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Research Article

An Analytical Study of Dispersion and Wall Absorption with Effect of Viscoelasticity and Magnetic Field

Venkataswamy K. V., Jagadeesha S.* and Indira Ramarao

Department of Mathematics, Nitte Meenakshi Institute of Technology, Bangalore, India
***Corresponding author:** jagadeeshas31@gmail.com

Abstract. In the present study an unsteady convective diffusive mass transfer in a flow of viscoelastic fluid flow in a concentric annulus with applied magnetic field is considered. The velocity is analytically obtained using no-slip condition. The species equation is solved by adopting a dispersion model used by Gill and Sankarasubramanian approach. The parameters like dispersion and convection coefficients which arise in the analysis are plotted against absorption parameter for different values of Hartmann number and viscoelastic parameter. The effect of viscoelastic parameter is to increase the convective coefficient and dispersion coefficient. Dispersion increases with absorption but convection decreases. The results are numerically evaluated and graphically depicted.

Keywords. Viscoelastic fluid; Magnetic field; Wall absorption; Catheter; Concentric annulus

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1. Introduction

Many researchers have analysed dispersion of solute in physiological fluids involving interphase mass transfer. Taylor [14] and Aris [2] have studied the dispersion of passive traces in circular tube. Sankarasubramanian and Gill [12] have used analytical methods to study dispersion with interphase mass transfer. DeGance and Johns [6] have shown that transport coefficients were functions of time. Lungu and Moffat [9], Clifford *et al*. [4], and Boddington and Clifford [3] have analysed the solute transfer by considering straight tubes.

A study of dispersion of a solute is done by Jayaraman *et al*. [7], where a curved tube with absorbing wall is considered. Nagarani [11], Agarwal and Jayaraman [1] and Sharp [13] have studied the dispersion in non-Newtonian fluids. Gill and Sankarasubramanian model [12] is used by Dash *et al*. [5] to study shear augmented dispersion in casson fluid flow. Jiang and Grot Berg [8] have analysed the effect of oscillatory field on tube.

In the present study, an analytical solution has been obtained for species equation with effect of viscoelastic fluid and magnetic field applied on a concentric annulus.

2. Mathematical Formulation

Physical configuration consists of a catheter of radius kR is inserted in the artery of radius R as given in Figure 1. The flow is assumed to be fully developed and $\frac{r}{2} \ll 1$.

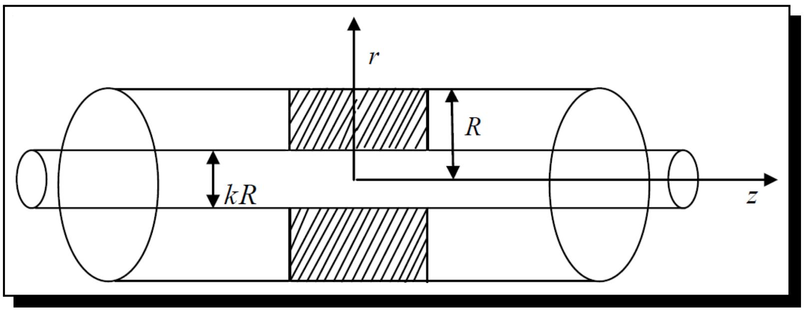


Figure 1. Physical configuration

The stress obeys the constitutive equation,

$$s = \frac{\mu}{1+\lambda_1}\{\dot{r} + \lambda_2 \ddot{r}\}, \tag{2.1}$$

where μ is viscosity, $\dot{r}$ is rate of strain, λ_1 is ratio of relaxation and λ_2 is the ratio of retardation time.

Using non-dimensional parameters $r^* = \frac{r}{R}$, $u^* = \frac{u}{u_0}$ and assuming fully developed steady flow with low Reynolds number following Nadeem and Akbar [10], the governing equations for velocity will be,

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\mu r}{1+\lambda_1}\frac{\partial w}{\partial r}\right] - M^2 w = \frac{\partial p}{\partial z}, \tag{2.2}$$

subject to no-slip conditions at the boundaries.

