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Research Article

Modeling the Dynamics of Banana Weevil, *Cosmopolites Sordidus* (Germar), by Trapping with Holling Type II Response Function

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Abstract. A harvested logistic growth model that incorporates trapping of the adult banana weevils is formulated and solved both analytically and numerically. Key in the analysis of the model is the derivation of the critical trapping rate, that is, the threshold above which extinction of the banana weevils is a possibility. The existence of the saddle-node bifurcation is investigated. It is established that the stability of the equilibria is dependent on the relative sizes of the intrinsic growth rate and the carrying capacity. It is also found out that when the trapping rate exceeds the critical value, the model displays chaotic behavior and the trapping of the adult banana weevils in isolation can only provide short term relief. Thus a management package that targets all developmental stages of the banana weevils would be more effective. Numerical simulations are performed and agree with the analytical results.

Keywords. Asymptotic stability; Banana weevil; Holling type 2 functional response; Logistic growth model; Pseudostem traps; Saddle node bifurcation

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1. Introduction

Banana (*Musa spp)* is the fourth most important food crop in the world after rice, wheat and maize and is the world's most important fruit crop in terms of production, volume and trade

[7]. However, its production is constrained by a number of biotic and abiotic factors. Among the biotic factors, the banana weevil, *Cosmopolites Sordidus* (Germar), is the single most important constraint to banana production and yield worldwide. The banana weevil is found in all regions wherever banana, plantains and ensete are grown [9]. The banana weevil is believed to have originated along with the banana in the Indo-Malay region of South-East Asia. In Africa, the banana weevil has become a serious constraint to the production of the East African highland bananas which are particularly susceptible to the banana weevil. Banana weevils can cause up to 100% yield loss through sucker death, toppling, shortened plantation life time and reduced bunch size if not controlled [10, 16, 24, 25]. For example, in East Africa the banana weevil is credited with the decline and disappearance of bananas from their traditional growing areas of central Uganda and western Tanzania [24]. Banana weevils attack interfere with root initiation, kills existing roots, limits nutrient uptake, reduces plant vigor, delays flowering and increases susceptibility to other banana pests and diseases. The banana weevil is a 'K'-selected pest characterized by a long life span and low fecundity [21]. Adult banana weevils are widely believed to live up to 2 years with a mean oviposition rate of about 1-2 eggs per day. As a consequence, the banana population build up is slow and its effects become increasingly important in successive crop cycles (ratoons). The adult banana weevils feed on rotting banana tissue and occasionally on young suckers [6] . The banana weevil has limited dispersal capacity and is most often disseminated through infested planting materials.

Control of the banana weevil has largely been elusive in part because of its cryptic nature [13]. The adult banana weevil is rarely seen as it is nocturnally active and feigns death when touched while the immature (larva and pupa) stages reside well inside the corms and so are largely inaccessible [8, 10]. This notwithstanding, a number of control measures have been ranged against the banana weevil with varying degrees of success. These include cultural control practices, use of resistant varieties, biological control and pesticide use.

Application of pesticide is always the last line of defense against the banana weevil given that it is largely unaffordable by most small scale farmers especially in the developing countries, in addition to having adverse effects to both the environment and the farmers.

Cultural control practices against the banana weevil, regarded as the first line of defense, include crop sanitation, use of clean planting materials when starting a new stand, appropriate agronomic practices that promote plant vigor and trapping. Among the cultural control practices, crop sanitation is highly recommended and practiced and involves the destruction of crop residues thereby denying adult banana weevils especially gravid females hiding, ovipositing and breeding sites.

According to Kiggundu [14], a number of clones have shown resistance to banana weevil infestation. While the very popular East African highland cooking bananas are highly susceptible, some cultivars such as *Tereza*, *Nalukira* and *Nsowe* have been found to have intermediate resistance to banana weevils whereas some degree of resistance has been identified with 'Gros Michel' (AAA). The beer varieties Pisang Awak (Kayinja (AAB)), Kisubi (AB) and Yangambi KM5 are highly resistant [14].

Biological control efforts against the banana weevil have included the use of exotic natural enemies (classical biological control), endemic natural enemies, secondary host associations and microbial control [11]. Biological control of banana weevil with arthropod natural enemies is considered to be one of the best options for the management of the pest [9].

There are no known parasitoids for the banana weevil presently . However, a number generalist predators both classical and endemic have been identified as potential control agents of the banana weevil. In Cuba and Fiji respectively, myrmicine ants and the predatory beetle *Plaesius javanus* reportedly successfully controlled the banana weevil infestation [11, 20, 22, 26]. However, attempts to import these classical biological agents elsewhere have not been successful probably due to low release numbers and failure to establish.

A number of endemic predatory species have been identified as potential control agents for the banana weevil. The search for natural enemies of the banana weevil in East Africa has been intensive and a subject of interest to many authors. In Kenya, Koppenhofer *et al.* [15] identified 12 species of banana weevil predators. In a study of ant composition and relative abundance conducted in Uganda to identify potential predators of the banana weevil, it was suggested that ants were important foragers in banana plantations [1]. The study was carried out on 39 farms in four regions revealed that ants as the major predators with 55 species from pitfall traps and 24 species in fish and honey baits. There were also 17 ants species in banana pseudostems and 34 in banana corms of live banana plants and residues of the harvested banana plants [1]. While other generalist predators (feed on a wide variety of food materials such as nectar, sugar, honeydew etc.) are unable to find banana weevil eggs, larvae and pupae concealed within the corms, ants are able to dig through the soil, locate the eggs on the corm surface and forage inside corm tunnels. In addition, ants are abundant, recruit one another to productive food sources and are not affected by satiation since individual ants do not forage for themselves but rather for the colony [23].

