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Research Article

A Study of Psi-Function

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Abstract. The aim of this paper is to introduce a new generalization of the well-known, interesting and useful Fox H-function and I-function into generalized function, namely, the Psi-function. From which authors obtained I-function defined by Saxena [17] and Rathie [8]. Convergent conditions, elementary properties, and special cases have also been given.

Keywords. I-function; H-function; Mellin transform; Laplace transform; General class of polynomials; Struve's function

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1. Introduction

Recently, Saxena [17] has introduced an I-function, which can be useful to solve general type of dual integral equations [16]. Also, Rathie [8] has given generalize H-function which is useful in testing hypothesis from statistics as special cases [1]. In the present paper, a new Psi-function is introduce, namely Ψ-function from which both I-function and generalized H-function can obtained as special cases. We shall utilize the following formulae in the present investigation. The I-function of one variable given by Saxena [17].

$$I^{m,n}_{p_i,q_i;r}\left[z \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix}\right] = \frac{1}{2\pi i}\int_L \phi(s) z^s \, ds, \tag{1.1}$$

where

$$\phi(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right]}.$$

The detailed conditions given in [17]. The I-function of one variable given by Rathie [8]

$$I_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L\phi(s)z^s\,ds, \tag{1.2}$$

where

$$\phi(s)=\frac{\prod_{j=1}^{m}\Gamma^{B_j}(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma^{B_j}(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma^{A_j}(a_j-\alpha_j s)}.$$

The detailed conditions given in [8]. According to Luke [6]

$$\lim_{|y|\to\infty}|\Gamma(x+iy)|=\lim_{|y|\to\infty}\sqrt{2\pi}e^{-\frac{\pi}{2}|y|}|y|^{x-\frac{1}{2}}. \tag{1.3}$$

According to Eredelyi [2, p. 370]

$$\int_0^\infty x^{s-1}\left[\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}g(s)x^{-s}ds\right]dx=g(s). \tag{1.4}$$

The Mellin transform of the function $f(x)$ is defined as Satyanaraya *et al*. [11], [13]

$$M\{f(x);s\}=\int_0^\infty x^{s-1}f(x)\,dx,\quad Re(s)>0. \tag{1.5}$$

If Laplace transform of $f(t)$ is $F(p)$ and $G(s)$ is Mellin transform, then [11], [13].

$$L\{f(t);s\}=F(p)=\sum_{s=0}^{\infty}\frac{(-p)^s}{s!}G(s+1). \tag{1.6}$$

General class of polynomials Kumar *et al*. [9], [15].

$$S_n^m[x]=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k,\quad n=0,1,2,\dots, \tag{1.7}$$

where m is an arbitrary positive integer and the coefficients $A_{n,k}$ $(n,k\ge 0)$ are arbitrary constants. Struve's function defined as Satyanarayana *et al*. [9], [10], [11].

$$H_{v,y,u}^{\lambda,k}[z]=\sum_{m=0}^{\infty}\frac{(-1)^m(z/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)},\quad Re(k)>0,Re(\lambda)>0,Re(y)>0,Re(v+u)>0. \tag{1.8}$$

From table of integrals we have [4, p. 314, eq. (3)]

$$\int_{-1}^{1}(1-x)^p(1+x)^q\,dx=2^{p+q+1}B(p+1,q+1),\quad Re(p+1)>0,Re(q+1)>0, \tag{1.9}$$

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}\Gamma(p+\frac{1}{2})}{2a(4ab+c)^{p+\frac{1}{2}}\Gamma(p+1)},\quad Re(p)+1/2>0. \tag{1.10}$$

2. The Psi-Function Ψ and Existence Conditions

In this section, the authors introduce a new Psi-function as,

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L\phi(s)z^s\,ds, \tag{2.1}$$

where

$$\phi(s)=\frac{\prod\limits_{j=1}^{m}\Gamma^{B_j}(b_j-\beta_j s)\prod\limits_{j=1}^{n}\Gamma^{A_j}(1-a_j+\alpha_j s)}{\sum\limits_{i=1}^{r}\left[\prod\limits_{j=m+1}^{q_i}\Gamma^{B_{ji}}(1-b_{ji}+\beta_{ji}s)\prod\limits_{j=n+1}^{p_i}\Gamma^{A_{ji}}(a_{ji}-\alpha_{ji}s)\right]}$$

(i) $z\neq 0$.

(ii) $i=\sqrt{-1}$.

(iii) p_i $(i=1,\dots,r)$, q_i $(i=1,\dots,r)$

$0\le n\le p_i$, $0\le m\le q_i$, r is finite.