Solving the above equation the velocity is obtained as

$$w = AI_0\left(M\sqrt{1+\lambda_1}\,r\right) + BK_0\left(M\sqrt{1+\lambda_1}\,r\right) + \frac{P}{M^2}, \tag{2.3}$$

where $A = \frac{B_1 - B_2}{B_2A_1 - B_1A_2}$, $B = \frac{A_1 - A_2}{A_2B_1 - A_1B_2}$, $A_1 = I_0\left(M\sqrt{1+\lambda_1}\,r\right)$, $A_2 = I_0\left(M\sqrt{1+\lambda_1}\,k\right)$, $B_1 = K_0\left(Mk\sqrt{1+\lambda_1}\right)$, $B_2 = K_0\left(M\sqrt{1+\lambda_1}\,k\right)$.

The species equation is

$$\frac{\partial c}{\partial t} + w\frac{\partial c}{\partial z} = D\left(\frac{\partial^2 c}{\partial z^2} + \frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r}\right), \tag{2.4}$$

subject to conditions

$$c(0,r,z)=c_0,$$

$$\frac{\partial c}{\partial r}=\begin{cases}0 & \text{at } r=kR\\ -\alpha c & \text{at } r=R.\end{cases} \tag{2.5}$$

In equation (2.5), negative sign is due to diffusion across the boundary resulting in loss of solute.

Non-dimensionalising the equations (2.4) and (2.5) using the quantities $c^*=\frac{c}{c_0}$, $t^*=\frac{t}{R^2/D}$, $r^*=\frac{r}{R}$ and $z^*=\frac{z}{D/R^2 w_0}$, we get

$$\frac{\partial c}{\partial t}+w\frac{\partial c}{\partial z}=\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r}+\frac{1}{Pe^2}\frac{\partial^2 c}{\partial z^2}, \tag{2.6}$$

subject to $c(0,r,z)=\frac{\delta(z)}{Pe}$, where $\delta(z)$ — Dirac delta function, Pe — Peclet number, such that

$$\frac{\partial c}{\partial r}=\begin{cases}0 & \text{at } r=k\\ -\beta_0 c & \text{at } r=1.\end{cases} \tag{2.7}$$

Following Jayaraman *et al*. [7], concentration is assumed as

$$c(r,t,z)=\sum f_n(t,r)\frac{\partial^n \theta_m}{\partial z^n} \tag{2.8}$$

and

$$\theta_m=\frac{\int_0^{2\pi}\int_k^1 rc\,dr\,d\theta}{\int_0^{2\pi}\int_k^1 r\,dr\,d\theta}=\frac{2}{(1-k^2)}\int_k^1 rc\,dr, \tag{2.9}$$

where θ_m is the mean concentration.

The generalised dispersion model of Sankarasubramanian and Gill [12], the governing equation in truncated form can be written by,

$$\frac{\partial \theta_m}{\partial t}=M_0(t)\,\theta_m+M_1(t)\frac{\partial \theta_m}{\partial z}+M_2(t)\frac{\partial^2 c}{\partial z^2}. \tag{2.10}$$

Using equation (2.10), substituting for $\frac{\partial \theta_m}{\partial t}$ from equation (2.8), we can find

$$\frac{\partial f_n}{\partial t}-\frac{\partial^2 f_n}{\partial r^2}-\frac{1}{r}\frac{\partial f_n}{\partial r}+w f_{n-1}-\frac{1}{Pe^2}\delta_{n,2}f_{n-2}+\sum_{i=0}^{n} f_{n-1}M_i=0 \tag{2.11}$$

and

$$M_n(t)=\frac{2}{(1-k^2)}\frac{\partial f_n}{\partial r}(t,1)+\frac{\delta_{n,2}}{Pe^2}-\frac{2}{(1-k^2)}\int_k^1 r w f_{n-1}\,dr. \tag{2.12}$$

Similarly, the boundary conditions becomes

$$\frac{\partial f_n}{\partial r}=\begin{cases}0 & \text{if } r=k\\ -\beta f_n & \text{if } r=1.\end{cases} \tag{2.13}$$