Trapping of the adult banana weevil is a widely recommended strategy to control the banana weevil worldwide. Control of insect pests by mass trapping assumes that a reduction in the pest population leads to a further population decrease in the next generation. A reduction in banana weevil population following mass trapping has been observed [2, 16, 18]. Trapping the banana weevil is influenced by environmental factors, biology, trapping density, trap placement, quality of traps, size of traps and frequency of collection [3]. Traditionally, the commonest types of traps are made from crop residues, that is, recently harvested corms and pseudostem such as disc-on-stump, split-pseudostem, corm disc, sandwich, wedge and canoe traps [9]. Adult banana weevils attracted to these traps for shelter, feeding or to lay eggs are either captured by hand and destroyed or when eggs hatch, their life cycle cannot continue as the cut pieces dry out and the grubs die from desiccation. These traps are however unpopular as they are labor intensive and require a constant supply of trapping materials which may not be easily forthcoming. Of late, though, enhanced trapping using infochemicals including pheromones and kairomones has been identified as a means that can be used to develop an effective method of trapping and control of the banana weevil. For example, in Costa Rica, traps baited with synthetic pheromones were found to be more effective and attractive than unbaited traps [19].

However, pheromone baited traps may be unaffordable by majority small holder farmers in developing countries as they require importation of the synthetic pheromones.

Presently, no single control measure has been successful in eradicating the banana weevil. Instead integrated pest management practices are being promoted worldwide to control the banana weevil.

In this study, a logistic growth model for the population dynamics of the banana weevil, *Cosmopolites Sordidus* (Germar), incorporating trapping and Holling type 2 functional response is formulated and analyzed to draw insights on the application and effectiveness of trapping the adult banana weevil.

This paper is organized as follows: In Section 2, the model is formulated and analyzed. In Section 3, numerical simulation of the model is carried out. The discussion of results and conclusion are done in Section 4.

2. Model Formulation and Analysis

In this section, a single species model for the population dynamics of the banana weevil, *Cosmopolites Sordidus* (Germar), is formulated and analyzed.

2.1 Model Formulation

The model is formulated based on the following assumptions.

(i) Trapping the adult banana weevil is adopted as the harvesting strategy.

(ii) In absence of trapping, the banana weevils are assumed to grow logistically.

(iii) A closed agroecosystem is assumed with migration of banana weevils to and from adjacent banana farms considered negligible.

(iv) The harvesting function is assumed to follow Holling type 2 functional response.

Let $N(t)$ be the banana weevil population size at time t. Then assuming logistic growth, the population dynamics of the banana weevil without trapping is represented by the following differential equation

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right), \tag{1}$$

where r is the banana weevil intrinsic growth rate and K is the carrying capacity. The environmental carrying capacity, K, is assumed to depend on the size of the plot or farm and on the number of mats.

Trapping the adult banana weevil is incorporated in the logistic growth model by subtracting the term $g(N) = \frac{hN}{A+N}$ from equation (1). This trapping function is similar to a Holling type 2 functional response term proposed by Holling [12]. It is a more realistic form of trapping the adult banana weevil and its key features are as follows:

(i) It has an equilibrium point $N = 0$ for all parameters.

(ii) The rate of trapping decreases with banana weevil population size N.

(iii) When N is large, $\lim_{N\to\infty} \frac{hN}{A+N} = h$, the maximum trapping rate.

Thus to study the population dynamics of banana weevil, *Cosmopolites Sordidus* (Germar), with trapping as a harvesting strategy, the following differential equation is adopted.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - \frac{hN}{A+N}, \tag{2}$$

where h is the trapping rate and A is the half-capturing or trapping constant since $g(A) = \frac{h}{2}$ when $A = N$. The half-capturing or trapping rate is a sort of threshold value because below it, trapping is not intensive while above it, trapping is intensive. The trapping rate h is measured in terms of the number of traps set and the trapping frequency.

2.2 Steady states

The right-hand side of equation (2) is a cubic polynomial and can have at most three roots. One steady state is always $N = 0$ and the others are obtained both analytically and graphically from the equation

$$r\left(1-\frac{N}{K}\right) - \frac{h}{A+N} = 0. \tag{3}$$

Analytically, equation (3) is expressed as a quadratic equation in N as

$$N^2 + (A-K)N + K\left(\frac{h-Ar}{r}\right) = 0. \tag{4}$$

The roots of equation (4) are

$$N_{1,2} = \frac{-(A-K) \pm \sqrt{[(A+K)^2 - \frac{4hK}{r}]}}{2}. \tag{5}$$

Equation (5) has real and distinct roots if and only if $(A+K)^2 > \frac{4hK}{r}$ or

$$h \le \frac{r}{4K}(A+K)^2$$

and since $h \ge 0$, then $0 \le h \le \frac{r}{4K}(A+K)^2$.

2.2.1 Critical trapping rate

There is a harvesting rate called the critical trapping rate, h_c, represented by the equation

$$h_c = \frac{r}{4K}(A+K)^2 \tag{6}$$

with the following properties.

(i) At a trapping rate, $h = h_c$, equation (4) has a repeated root and the banana weevil population size, N, approaches the equilibrium level $\frac{-(A-K)}{2}$ provided $K > A$, that is, the environmental carrying capacity, K, exceeds the half-capturing constant, A.

(ii) For a trapping rate, $h > h_c$, both roots of equation (4) are complex, $N'(t) < 0$ for all N and every solution crashes, hitting zero in finite time. The system is then said to have collapsed. In this situation, trapping drives the banana weevil population to extinction.