(iv) α_j, β_j, α_{ji} and β_{ji} are positive integers and A_j, B_j, A_{ji} and B_{ji} are non-negative integers.

(v) a_j, b_j, a_{ji} and b_{ji} are complex numbers such that no singularities of $\Gamma^{B_j}(b_j-\beta_j s)$, $j=1,\dots,m$ coincides with any singularities of $\Gamma^{A_j}(1-a_j+\alpha_j s)$, $j=1,\dots,n$.

(vi) L is contour running from $\sigma-i\infty$ to $\sigma+i\infty$ in complex s-plane so that all the singularities of $\Gamma^{B_j}(b_j-\beta_j s)$, $j=1,\dots,m$ lie to the right of L, and all the singularities of $\Gamma^{A_j}(1-a_j+\alpha_j s)$, $j=1,\dots,n$ lie to the left of L.

Convergent conditions

The integrand for L, defined by (2.1) converges when

$$|\arg z|<\Delta\frac{\pi}{2},\quad \text{if }\ \Delta>0, \tag{2.2}$$

where

$$\Delta=\sum_{j=1}^{m}\beta_jB_j+\sum_{j=1}^{n}\alpha_jA_j-\max_{1\le i\le r}\left\{\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}+\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right\}.$$

If $|\arg z|=\Delta\frac{\pi}{2}$ if $\Delta\ge 0$ the integrand converges absolutely when

(i) $\mu=0$ or $\sigma=0$ if $\nabla>1$, where

$$\nabla=\sum_{j=1}^{n}\left(Re(a_j)-\frac{1}{2}\right)A_j-\sum_{j=1}^{m}\left(Re(b_j)-\frac{1}{2}\right)B_j$$
$$+\min_{1\le i\le r}\left\{\sum_{j=n+1}^{p_i}\left(Re(a_{ji})-\frac{1}{2}\right)A_{ji}-\sum_{j=m+1}^{q_i}\left(Re(b_{ji})-\frac{1}{2}\right)B_{ji}\right\} \tag{2.3}$$

and

$$\mu=\sigma\left[\sum_{j=1}^{m}\beta_jB_j-\sum_{j=1}^{n}\alpha_jA_j\right]+\min_{1\le i\le r}\left\{\sigma\left[\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}-\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right]\right\}. \tag{2.4}$$

(ii) $\mu\neq 0$, σ is chosen, if $(\nabla+\mu)>1$ with $s=\sigma+it$; σ, t are real.

Proof. From Luke [6]

$$\lim_{|y|\to\infty} |\Gamma(x+iy)| = \lim_{|y|\to\infty} \sqrt{2\pi} e^{-\frac{\pi}{2}|y|} |y|^{x-\frac{1}{2}}$$

and take $s=\sigma+it$, $t\to\infty$, $z=Re^{i\theta}$ in (2.1), we get

$$\lim_{|t|\to\infty} \Psi^{m,n}_{p_i,q_i;r}[|z|] = \lim_{|t|\to\infty} C \frac{e^{-\frac{\pi}{2}\left(\sum_{j=1}^{m}\beta_j B_j+\sum_{j=1}^{n}\alpha_j A_j\right)|t|}}{\sum_{i=1}^{r}\left[e^{-\frac{\pi}{2}\left(\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}+\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right)|t|}\right]} \cdot \frac{|t|^{\left\{\sum_{j=1}^{m}(b_j-\sigma\beta_j-\frac{1}{2})B_j+\sum_{j=1}^{n}(-a_j+\sigma\alpha_j+\frac{1}{2})A_j\right\}} e^{|-\theta t|}}{|t|^{\left\{\sum_{j=m+1}^{q_i}(-b_{ji}+\sigma\beta_{ji}+\frac{1}{2})B_{ji}+\sum_{j=n+1}^{p_i}(a_{ji}-\sigma\alpha_{ji}-\frac{1}{2})A_{ji}\right\}}}, \tag{2.5}$$

where C is independent of t.

Let $A=\max\left\{\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}+\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right\}$ for all $i=1,\dots,r$.

$$B=\min\left\{\sum_{j=m+1}^{q_i}\left(-b_{ji}+\sigma\beta_{ji}+\tfrac{1}{2}\right)B_{ji}+\sum_{j=n+1}^{p_i}\left(a_{ji}-\sigma\alpha_{ji}-\tfrac{1}{2}\right)A_{ji}\right\} \quad \text{for all } i=1,\dots,r.$$