Solving equations (2.10) and (2.11), the exchange coefficient takes the form

$$M_0(t)=\frac{2}{(1-k^2)}\left(\frac{\partial f_0}{\partial r}\right)_{r=1} \tag{2.14}$$

and

$$\frac{\partial f_0}{\partial t}-\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial f_0}{\partial r}r\right)+f_0 M_0=0. \tag{2.15}$$

Using $f_0(t,r) = e^{-\int_0^t M_0(\eta)g_0(\eta,r)d\eta}$ and solving the resulting equation using separation of variables method, we get

$$g_0(t,r) = \sum \frac{A_n}{J_1(\mu_n k)} e^{-\mu_n^2 t}[a_1 - a_2] \tag{2.16}$$

and

$$A_n = \frac{J_1(\mu_n k)[1-k^2]\mu_n^2 \int_k^1 r E_n(\mu_n r) B_1(r) dr}{(\mu_n^2+\beta^2)[E_n(\mu_n)]^2 - \mu_n^2 k^2 [E_n(\mu_n k)]^2 \int_k^1 r B_1(r) dr}, \tag{2.17}$$

where $E_n(\mu_n r) = a_1 - a_2$, and μ_n are eigen values of the equation,

$$\mu_n[a_3 - a_4] + \beta_0[a_5 - a_6] = 0. \tag{2.18}$$

Using equations (2.16) and (2.17), we get

$$M_0(t) = \frac{-\sum \frac{A_n}{J_1(\mu_n k)} e^{-\mu_n^2 t} \mu_n [a_4 - a_3]}{\sum \frac{A_n}{J_1(\mu_n k)} e^{-\mu_n^2 t}[a_4 - a_3]}, \tag{2.19}$$

where $a_1 = Y_0(\mu_n r)J_1(\mu_n k)$, $a_2 = Y_1(\mu_n k)J_0(\mu_n r)$, $a_3 = Y_1(\mu_n k)J_1(\mu_n)$, $a_4 = Y_1(\mu_n)J_1(\mu_n k)$, $a_5 = Y_0(\mu_n)J_1(\mu_n k)$, $a_6 = Y_1(\mu_n k)J_0(\mu_n)$.

At large time $t \to \infty$, following asymptotic values are obtained

$$M_0(\infty) = -\mu_0^2. \tag{2.20}$$

Solving for f_1 assuming large time, we have

$$\frac{\partial^2 f_1}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \mu_0 f_1 = w f_0 + M_1 f_0 \tag{2.21}$$

and

$$M_1 = -\frac{1}{(1-k)^2}\left\{\beta f_1(1) + \int_k^1 r w f_0 \, dr\right\}, \tag{2.22}$$

subjected to the conditions $\frac{\partial f_1}{\partial r}(r) = -\beta f_1(r)$ and $\frac{\partial f_1}{\partial r}(k) = 0$.

Multiplying equation (2.22) by $r E_0(M_0 r)$ and integrating from k to 1 with respect to r, we get

$$f_1(r) = \sum_{n=0}^{\infty} \frac{A_{1n} E_n(\mu_n r)}{J_1(\mu_n k)}, \tag{2.23}$$

$$M_1 = \frac{-4\mu_0[b_1 - b_2]\int_k^1 r w E_0(\mu_0 r) dr}{(1-k^2)[(\mu_0^2+\beta^2)\{E_0(\mu_0)\}^2 - k^2\mu_0^2\{E_0(\mu_0 k)\}^2]}, \tag{2.24}$$

where $b_1 = Y_1(\mu_0)J_1(\mu_0 k)$, $b_2 = J_1(\mu_0)Y_1(\mu_0 k)$, and

$$A_{1n} = \begin{cases} \frac{\int_k^1 [w(r)+\mu_0] r E_n(\mu_n r) f_0(r) dr}{J_1(\mu_n k)[\mu_n^2 - \mu_0^2]} & \text{for } n \geq 1 \\ \frac{-J_1(\mu_0 k)}{\int_k^1 r E_0(\mu_0 r) f_0(r) dr} \sum \frac{A_{1n}}{J_1(\mu_n k)} \int_k^1 r E_n(\mu_n r) dr & \text{for } n = 0. \end{cases} \tag{2.25}$$