(iii) At a trapping rate, $h < h_c$, equation (4) has two real and distinct roots, N_1, N_2, as given in equation (5). N_1 increases from 0 to $\frac{-(A-K)}{2}$ as h increases from 0 to $\frac{r}{4K}(A+K)^2$ while N_2 decreases from $-(A-K)$ to $\frac{-(A-K)}{2}$ as h decreases.

It is deduced from equation (6) that the critical trapping rate, h_c, is directly proportional to; the banana weevil intrinsic growth rate r, the environmental carrying capacity K and the half-capturing constant, A.

It is also deduced that as the trapping rate, h, approaches the critical value h_c, there is a discontinuity as the two equilibria (one stable and the other unstable) collide with each other and self-destruct. This is called a saddle-node bifurcation because the two colliding fixed points are a stable node and a saddle (which is unstable). This discontinuity is called a mathematical catastrophe and heralds the extinction of the banana weevil [5, 27], for which trapping is regarded as successful in eradication the banana weevil. For $h < h_c$, the banana weevil population tends to an equilibrium size that approaches $\frac{-(A-K)}{2}$ as $h \to h_c$ (provided the initial population size is at least N_1) but for $h > h_c$, the population size approaches zero in finite time for all initial population sizes.

2.2.2 Stability Analysis of the Steady States

The following results, namely Theorem 2.1 and its Corollary cor1.1 in Brauer and Castillo-Chavez [4], are essential in determining the stability of the equilibrium points of equation (2).

Theorem 2.1. *Consider a first order autonomous differential equation*

$$x' = f(x). \tag{7}$$

If all solutions of the linearization of equation (7) *at an equilibrium* x^* *tend to zero as* $t \to \infty$, *then all solutions of equation* (7) *with* $x(0)$ *sufficiently close to* x^* *tend to the equilibrium* x^* *as* $t \to \infty$.

Corollary 2.1. *An equilibrium* x^* *of equation* (7) *with* $f'(x^*) < 0$ *is asymptotically stable while an equilibrium* x^* *with* $f'(x^* > 0)$ *is unstable.*

Now, rewriting equation (2) as $f(N) = rN\left(1 - \frac{N}{K}\right) - \frac{hN}{A+N}$, then

$$f'(N) = r\left(1 - \frac{2N}{K}\right) - \frac{hA}{(A+N)^2}, \tag{8}$$

such that

$$f'(0) = r - \frac{h}{A}. \tag{9}$$

Clearly, the steady state $N = 0$ is linearly stable if $r < \frac{h}{A}$ and unstable otherwise. It therefore follows that the model exhibits a transcritical bifurcation at the point $r = \frac{h}{A}$. It is worth pointing out that the condition $r < \frac{h}{A}$ for the linear stability of this steady state in effect means that the trapping rate of the adult banana weevils exceeds their intrinsic growth rate This is unsustainable in the long run and can only result in the exhaustion of the banana weevil population size. The point $N = 0$ therefore corresponds to the situation whereby trapping the adult banana weevil as a control option succeeds in suppressing the banana weevil population size and is therefore regarded a success.

In practice though, it may not be possible to trap the adult banana weevils to extinction. Rather, it is possible that trapping reduces the banana weevil population size to a point where they do not cause significant economic damage after which trapping may then be relaxed. At this point either the Allee effect sets in whereby survival and reproduction rates of the banana weevil drastically fall due to small population size leading to possible extinction of the banana weevil or a slow banana weevil population build up begins. With low fecundity, banana weevil population build up is known to be slow and several crop cycles (or ratoons) are required before banana weevil population is fully established and damage is more pronounced [10]. This may well explain the observed periodic outbreaks of the banana weevil in banana growing areas.

Similarly, by rewriting equation (4) as

$$f(N) = N^2 + (A-K)N + K\left(\frac{h-Ar}{r}\right), \tag{10}$$

then

$$f'(N) = 2N + (A-K). \tag{11}$$

By Theorem 2.1, an equilibrium point N^* is asymptotically stable if $f'(N^*) < 0$ or equivalently $N^* < -\frac{(A-K)}{2}$. Accordingly, $N_1 < -\frac{(A-K)}{2}$ and so is asymptotically stable while $N_2 > \frac{-(A-K)}{2}$ and so is unstable.

Graphically, the solution to equation (3) is obtained by sketching the curves $f_1(N) = r(1-\frac{N}{K})$ and $f_2(N) = \frac{h}{A+N}$ on the same axes. $f_1(N)$ is a straight line with intercepts at r and K and the gradient is equal to $\frac{-1}{K}$ while $f_2(N)$ is a curve asymptotic to the line $N = -A$. The points of intersection of the two curves represents the two non-zero equilibrium points.

For simplicity, rescaling equation (2) by letting $u = \frac{N}{A}$, $q = \frac{K}{A}$, $\tau = \frac{h}{A}t$ and $p = \frac{rA}{h}$ and repeatedly applying the chain rule transforms equation (2) to

$$\frac{du}{d\tau} = pu\left(1-\frac{u}{q}\right) - \frac{u}{1+u} \tag{12}$$

and reduces equation (3) to

$$p\left(1-\frac{u}{q}\right) = \frac{1}{1+u}. \tag{13}$$

Equation (12) now has only two parameters p and q compared to four for equation (2) and is easier to manipulate. Taking A as a proportionality constant, it is deduced that parameter p represents the ratio of the banana weevil intrinsic growth rate to the trapping rate. Similarly, both u and q differ from N and K respectively by a constant factor of $\frac{1}{A}$ and are regarded as representing the rescaled banana weevil population size, N, and carrying capacity, K. The interpretation of equation (13) is as follows: The LHS represents the per capita growth rate of the scaled variable u with respect to the scaled time τ while the RHS represents the per capita trapping rate of the banana weevil due to trapping also in rescaled variables. The points of intersection of the two curves $f_1(u) = \frac{1}{1+u}$ and $f_2(u) = p(1-\frac{u}{q})$ plotted on the same axes give the non-zero equilibria for u or equivalently N. $f_1(u)$ is a hyperbola in the first quadrant while $f_2(u)$ is a straight line with gradient $-\frac{1}{q}$ and intercepts at p and q.