Then, from (2.5), we have

$$\lim_{|t|\to\infty} \Psi^{m,n}_{p_i,q_i;r}[|z|] = \lim_{|t|\to\infty} C \frac{e^{-\frac{\pi}{2}\left(\sum_{j=1}^{m}\beta_j B_j+\sum_{j=1}^{n}\alpha_j A_j-A\right)|t|}}{\sum_{i=1}^{r}\left[e^{-\frac{\pi}{2}\left(\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}+\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}-A\right)|t|}\right]} \cdot \frac{|t|^{\left\{\sum_{j=1}^{m}(b_j-\sigma\beta_j-\frac{1}{2})B_j+\sum_{j=1}^{n}(-a_j+\sigma\alpha_j+\frac{1}{2})A_j-B\right\}} e^{|-\theta t|}}{|t|^{\left\{\sum_{j=m+1}^{q_i}(-b_{ji}+\sigma\beta_{ji}+\frac{1}{2})B_{ji}+\sum_{j=n+1}^{p_i}(a_{ji}-\sigma\alpha_{ji}-\frac{1}{2})A_{ji}-B\right\}}}. \tag{2.6}$$

Hence, $|\phi(s)z^s| = Ce^{-(\frac{\pi}{2}\Delta-\theta)|t|}|t|^{-\nabla-\mu}$.

As $t\to\infty$ the convergent conditions as follows. Where Δ, ∇ and μ defined in eq. (2.2), (2.3) and (2.4), respectively

Subcase: If σ is positive real and $\left(\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}-\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right)>0$ then

$$|\phi(s)z^s| = Ce^{-(\frac{\pi}{2}\Delta-\theta)|t|}|t|^{-\nabla-\sigma\mu},$$

where Δ, ∇ is defined as in equation (2.2), (2.3), respectively and

$$\mu=\left[\sum_{j=1}^{m}\beta_j B_j-\sum_{j=1}^{n}\alpha_j A_j\right]+\min_{1\le i\le r}\left\{\left[\sum_{j=m+1}^{q_i}\beta_{ji}B_{ji}-\sum_{j=n+1}^{p_i}\alpha_{ji}A_{ji}\right]\right\}. \tag{2.7}$$

Then the integrand in L.H.S of (2.1) is convergent.

If $|\arg z|=\Delta\frac{\pi}{2}$, if $\Delta\ge 0$ the integrand converges absolutely when

(i) $\mu=0$ if $\nabla>1$,

(ii) $\mu\ne 0$, σ is chosen, if $(\nabla+\mu)>1$,

with $s=\sigma+it$; σ, t are real.

3. Some Simple Properties

(i) While defining Psi-function Ψ in (2.1), α_j, β_j, α_{ji}, β_{ji}, A_j, B_j, A_{ji} and B_{ji} are positive integers. However, one can see that this function has meaning even if some quantities are zero.

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a,0,0),(a_j,\alpha_j;A_j)_{2,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\Gamma(1-a)\Psi^{m,n-1}_{p_i-1,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{2,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\tag{3.1}$$

Provided $Re(1-a)>0$ and $n\geq 1$

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i-1},(a,0,0)\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\frac{1}{\Gamma(a)}\Psi^{m,n}_{p_i-1,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i-1}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\tag{3.2}$$

Provided $Re(a)>0$ and $p_i>n$ for $i=1,\dots,r$

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b,0,0),(b_j,\beta_j;B_j)_{2,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\Gamma(b)\Psi^{m-1,n}_{p_i,q_i-1;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{2,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\tag{3.3}$$

Provided $Re(b)>0$ and $m>1$

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{2,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i-1},(b,0,0)\end{matrix}\right.\right]$$
$$=\frac{1}{\Gamma(1-b)}\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{2,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i-1}\end{matrix}\right.\right]\tag{3.4}$$

(ii) Provided $Re(1-b)>0$ and $q_i>m$ for $i=1,\dots,r$

$$\Psi^{m,n}_{p_i,q_i;r}\left[z^k\left|\begin{matrix}(a_j,k\alpha_j;A_j)_{1,n};(a_{ji},k\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,k\beta_j;B_j)_{1,m};(b_{ji},k\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\frac{1}{k}\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\tag{3.5}$$

(iii) where $k>0$

$$z^{\sigma}\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j+\sigma\alpha_j,\alpha_j;A_j)_{1,n};(a_{ji}+\sigma\alpha_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j+\sigma\beta_j,\beta_j;B_j)_{1,m};(b_{ji}+\sigma\beta_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\tag{3.6}$$

(iv) where σ is complex number

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$
$$=\Psi^{n,m}_{q_i,p_i;r}\left[z^{-1}\left|\begin{matrix}(1-b_j,\beta_j;B_j)_{1,m};(1-b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\\(1-a_j,\alpha_j;A_j)_{1,n};(1-a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\end{matrix}\right.\right]\tag{3.7}$$