Similarly, solving for M_2, we get

$$M_2 = \frac{1}{Pe^2} - \frac{\int_k^1 r(w + M_1) f_1 E_0(\mu_0 r) dr}{\int_k^1 r f_0 E_0(\mu_0 r) dr}. \tag{2.26}$$

3. Results and Discussions

Topical study deals with chemically active traces in the fluid flow through concentric annular region bounded by relative boundary. The effects of magnetic field, viscoelasticity on convection and dispersion coefficients are analysed. M_0 is assumed to be independent of velocity.

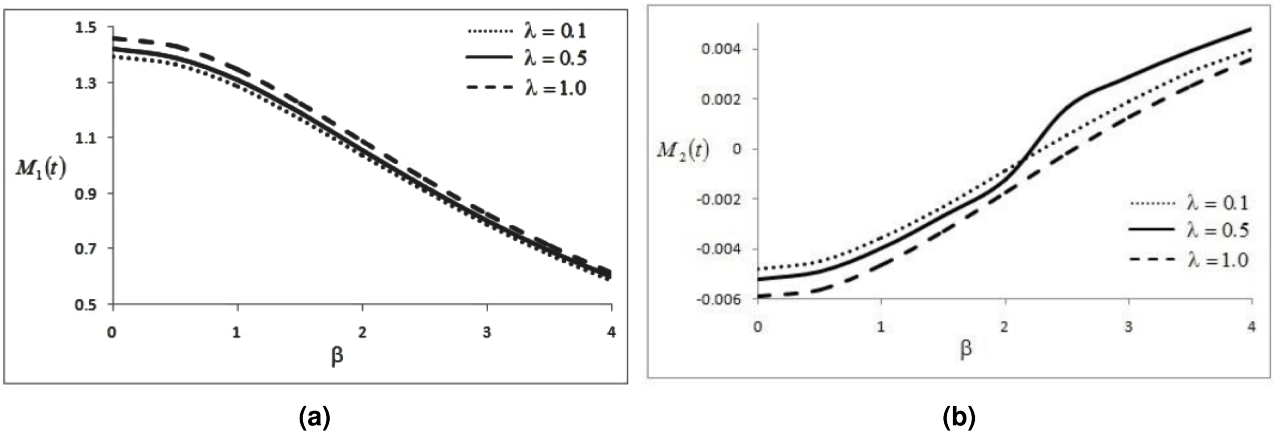


Figure 2. Plot of Exchange coefficient vs. Absorption parameter

Figure 2(a) gives the plot of convection coefficient against the absorption coefficient β for different values of relaxation parameter λ. The convection coefficient decreases with increasing absorption parameter β. As λ increases fluid looses the elasticity there-by increasing velocity results in more solute getting convected. $\lambda = 0.5$ shows both properties of viscosity and elasticity, hence initially the value is less and then shoots up faster.

The dispersion coefficient is plotted against β in Figure 2(b), which shows different pattern. Moderate relaxation parameter shows higher dispersion for large absorption parameter β, but when β is small, values of $M_1(t)$ for $\lambda = 0.5$ is in between $\lambda = 1.0$ and $\lambda = 0.1$ but later $\beta > 2$ shows high value compared to $\lambda = 1.0$ or $\lambda = 0.1$. As λ increases to 1.0 the dispersion coefficient decreases for all values of β showing effect of convection. Dispersion is less due to the fact that more solute gets convected to the wall and there is a loss of solute.