The non-zero equilibria obtained by solving equation (13) are

$$u_{1,2} = \frac{-(p-pq) \pm \sqrt{[(p+pq)^2 - 4pq]}}{2p}.$$

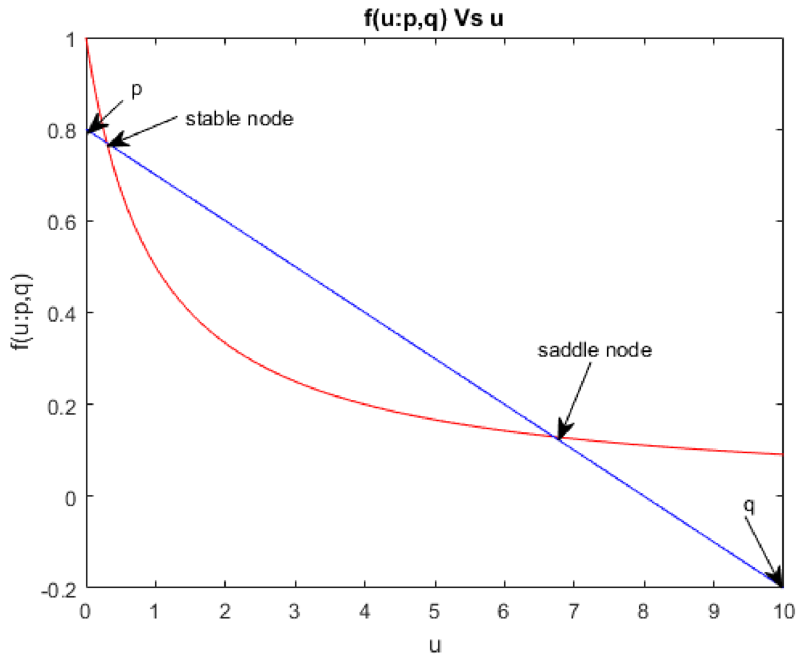

Figure 1. Illustration of the non-zero equilibrium points for the rescaled model for the banana weevil population dynamics with p and q the vertical and horizontal asymptotes respectively

Writing equation (13) as $f(u) = pu^2 + u(p - pq) + q(1 - p)$, then clearly $f'(u) = 2up + (p - pq)$ or equivalently $u^* < \frac{-(p-pq)}{2p}$ for any equilibrium point u^*. Again by Theorem 2.1, asymptotic stability requires that $f'(u^*) < 0$. Accordingly, $u_1 = \frac{-(p-pq)-\sqrt{[(p+pq)^2-4pq]}}{2p} < \frac{-(p-pq)}{2p}$ is always asymptotically stable whereas $u_2 = \frac{-(p-pq)+\sqrt{[(p+pq)^2-4pq]}}{2p} > \frac{-(p-pq)}{2p}$ is always unstable.

The number and stability of the equilibrium points depends on the relative positions of the parameters p and q representing the ratio of the intrinsic growth rate to the trapping rate and the rescaled carrying capacity respectively. This can be illustrated as follows: Suppose, as shown in Figure 1, p is slowly increased while keeping q fixed in position, that is, the line is rotated clockwise about the intercept q. Then, u_1-the lower equilibrium decreases slowly in value while u_2 the higher equilibrium increases. When u_1 hits zero, p attains its maximum value of 1 and any further increase results in the loss of the stable node and consolidation of the unstable node. On the other hand reducing p while keeping q fixed in position results in a progressive increase in u_1 and a corresponding decrease in u_2 until $u_1 = u_2$ when both coalesce and annihilate each other. Thus, at low values of p and for a fixed q, the system has one equilibrium point $u = \frac{-(1-q)}{2}$ while at high values of p and for a fixed q, the system has one equilibrium u_2 which is a saddle node. And, at intermediate values of p, the system exhibits two equilibria-the lower equilibrium u_1 which is stable and the higher equilibrium u_2 which is unstable. From the rescaling, it is adduced that p is directly proportional to the banana weevil intrinsic growth rate, r, and inversely proportional to the trapping rate, h, if A is considered a proportionality constant. This implies that low values of the intrinsic growth rate, r, correspond to the stable equilibrium, u_1, while high rates of the intrinsic growth rate are aligned with the saddle node, u_2. This makes sense since large banana weevil population size is credited with low banana yield and production and vice-versa.

Similarly, fixing p in position and reducing q progressively reduces u_2 while slightly increasing u_1 until both coalesce into a single equilibrium, $u = \frac{-(1-q)}{2}$. However, as $f(u)$ is always positive for all $u \geq 0$, increasing q while keeping p fixed in position will always result in the two equilibria u_1 and u_2. Low values of q result in one equilibrium otherwise there will always be two equilibria. Again from rescaling, it is observed that q is directly proportional to carrying capacity, K, with a constant of proportionality A. Low values of the carrying capacity favors the stable equilibrium while at high values, there will always be both the stable and unstable equilibria u_1 and u_2 respectively. Related analysis is carried out in Ludwig *et al.* [17] for the case of the spruce bud worm that devastates balsam fir forest but is regulated by predatory birds.