4. Special Cases

(i) In equation (2.1), take $A_j = A_{ji} = B_j = B_{ji} = 1$, then the equation reduced to I-function defined by Saxena [17]

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j;1)_{1,n};(a_{ji},\alpha_{ji};1)_{n+1,p_i}\\(b_j,\beta_j;1)_{1,m};(b_{ji},\beta_{ji};1)_{m+1,q_i}\end{matrix}\right] = I^{m,n}_{p_i,q_i;r}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right]. \tag{4.1}$$

(ii) In the equation (2.1), take $r = 1$, $\alpha_{ji} = \alpha_j$, $\beta_{ji} = \beta_j$, $A_{ji} = A_j$, $B_{ji} = B_j$ then the equation will reduce to I-function defined by Arjun Rathie K [8].

$$\Psi^{m,n}_{p_i,q_i;1}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right] = I^{m,n}_{p_i,q_i;r}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix}\right]. \tag{4.2}$$

(iii) In (4.1), put $r = 1$, $\alpha_{ji} = \alpha_j$, $\beta_{ji} = \beta_j$, we get Fox H-function [3]

$$\Psi^{m,n}_{p_i,q_i;1}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j;1)_{1,n};(a_{ji},\alpha_{ji};1)_{n+1,p_i}\\(b_j,\beta_j;1)_{1,m};(b_{ji},\beta_{ji};1)_{m+1,q_i}\end{matrix}\right] = H^{m,n}_{p,q}\left[z\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right]. \tag{4.3}$$

(iv) In (4.1), substitute $\alpha_{ji} = \alpha_j = \beta_{ji} = \beta_j = 1$, then we have a function defined as I_G [17]

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\,\middle|\,\begin{matrix}(a_j,1;1)_{1,n};(a_{ji},1;1)_{n+1,p_i}\\(b_j,1;1)_{1,m};(b_{ji},1;1)_{m+1,q_i}\end{matrix}\right] = I^{m,n}_{p_i,q_i;r}\left[z\,\middle|\,\begin{matrix}(a_j,1)_{1,n};(a_{ji},1)_{n+1,p_i}\\(b_j,1)_{1,m};(b_{ji},1)_{m+1,q_i}\end{matrix}\right]. \tag{4.4}$$

(v) In (4.3), put $\alpha_j = \beta_j = 1$, then we will get Maijer's G-function [3]

$$\Psi^{m,n}_{p_i,q_i;1}\left[z\,\middle|\,\begin{matrix}(a_j,1;1)_{1,n};(a_{ji},1;1)_{n+1,p_i}\\(b_j,1;1)_{1,m};(b_{ji},1;1)_{m+1,q_i}\end{matrix}\right] = G^{m,n}_{p,q}\left[z\,\middle|\,\begin{matrix}(a_j)_{1,p}\\(b_j)_{1,q}\end{matrix}\right]. \tag{4.5}$$

5. Some Simple Differential Formulas

Notations:

(i) $D_x = \frac{d}{dx}$

(ii) $D_x^r[f(x)] = \frac{d^r}{dx^r}[f(x)]$

(iii) $(xD_x)^r[f(x)] = \left(x\frac{d}{dx}\right)^r[f(x)]$

(iv) $(D_x x)^r[f(x)] = \left(\frac{d}{dx}x\right)^r[f(x)]$

Formula 1.

$$D_x^l\{\Psi[z\,x^\sigma]\} = x^{-l}\,\Psi^{m,n}_{p_i,q_i;r}\left[zx^\sigma\,\middle|\,\begin{matrix}(0,\sigma;1),(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(l,\sigma;1)\end{matrix}\right] \tag{5.1}$$

where $\sigma > 0$.

Proof. Use the notation of derivative and evaluate. Then integrand becomes

$$\frac{1}{2\pi i}\int_L \phi(s)\prod_{j=0}^{l-1}(\sigma s - j)x^{\sigma s - l}z^s\,ds$$

and using $\prod\limits_{j=0}^{l-1}(\sigma s - j) = \frac{\Gamma((1+\sigma s)}{\Gamma(1+\sigma s - l)}$ to get the required result. □

Formula 2.

$$(xD_x - k_1)(xD_x - k_2)\dots(xD_x - k_l)\{\Psi[zx^\sigma]\}$$
$$= \Psi^{m,n+l}_{p_i+l,q_i+l;r}\left[zx^\sigma \left| \begin{matrix} (k_l,\sigma;1)_{1,l},(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i} \\ (b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(1+k_l,\sigma;1)_{1,l} \end{matrix} \right.\right] \tag{5.2}$$