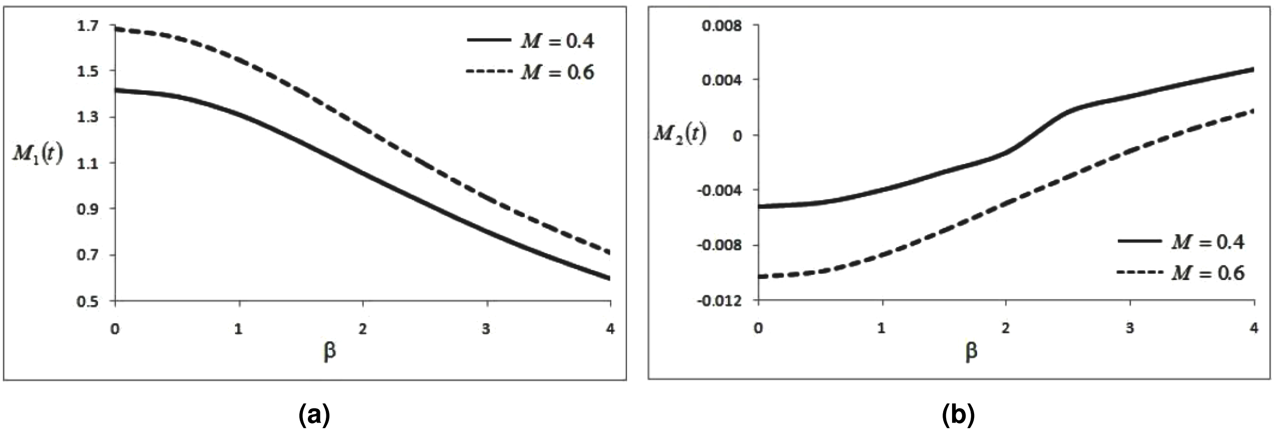
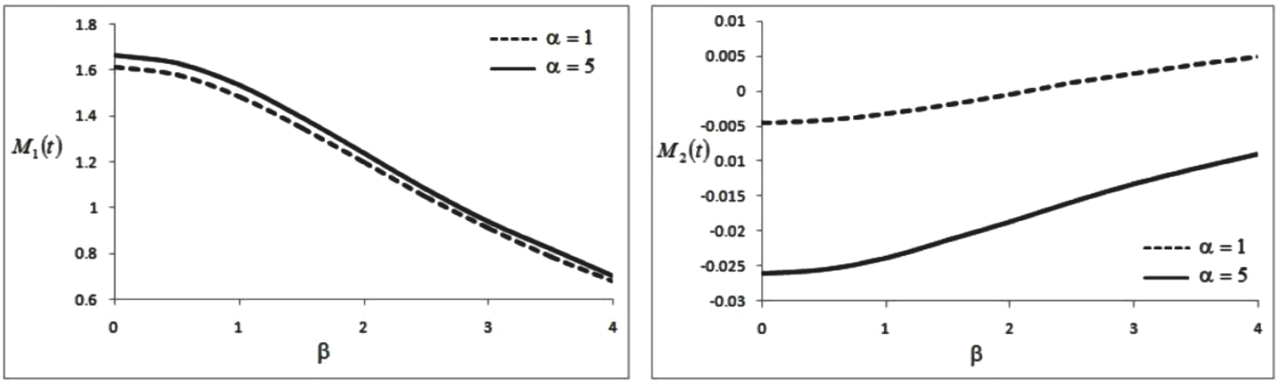


Figure 3. Plot of Exchange coefficient vs. Absorption parameter

Figures 3(a) and 3(b) shows the effect of magnetic field on convection and dispersion coefficients respectively against absorption parameter β. As M increases the convection increases and dispersion decreases. Magnetic field results in making velocity pulsatile which reduces convection but due to accumulation of solute, dispersion will be more.



(a) (b)

Figure 4. Plot of Exchange coefficient vs. Absorption parameter

Figures 4(a) and 4(b) depicts the effect of reaction parameter. The effect is to decrease convection and increase in dispersion as increase in α results in loss of solute. More solute gets convected to the outer boundary as α increases due to osmotic pressure.

4. Conclusions

Diffusion coefficient is not affected by velocity. Hence the study does not focus on this. Effect of viscoelasticity is more on dispersion coefficient than convection coefficient. Magnetic field affects both convection and dispersion coefficients but the effect is reverse. Reaction rate at the wall increases dispersion.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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