The bifurcation points are obtained by requiring that the curves $f_1(u) = \frac{1}{1+u}$ and $f_2(u) = p(1-\frac{u}{q})$ are equal in both value and derivative viz:

$$p\left(1-\frac{u}{q}\right) = \frac{1}{1+u}, \tag{14}$$

$$\frac{d}{du}\left[p\left(1-\frac{u}{q}\right)\right] = \frac{d}{du}\left[\frac{1}{1+u}\right],$$

$$-\frac{p}{q} = -\frac{1}{(1+u)^2}. \tag{15}$$

Equation (14) can be rewritten as

$$p = \frac{pu}{q} + \frac{1}{(1+u)}. \tag{16}$$

In equation (15), $\frac{p}{q}$ is a function of u and substituting it into equation (16) gives

$$p = \frac{1+2u}{(1+u)^2}. \tag{17}$$

Finally, substituting equation (17) into equation (15) yields

$$q = (1+2u). \tag{18}$$

Thus, the bifurcation points can implicitly be expressed as functions of u as $p = \frac{1+2u}{(1+u)^2}$ and $q = (1+2u)$.

The stability analysis in this section can be summarized in the following theorem:

Theorem 2.2. *The harvested logistic equation* (2) *or its rescaled equation* (12) *has three equilibria whose stability is as follows:*

(i) *the trivial equilibrium is always unstable whenever $r < \frac{h}{A}$. In addition, there is a transcritical bifurcation for $r = \frac{h}{A}$;*

(ii) *there are two nontrivial equilibria with the lower equilibrium, N_1, a stable node and the upper equilibrium, N_2, a saddle;*

(iii) *there is a saddle-node bifurcation when the trapping rate h approaches a critical value, h_c.*

3. Numerical Simulation of the Model

In this section, a numerical simulation of the model is carried out for parameter values gleaned from literature and using Matlab computer software program.

3.1 Parameter Estimation

The elusive nature of the banana weevil makes determining the model parameters rather an audacious task. The adult banana weevil is elusive [13] and rarely observed since it is nocturnally active. The immature stages live within the corm and are largely inaccessible. This notwithstanding, some parameters have been estimated from previous studies as follows: For example, it is known that once introduced in a virgin banana plantation, the banana weevil population build up is slow occasioned by low fecundity rates and low egg to adult banana weevil survival rates [9]. Thus, assuming a $4-5$ fold increase in banana weevil population per year, then for continuous time population, the intrinsic growth rate can be estimated from $e^r = 4$ or $e^r = 5$. In this case $1.4 < r < 1.6$ per year.

The environmental carrying capacity, K, is assumed to depend on the plot (or farm) size and the number of mats contained in it and is influenced by a number of factors such as the cultivar planted and the spacing between mats which may well depend on the farmers' economic status and the farming system employed. In commercial farms, spacing between mats and the number of pseudostems on a particular mat are set to a given standard and followed which may not be the case with traditional household farms especially by resource limited small holder farmers in developing countries whose interest may be in quantity of produce rather than quality.

In this study, it is assumed without loss of generality that there are about $200 < K < 300$ stands in a typical field.

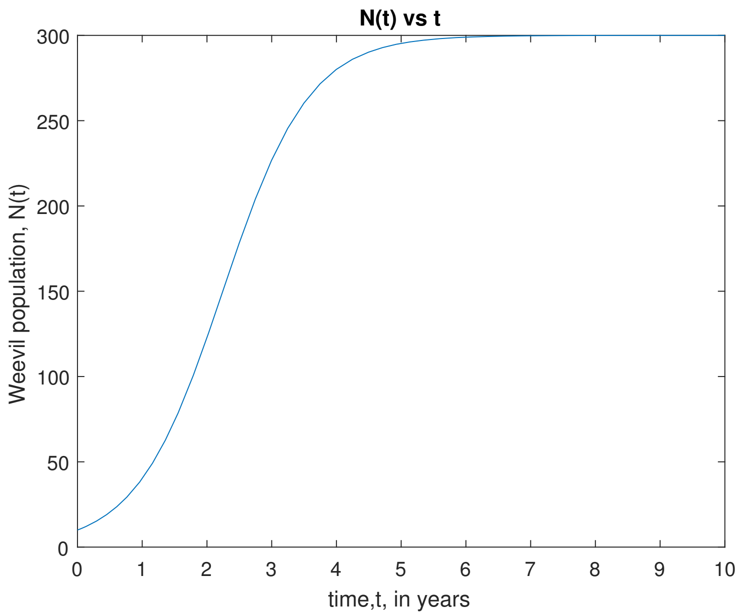

Figure 2. Without trapping the banana weevil follows logistic growth with the population eventually levelling off at its carrying capacity; $r = 1.5$ and $K = 300$