Proof. Use differentiation notation and evaluate. Then integrand becomes

$$\frac{1}{2\pi i}\int_L \phi(s)\prod_{j=1}^{l}(\sigma s - k_l)x^{\sigma s}z^s\,ds,$$

then by using $\prod_{j=1}^{l}(\sigma s - k_l) = \frac{\Gamma((\sigma s - k_l+1)}{\Gamma(\sigma s - k_l)}$ to get the formula. □

Formula 3.

$$(D_x x - k_1)(D_x x - k_2)\dots(D_x x - k_l)\{\Psi[zx^\sigma]\}$$
$$= \Psi^{m,n+l}_{p_i+l,q_i+l;r}\left[zx^\sigma \left| \begin{matrix} (k_l-1,\sigma;1)_{1,l},(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i} \\ (b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(k_l,\sigma;1)_{1,l} \end{matrix} \right.\right] \tag{5.3}$$

Proof. Proof of (5.3) is similar as that of (5.2). □

Formula 4.

$$D_x^l\{\Psi[(cx+d)^\sigma]\}$$
$$= \left(\frac{c}{cx+d}\right)^l \Psi^{m,n+1}_{p_i+1,q_i+1;r}\left[(cx+d)^\sigma \left| \begin{matrix} (0,\sigma;1),(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i} \\ (b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(l,\sigma;1) \end{matrix} \right.\right] \tag{5.4}$$

where c, d are complex numbers and σ is positive real.

Proof. Proof is same as that of (5.1). □

6. Mellin and Laplace Transform

(i) In this section, authors apply Mellin and Laplace transforms to Psi-function (2.1)

(a) Mellin Transform

$$\int_0^\infty x^{s-1}\Psi^{m,n}_{p_i,q_i;r}\left[ax^\sigma \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i} \\ (b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i} \end{matrix} \right.\right]$$
$$= \frac{a^{-s/\sigma}}{\sigma}\frac{\prod_{j=1}^{m}\Gamma^{B_j}(b_j+\beta_j(s/\sigma))\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j-\alpha_j(s/\sigma))}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}(1-b_{ji}-\beta_{ji}(s/\sigma))\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}(a_{ji}+\alpha_{ji}(s/\sigma))\right]} \tag{6.1}$$

provided

(a1) $\sigma > 0$
(a2) $Re(b_j + \beta_j(s/\sigma)) > 0$
(a3) $Re(1 - a_j - \alpha_j(s/\sigma)) > 0$
(a4) $\Delta > 0$, $|\arg a| < \frac{\pi}{2}\Delta$
(a5) $\Delta \ge 0$, $|\arg a| \ge \frac{\pi}{2}\Delta$, $(\nabla + \mu) > 1$

where Δ, ∇ and μ are defined in (2.2), (2.3) and (2.4), respectively.

(b) Laplace Transform

$$L\left[\Psi[ax^{\sigma}];s\right]=\sum_{s=0}^{\infty}\frac{(-p)^s}{s!}\frac{a^{-(s+1)/\sigma}}{\sigma} \cdot\frac{\prod_{j=1}^{m}\Gamma^{B_j}\left(b_j+\beta_j\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j-\alpha_j\left(\frac{s+1}{\sigma}\right)\right)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}\left(1-b_{ji}-\beta_{ji}\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}\left(a_{ji}+\alpha_{ji}\left(\frac{s+1}{\sigma}\right)\right)\right]} \tag{6.2}$$

Proof of (a) and (b) can easily obtain from equations (1.4) and (1.6), respectively.

(ii) Mellin and Laplace transform of product of general class of polynomials, Struve's function and Psi-function of one variable.

(a) Mellin Transform

$$\int_0^{\infty}x^{s-1S_p^q[ax^h]H^{\lambda,\mu}_{v,y,\delta}[bh^g}]\Psi^{m,n}_{p_i,q_i;r}\left[cx^{\sigma}\right]dx$$

$$=\frac{1}{\sigma}\sum_{k=0}^{[p/q]}\frac{(-p)_{qk}}{k!}A_{p,k}a^k\sum_{t=0}^{\infty}\frac{(-1)^t\left[\frac{b}{2}\right]^{v+2t+1}}{\Gamma(\mu t+y)\Gamma(v+\lambda t+\delta)}c^{-\frac{A}{\sigma}} \cdot\frac{\prod_{j=1}^{m}\Gamma^{B_j}(b_j+\beta_j(A/\sigma))\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j-\alpha_j(A/\sigma))}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}(1-b_{ji}-\beta_{ji}(A/\sigma))\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}(a_{ji}+\alpha_{ji}(A/\sigma))\right]} \tag{6.3}$$

where $A=(s+hk+g(v+2t+1))/\sigma$, provided

(a1) $\sigma>0$; g, h are complex numbers;
$p=0,1,2,\dots$; $q=1,2,\dots$.

