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$ are positive implicative hyper BCK-ideal of types $(\ll, \subseteq, \subseteq)$, $(\ll, \ll, \subseteq)$ and $(\subseteq, \ll, \subseteq)$, respectively. As demonstrated, in the part of this paper, every positive implicative hyper BCK-ideal of any type $(\ll, \subseteq, \subseteq)$ or $(\subseteq, \subseteq, \subseteq)$ or $(\subseteq, \ll, \subseteq)$ is (are) a hyper BCK-ideal. In addition, for any fuzzy soft set $(\widetilde{\lambda}, A)$ over H, if a level set $U(\widetilde{\lambda}[u];t)$ is reflexive $\forall\ t \in [0,1]$ and $(\widetilde{\lambda}, A)$ is a fuzzy soft positive implicative hyper BCK-ideal of H of type $(\ll, \ll, \subseteq)$, then the set $\widetilde{\lambda}[u]_c := \{a \in H : a \circ c \subseteq (\widetilde{\lambda}[\mu];t)\}$ is a (weak) hyper BCK-ideal of H for all $c \in H$.

Acknowledgement

The authors command to thank the anonymous reviewers for their valuable suggestions.In addition, the author would like to express the special thanks to my lovely wife, Ms. Tigist Tamirat for her support and being with me in every situation.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

References

[1] A. Aygünoğlu and H. Aygün, Introduction to fuzzy soft groups, *Computers & Mathematics with Applications* **58**(6) (2009), 1279 – 1286, DOI: 10.1016/j.camwa.2009.07.047.

[2] J. Y. Bae, X. X. Long, M. M. Zahedi and R. E. Hwan, Strong hyper BCK-ideals of hyper BCK-algebras, *Mathematica Japonicae* **51**(3) (2000), 493 – 498, `https://ci.nii.ac.jp/naid/10010623326/en/`.

[3] H. Bordbar, S. Khademan, R. A. Borzooei, M. M. Zahedi and Y. B. Jun, Double-framed soft set theory applied to hyper BCK-algebras, *New Mathematics and Natural Computation* **17**(1) (2021), 215 – 228, DOI: 10.1142/S1793005721500113.

[4] H. Bordbar, S. Z. Song, M. R. Bordbar and Y. B. Jun, Fuzzy soft set theory with applications in hyper BCK-algebras, *Journal of Intelligent & Fuzzy Systems 38*(2) (2020), 1789 – 1797, DOI: 10.3233/JIFS-190103.

[5] R. A. Borzooei and M. Bakhshi, On positive implicative hyper BCK-ideals, *Scientiae Mathematicae Japonicae* **59**(3) (2004), 505 – 516, `https://ci.nii.ac.jp/naid/10013664130/en/`.

[6] N. Çağman, F. Çıtak and S. Enginoğlu, Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy System* **1**(1) (2010), 21 – 35.

[7] N. Cagman and S. Enginoglu, Fuzzy soft matrix theory and its application in decision making, *Iranian Journal of Fuzzy Systems 9*(1) (2012), 109 – 119, DOI: 10.22111/ijfs.2012.229.

[8] P. Corsini and V. Leoreanu, *Applications of Hyper structure Theory. Advances in Mathematics*, Kluwer Academic Publishers, Dordrecht (2003).

[9] F. Fatimah and J. C. R. Alcantud, The multi-fuzzy N-soft set and its applications to decision-making, *Neural Computing and Applications* **33** (2021), 11437 – 11446, DOI: 10.1007/s00521-020-05647-3.

[10] S. Khademan, M. M. Zahedi, R. A. Borzooei and Y. B. Jun, Fuzzy soft positive implicative hyper BCK-ideals of several types, *Miskolc Mathematical Notes* **22**(1) (2021), 299 – 315, DOI: 10.18514/MMN.2021.2855.

[11] S. Khademan, M. M. Zahedi, Y. B. Jun and R.A. Borzooei, Fuzzy soft positive implicative hyper BCK-ideals in hyper BCK-algebras, *Journal of Intelligent & Fuzzy Systems* **36**(3) (2019), 2605 – 2613, DOI: 10.3233/JIFS-181755.

[12] S. Khademan, M. M. Zahedi, R. A. Borzooei and Y. B. Jun, Neutrosophic hyper BCK-ideals, *Infinite Study* **27** (2019), 201 – 217.

[13] Y. B. Jun and C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, *Information Sciences* **178**(11) (2008), 2466 – 2475, DOI: 10.1016/j.ins.2008.01.017.

[14] Y. B. Jun and W. H. Shim, Fuzzy structures of PI $(\ll,\subseteq,\subseteq)$ BCK-ideals in hyper BCK-algebras, *International Journal of Mathematics and Mathematical Sciences* **2003** (2003), Article ID 471490, DOI: 10.1155/S0161171203110277.

[15] V. Leoreanu-Fotea and B. Davvaz, Join n-spaces and lattices, *Journal of Multiple-Valued Logic & Soft Computing* **15** (2009), 421 – 432.

[16] V. Leoreanu-Fotea and B. Davvaz, n-Hypergroups and binary relations, *European Journal of Combinatorics* **29**(5) (2008), 1207 – 1218, DOI: 10.1016/j.ejc.2007.06.025.

[17] F. Marty, Sur uni generalization de la notion de group, *8th Congress Math. Scandienaves*, Stockholm (1934), 45 – 49.

[18] R. K. Maji, A. R. Roy and R. Biswas, Fuzzy soft sets, *Journal of Fuzzy Mathematics* **9**(3) (2001), 589 – 602.

[19] D. Molodtsov, Soft set theory — first results, *Computers & Mathematics with Applications* **37**(4-5) (1999), 19 – 31, DOI: 10.1016/S0898-1221(99)00056-5.

[20] G. Muhiuddin and D. Al-Kadi, Bipolar fuzzy implicative ideals of BCK-algebras, *Journal of Mathematics* **2021** (2021), Article ID 6623907, DOI: 10.1155/2021/6623907.

[21] Y. J. Seo, H. S. Kim, Y. B. Jun and S. S. Ahn, Multipolar intuitionistic fuzzy hyper BCK-ideals in hyper BCK-algebras, *Mathematics* **8**(8) (2020), 1373, DOI: 10.3390/math8081373.

[22] B. J. Young and L. X. Xiao, Positive implicative hyper BCK-algebras, *Scientiae Mathematicae Japonicae* **55**(1) (2002), 97 – 106, https://ci.nii.ac.jp/naid/10010238729/en/.