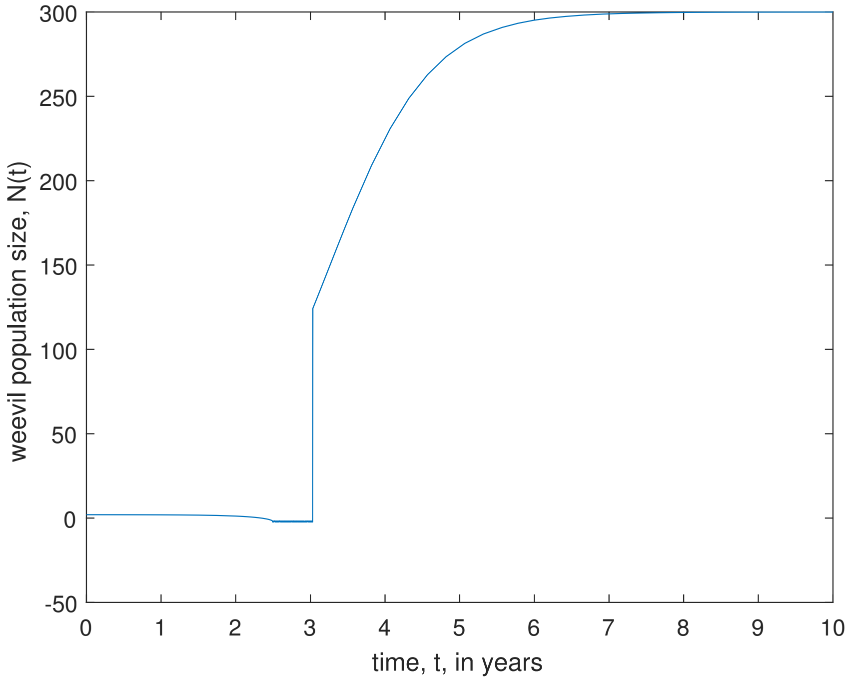

Figure 3. Schematic representation of the banana weevil model at moderate trapping rates; the equilibrium population size is lower with trapping than without. Here $r = 1.5$, $K = 300$, $A = 2$ and $h = 10$

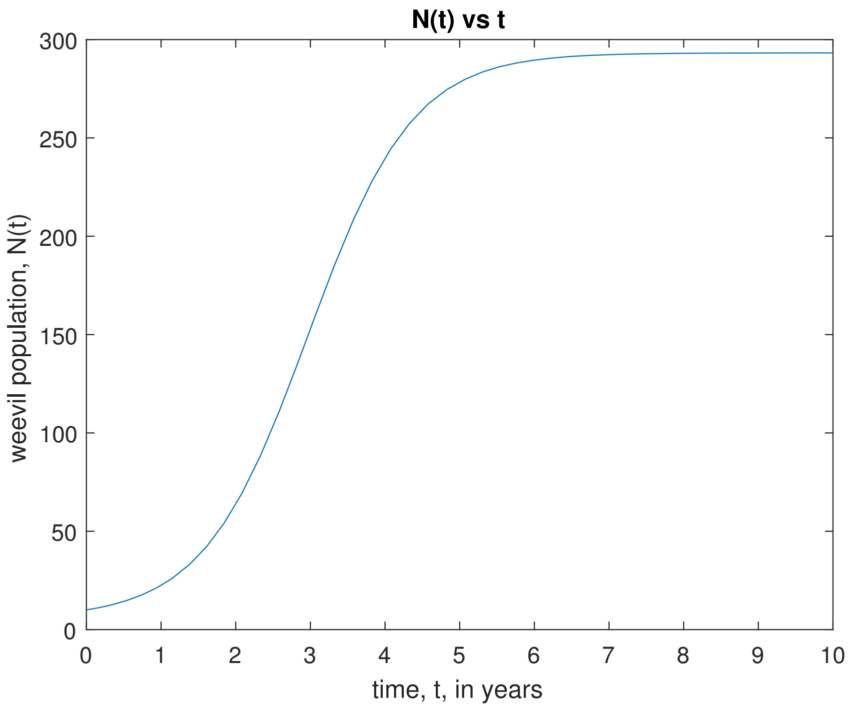

Figure 4. At higher trapping rates, the banana weevil population size is brought close to exhaustion. Here $r = 1.5$, $K = 300$, $A = 2$ and $h = 12$

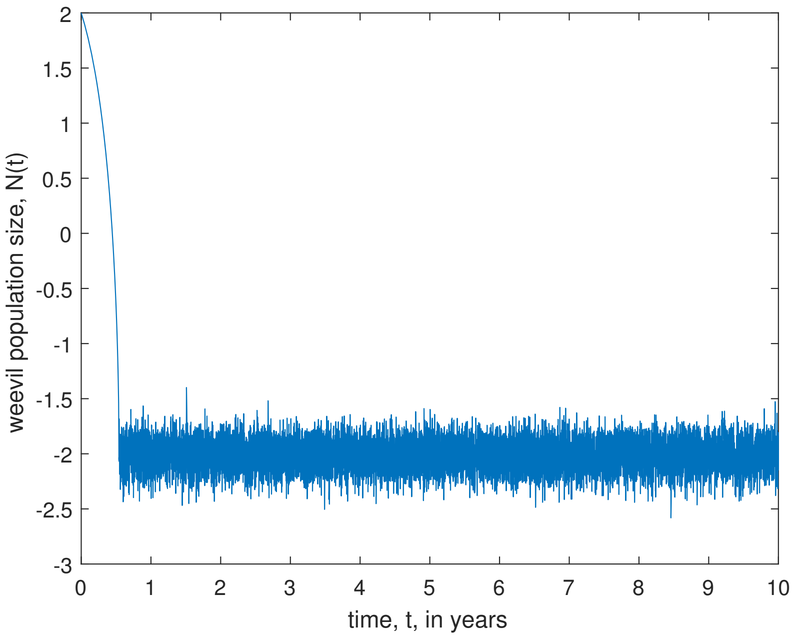

Figure 5. When the trapping rate exceeds the critical value, the banana weevil population size is exhausted in finite time and the banana weevil dynamics display chaotic behavior. Here $r = 1.5$, $K = 300$, $A = 2$ and $h = 20$

Trapping intensities are variable and reflective of farmers' economic status and the production system used. Trap captures are influenced by such factors as trap placement, banana clones used as trapping materials and trapping intensities [3, 8]. Trapping rates can be measured in terms of the number of traps per acre and/or the frequency of trapping which reflect the infestation status of the banana weevil and the farmer's level of interest in fighting the banana weevil. Recommended trap densities for banana weevil monitoring range from ≤ 10 traps per hectare to 20-30 traps per hectare to 40-60 traps per hectare. There is no standard value of the trapping rate and this study adopts a trapping rate in the range of $2 < h < 20$ traps per hectare. The numerical simulations run by Matlab computer software program are shown in Figures 2-5.