(a2) $Re(b_j+\beta_j(A/\sigma))>0$

(a3) $Re(1-a_j-\alpha_j(A/\sigma))>0$

(a4) $\Delta>0$, $|\arg c|<\frac{\pi}{2}\Delta$

(a5) $\Delta\geq 0$, $|\arg c|\geq\frac{\pi}{2}\Delta$, $(\nabla+\mu)>1$

where Δ, ∇ and μ are defined in (2.2), (2.3) and (2.4), respectively.

Proof. Replace general class of polynomials, Struve's function and Ψ-function of one variable. Using (1.7), (1.8) and (2.1) in (6.3) and apply (1.4), we get required result. □

Special cases:

(i) Take $g=0$, $t=0$, $b=2$, $v=1-\delta$ in (6.3), we obtain

$$\int_0^{\infty}x^{s-1}S_p^q[ax^h]\Psi^{m,n}_{p_i,q_i;r}\left[cx^{\sigma}\right]dx$$

$$=\frac{1}{\sigma}\sum_{k=0}^{[p/q]}\frac{(-p)_{qk}}{k!}A_{n,k}a^kc^{-\frac{(s+hk)}{\sigma}}$$

$$\frac{\prod_{j=1}^{m}\Gamma^{B_j}(b_j+\beta_j(s+hk/\sigma))\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j-\alpha_j(s+hk/\sigma))}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}(1-b_{ji}-\beta_{ji}(s+hk/\sigma))\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}(a_{ji}+\alpha_{ji}(s+hk/\sigma))\right]}$$

provided conditions given in (6.3).

(ii) Apply $h=0$, $a=1,k=0$, we have

$$\int_0^\infty x^{s-1}H_{v,y,\delta}^{\lambda,\mu}[bh^g]\Psi_{p_i,q_i;r}^{m,n}\left[cx^\sigma\right]dx$$

$$=\frac{1}{\sigma}\sum_{t=0}^{\infty}\frac{(-1)^t\left[\frac{b}{2}\right]^{v+2t+1}}{\Gamma(\mu t+y)\Gamma(v+\lambda t+\delta)}c^{-\frac{A}{\sigma}}$$

$$\cdot\frac{\prod_{j=1}^{m}\Gamma^{B_j}\left(b_j+\beta_j\left(\frac{s+g(v+2t+1)}{\sigma}\right)\right)\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j-\alpha_j\left(\frac{s+g(v+2t+1)}{\sigma}\right)\right)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}\left(1-b_{ji}-\beta_{ji}\left(\frac{s+g(v+2t+1)}{\sigma}\right)\right)\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}\left(a_{ji}+\alpha_{ji}\left(\frac{s+g(v+2t+1)}{\sigma}\right)\right)\right]}$$

provided conditions given in (6.3).

(iii) Take $A_j=A_{ji}=B_j=B_{ji}=1$ in (6.3), we get Mellin transform containing product of general class of polynomials, Struve's function and I-function of one variable given by Saxena [17].

(iv) Assign $r=1$, $\alpha_{ji}=\alpha_j$, $\beta_{ji}=\beta_j$, $A_{ji}=A_j$, $B_{ji}=B_j$ in (6.3), we have Mellin transform containing product of general class of polynomials, Struve's function and I-function of one variable given by Arjun Rathie [8].

Note. By using (1.6) one can easily get Laplace transform of product of general class of polynomials, Struve's function and Psi-function Ψ of one variable with all above special cases.

7. Some Integrals Containing General Class of Polynomials and Struve's Function

Theorem 7.1. *Prove that*

$$\int_{-1}^{1}(1-x)^p(1+x)^q S_e^f\left[a(1-x)^u(1+x)^v\right]H_{l,y,\delta}^{\lambda,\mu}\left[b(1-x)^g(1+x)^h\right]\Psi\left[z(1-x)^\rho(1+x)^\sigma\right]dx$$

$$=2^{p+q+1}\sum_{k=0}^{[e/f]}\frac{(-e)_{fk}}{k!}A_{e,k}\left(a\,2^{u+v}\right)^k\sum_{t=0}^{\infty}(-1)^t\left(b\,2^{g+h-1}\right)^{l+2t+1}$$