4. Discussion

A harvested logistic growth model describing the dynamics of the banana weevil, *Cosmopolites Sordidus* (Germar) that incorporates trapping of the adult banana weevil as a control measure has been formulated and analyzed both analytically and numerically. The model equilibria and their stability properties were determined both analytically and geometrically as shown in Figure 1. In addition, some possible bifurcations were determined. Key in the analysis is the derivation of the expression for the critical trapping rate-the threshold that governs the exhaustion or establishment of the banana weevil population. Numerical simulations were carried out using Matlab computer software program with parameter values gleaned from literature.

It was established that the critical trapping rate is directly proportional to the banana weevil intrinsic growth rate, r, the environmental carrying capacity, K, and the half-capturing constant, A. High intrinsic growth rates translate into a large weevil infestation status, which in absence of other control options other than trapping, would require high trapping rates to suppress the banana weevil population. Similarly, large environmental carrying capacities imply that a given plot or farm is able to support a large population of banana weevils. This in turn translates into a high banana weevil infestation status that would, in essence, require high trapping rates to bring down the pest numbers. Since trapping rates are measured in terms of the frequency of trapping or the number of traps set, at high pest infestation levels, a number of traps and/or the frequency of trapping are assumed to accordingly increase at high intrinsic growth rates and environmental carrying capacity values. In absence of trapping, the banana weevil dynamics are best described by a logistic growth with the population eventually leveling off at the carrying capacity (see Figure 2). Trapping removes adult banana weevil that would otherwise grow logistically. With trapping, as shown in Figure 3, the equilibrium level of the banana weevil population is otherwise lower than in the case without trapping. This implies that progressively removing the adult banana weevils that are responsible for the next generation of the adult banana weevils- continually lowers the banana weevil intrinsic growth rates until the banana weevil population size falls to insignificant amounts at which they cause minimal economic damage. This situation is well depicted in Figure 4 where is shown that at high trapping rates, the banana weevil population size is brought close to exhaustion. If trapping is then stopped at the point, a slow banana weevil population build-up will ensue and after sometime it will reach infestation level, growing logistically and again level off at the environmental carrying capacity much later. This scenario explains to periodic outbreaks of the banana weevil in banana growing areas. The sudden jump in the banana weevil population size shown in Figure 4 can also be explained by assuming that after the adult banana weevil population size has been trapped and exhausted, only the immature banana weevils (i.e. the eggs, larvae and pupae) remain which will eventually develop into adult banana weevils. When the trapping rate exceeds the critical trapping rate, the banana weevil population size falls below zero in finite time and extinction results and the dynamics are described by chaotic behavior as shown in Figure 5.

In hindsight, it is clear that to fight the banana weevil, trapping the adult banana weevils per se will only provide temporary relief. After sometime, the banana weevils in the juvenile stages will mature into adult banana weevils and the cycle will start all over again. Therefore a management package that targets all developmental stages of the banana weevils would be best placed to provide long lasting relief against the banana weevil and should be promoted in banana growing areas.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] A. M. Abera-Kalibata, C. S. Gold, R. G. Van Driesche, and P. E. Ragama, Composition, distribution, and relative abundance of ants in banana farming systems in Uganda, *Biological Control* **40**(2) (2007), 168 – 178, DOI: 10.1016/j.biocontrol.2006.11.006.

[2] R. Bakyalire and M. W. Ogenga-Latigo, Aspects of the life cycle and behavior of the banana weevil, *Cosmopolites Sordidus* Germar (Coleoptera: Curculionidae), in *Recent Contribution to Banana Entomology in Uganda* (1990-1992), M. W. Ogenga-Latigo (ed.), Department of Crop Sciences, Makerere University, Uganda (1992).

[3] R. Bakyalire, A study of the life cycle and behavior of the banana weevil *Cosmopolites Sordidus* Germar in Uganda, 118pp, *Masters Thesis*, Makerere University, Kampala, Uganda (1992).

[4] F. Brauer and C. Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology*, Springer (2000), URL: `https://link.springer.com/book/10.1007/978-1-4614-1686-9`.

[5] F. Brauer and D. A. Sanchez, Constant rate population harvesting: equilibrium and stability, *Theoretical Population Biology* **8**(1975), 12 – 30, DOI: 10.1016/0040-5809(75)90036-2.

[6] W. J. Budenberg, I. O. Ndiege and F. W. Karago, Evidence for volatile male-produced pheromone in banana weevil *Cosmopolites sordidus*, *Journal of Chemical Ecology* **19** (1993), 1905 – 1916, DOI: 10.1007/BF00983795.

[7] FAOSTAT, *Banana and Plantain Surface and Production in 2013*, available at `http//faostat.fao.org/site/339/default.aspx` (2014).