$$\times\Psi_{p_i+2,q_i+3:r}^{m,n+2}\left[z\,2^{\rho+\sigma}\left|\begin{array}{c}(-A,\rho:1),(-B,\sigma:1),(a_j,\alpha_j;A_j)_{1,n};(a_{ij},\alpha_{ij};A_{ij})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ij},\beta_{ij};B_{ij})_{m+1,q_i},(-(A+B+1),\rho+\sigma;1),\\(1-(\mu t+y),0;1),(1-(l+\lambda t+\delta),0;1)\end{array}\right.\right] \quad (7.1)$$

where $A=p+uk+g(l+2t+1)$ *and* $B=q+vk+h(l+2t+1)$, *provided*

(i) $Re(p+1)>0$, $Re(q+1)>0$, u,v,g,h *are complex numbers,*

(ii) $\rho>0$, $\sigma>0$, $Re(\mu)>0$, $Re(l+\delta)>0$, $Re(\lambda)>0$,

(iii) $A_{e,k}$ *are arbitrary constants;* $e = 0,1,2,\ldots$; $f = 1,2,\ldots$,

(iv) $\Delta > 0$, $|\arg z| < \frac{\pi}{2}\Delta$,

(v) $\Delta \geq 0$, $|\arg z| \geq \frac{\pi}{2}\Delta$, $(\nabla + \mu) > 1$, (7.2)

where Δ, ∇ *and* μ *are defined in* (2.2), (2.3) *and* (2.4), *respectively.*

Proof. Using (1.7), (1.8) and (2.1) in L.H.S of (2.1), we get

$$\sum_{k=0}^{[e/f]} \frac{(-e)_{fk}}{k!} A_{e,k} a^k \sum_{t=0}^{\infty} \frac{(-1)^t (b/2)^{l+2t+1}}{\Gamma(\mu t + y)\Gamma(l + \lambda t + \delta)} \int_{-1}^{1} (1-x)^{p+uk+g(l+2t+1)} (1+x)^{q+vk+h(l+2t+1)} dx \cdot \left[\frac{1}{2\pi i} \int_L \phi(s) z^s (1-x)^{\rho s} (1+x)^{\sigma s} ds \right].$$

Interchange the order of integration and using (1.9) to obtain required result.

Special cases:

(i) Put $p = q = g = h = 0$, $t = 0$, $b = 2$, $y = 1$, $\delta = 1 - l$, author get the following result

$$\int_{-1}^{1} S_e^f \left[a(1-x)^u (1+x)^v \right] \Psi \left[z(1-x)^\rho (1+x)^\sigma \right] dx$$
$$= 2 \sum_{k=0}^{[e/f]} \frac{(-e)_{fk}}{k!} A_{e,k} \left(a 2^{u+v} \right)^k \times \Psi^{m,n+2}_{p_i+2,q_i+1:r} \left[z 2^{\rho+\sigma} \left| \begin{matrix} (-uk, \rho : 1), (-vk, \sigma : 1), (a_j, \alpha_j; A_j)_{1,n}; (a_{ij}, \alpha_{ij}; A_{ij})_{n+1,p_i} \\ (b_j, \beta_j; B_j)_{1,m}; (b_{ij}, \beta_{ij}; B_{ij})_{m+1,q_i}, (-(u+v)k + 1, \rho + \sigma; 1) \end{matrix} \right. \right]$$

(ii) Take $a = 1$, $p = q = u = v = 0$ and $k = 0$ in (7.1), have the result

$$\int_{-1}^{1} H^{\lambda,\mu}_{l,y,\delta} \left[b(1-x)^g (1+x)^h \right] \Psi \left[z(1-x)^\rho (1+x)^\sigma \right] dx$$
$$= 2 \sum_{t=0}^{\infty} (-1)^t \left(b 2^{g+h-1} \right)^{l+2t+1} \times \Psi^{m,n+2}_{p_i+2,q_i+3:r} \left[z 2^{\rho+\sigma} \left| \begin{matrix} (-g(l+2t+1), \rho : 1), (-h(l+2t+1), \sigma : 1), (a_j, \alpha_j; A_j)_{1,n}; \\ (b_j, \beta_j; B_j)_{1,m}; (b_{ij}, \beta_{ij}; B_{ij})_{m+1,q_i}, (-(g+h)(l+2t+1)+1), \rho + \sigma; 1), \end{matrix} \right. \begin{matrix} (a_{ij}, \alpha_{ij}; A_{ij})_{n+1,p_i} \\ (1-(\mu t + y), 0; 1), (1-(l+\lambda t+\delta), 0; 1) \end{matrix} \right]$$

(iii) Substitute $A_j = A_{ji} = B_j = B_{ji} = 1$ in (7.1), get integral containing product of general class of polynomials, Struve's function and I-function of one variable by Saxena [17].