[8] C. S. Gold and M. I. Bagabe, Banana weevil, *Cosmopolites* Germar (Coleoptera, Curculionidae) infestation of cooking and beer bananas in adjacent stands in Uganda, *African Entomology* **5**(1) (1997), 103 – 108, URL: `https://www.ingentaconnect.com/content/sabinet/ento/1997/00000005/00000001/art00017`.

[9] C. S. Gold, J. E. Pena and E. B. Karamura, Biology and integrated pest management for the banana weevil *Cosmopolites Sordidus* (Germar) (Coleoptera Curculionidae), *Integrated Pest Management Reviews* **6**(2001), 75 – 155, DOI: 10.1023/a:1023330900707.

[10] C. S. Gold,G. H. Kagezi, G. Night and P. E. Ragama, The effects of banana weevil, *Cosmopolites Sordidus* (Germar) damage on highland banana growth, yield and stand duration in Uganda, *Annals of Applied Biology* **145**(3)(2004), 263 – 269, DOI: 10.1111/j.1744-7348.2004.tb00382.x.

[11] D. J. Greathead, Opportunities for control of insect pests in tropical Africa, *Review of Zoology Africa (Revue de Zoologie Africaine)* **100**(1) (1986), 85 – 96, URL: `https://www.cabi.org/isc/abstract/19870546138`.

[12] C. S. Holling, Some characteristics of simple types of predation and parasitism, *The Canadian Entomologist* **91**(1959), 385 – 398, DOI: 10.4039/Ent91385-7.

[13] E. B. Karamura and C. S. Gold, The elusive banana weevil, *Cosmopolites sordidus* Germar, in *I International Symposium on Banana: I International Conference on Banana and Plantain for Africa* **540** (1996), 471 – 485, DOI: 10.17660/ActaHortic.2000.540.53.

[14] A. Kiggundu, W. Tushemereirwe and K. Kunert, Enhancing banana weevil resistance by plant genetic modification: a perspective. *African Journal of Biotechnology* **2**(12)(2003), 563 – 569, DOI: 10.5897/AJB2003.000-1109.

[15] A. M. Koppenhofer, K. V. Senshu-Reddy, G. Madel and M. C. Lubega, Predators of banana weevil *Cosmopolites Sordidus*, (Germar) (Col., Curculionidae), in western Kenya, *Journal of Applied Entomology* **114**(1992), 530 – 533, DOI: 10.1111/j.1439-0418.1992.tb01161.x.

[16] A. M. Koppenhofer, K. V. Seshu-Reddy and R. A. Sikora, Reduction of banana weevil populations with pseudostem traps, *International Journal of Pest Management* **40** (1994), 300 – 304, DOI: 10.1080/09670879409371903.

[17] D. Ludwig, D. D. Jones and C. S. Holling, Qualitative analysis of the insect outbreak systems: The spruce budworm and the forest, *The Journal of Animal Ecology* **47**(1) (1978), 315 – 332, DOI: 10.2307/3939.

[18] M. Masanza, *Effect of crop sanitation on banana weevil Cosmopolites Sordidus populations and associated damage*, PhD. Thesis, Wageningen University, The Netherlands (2003).

[19] I. O. Ndiege, S. Jarayama, A. C. Oehlschlager, L. Gonzalez, G. Alpizar and M. Fallas, Convenient synthesis and field activity of a male-produced aggregation pheromone of *Cosmopolites Sordidus*, *Naturwissenschafften* **83**(6) (1996), 280 – 282, DOI: 10.1007/BF01149603.

[20] I. Perfecto, Ants (Hymenoptera: Formicidae) as natural control agents of pests in irrigated maize in Nicaragua, *Journal of Economic Entomology* **84**(1) (1991), 65 – 70, DOI: 10.1093/jee/84.1.65.

[21] E. R. Pianka, On r-and K-selection, *The American Naturalist* **104**(940) (1970), 592 – 597, URL: `https://www.journals.uchicago.edu/doi/pdfplus/10.1086/282697`.

[22] R. Roche and S. Abreu, Control of the banana weevil *Cosmopolites Sordidus* by the ant *Tetramorium guineense*, *Ciencias de la Agricultura* **17**(1983), 41 – 49, URL: `https://www.cabdirect.org/cabdirect/abstract/19850524533`.

[23] P. M. Room, The relative abundance of ant species in Ghana's cocoa farms, *The Journal of Animal Ecology* **40** (1971), 735 – 751, DOI: 10.2307/3447.

[24] N. D. T. M. Rukazambuga, C. S. Gold and S. R. Gowen, Yield loss in East African highland banana (Musa spp., AAA-EA group) caused by the banana weevil, *Cosmopolites Sordidus* Germar, *Crop Protection* **17**(7) (1998), 581 – 589, DOI: 10.1016/S0261-2194(98)00056-8.

[25] R. A. Sikora, N. D. Bafokuzara, A. S. S. Mbwana, G. W. Oloo, B. Uronu and K. S. Reddy, Interrelationship between banana weevil, root lesion nematode and agronomic practices, and their importance for banana decline in the United Republic of Tanzania, *FAO Plant Protection Bulletin* **37**(4) (1989), 151 – 157, URL: `https://www.cabdirect.org/cabdirect/abstract/19911179818`.

[26] D. F. Waterhouse, *Biological control of Insect pests: South East Asia Prospects*, *ACIAR Monograph* No. **51**, Canberra, Australia (1998), URL: `https://swfrec.ifas.ufl.edu/hlb/database/pdf/00000382.pdf`.

[27] E. C. Zeeman, Catastrophe theory, *Scientific American* **234**(4) (1976), 65 – 83, DOI: 10.1038/scientificamerican0476-65.