(iv) Assign $r = 1$, $\alpha_{ji} = \alpha_j$, $\beta_{ji} = \beta_j$, $A_{ji} = A_j$, $B_{ji} = B_j$in (7.1), we have integral containing product of general class of polynomials, Struve's function and I-function of one variable by Rathie [8]. □

Theorem 7.2. *Prove that*

$$\int_0^\infty \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} S_p^q \left[d \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-u} \right] H^{\lambda,\mu}_{l,y,\delta} \left[e \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-v} \right]$$

$$\times\,\Psi\left[z\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\sigma}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+\frac{1}{2}}}\sum_{k=0}^{[p/q]}\frac{(-p)_{mk}}{k!}A_{p,k}\left[d(4ab+c)^{-u}\right]^k\sum_{t=0}^{\infty}(-1)^t\left[\frac{e}{2}(4ab+c)^{-v}\right]^{(l+2t+1)}$$

$$\times\,\Psi^{m,n+1}_{p_i+1,q_i+3:r}\left[z(4ab+c)^{-\sigma}\left|\begin{array}{c}(\frac{1}{2}-A,\sigma:1),(a_j,\alpha_j;A_j)_{1,n};(a_{ij},\alpha_j;A_j)_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ij},\beta_{ij};B_{ij})m+1,q_i,(-A,\sigma;1),\\(1-\mu t+y,0;1),(1-(l+\lambda t+\delta),0;1)\end{array}\right.\right] \tag{7.3}$$

where $A=\eta+uk+v(l+2t+1)$ *and provided*

(i) u, v *are complex numbers*

(ii) $Re(\eta)+\frac{1}{2}>0$, $\sigma>0$, $Re(\mu)>0$, $Re(l+\delta)>0$, $Re(\lambda)>0$

(iii) $A_{p,k}$ *are arbitrary constants;* $p=0,1,2,\dots$; $q=1,2,\dots$

(iv) $\Delta>0$, $|\arg a|<\frac{\pi}{2}\Delta$

(v) $\Delta\geq 0$, $|\arg a|\geq\frac{\pi}{2}\Delta$, $(\nabla+\mu)>1$,

where Δ, ∇ *and* μ *are defined in* (2.2), (2.3) *and* (2.4), *respectively.*

Proof. Using (1.7), (1.8) and (2.1) respectively in (7.3) and interchange the order of integration and apply (1.10), we will get the desired expression. □

Special cases:

(i) Take $v=0$, $t=0$, $e=2$, $y=1$, $\delta=1-1$ in (7.2), then we get integral contains product of class of polynomials and I-function of one variable

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\eta-1}S_p^q\left[d\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u}\right]\Psi\left[z\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\sigma}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+\frac{1}{2}}}\sum_{k=0}^{[p/q]}\frac{(-p)_{qk}}{k!}A_{p,k}\left[d(4ab+c)^{-u}\right]^k$$

$$\times\,\Psi^{m,n+1}_{p_i+1,q_i+1:r}\left[z(4ab+c)^{-\sigma}\left|\begin{array}{c}(\frac{1}{2}-(\eta+uk),\sigma:1),(a_j,\alpha_j;A_j)_{1,n};(a_{ij},\alpha_j;A_j)_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ij},\beta_{ij};B_{ij})m+1,q_i,(-(\eta+uk),\sigma;1)\end{array}\right.\right] \tag{7.4}$$

(ii) Put $d=1$, $u=0$, $k=0$, we get integral

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\eta-1}H^{\lambda,\mu}_{l,y,\delta}\left[e\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-v}\right]\Psi\left[z\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\sigma}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+\frac{1}{2}}}\sum_{t=0}^{\infty}(-1)^t\left[\frac{e}{2}(4ab+c)^{-v}\right]^{(l+2t+1)}$$

$$\times\,\Psi^{m,n+1}_{p_i+1,q_i+3:r}\left[z(4ab+c)^{-\sigma}\left|\begin{array}{c}(\frac{1}{2}-(\eta+v(l+2t+1),\sigma:1),(a_j,\alpha_j;A_j)_{1,n};(a_{ij},\alpha_j;A_j)_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ij},\beta_{ij};B_{ij})m+1,q_i,(-(\eta+v(l+2t+1),\sigma;1),\\(1-\mu t+y,0;1),(1-(l+\lambda t+\delta),0;1)\end{array}\right.\right] \tag{7.5}$$

(iii) Put $A_j = A_{ji} = B_j = B_{ji} = 1$ in (7.2), we get the result Saxena [17].

(iv) Assign $r = 1$, $\alpha_{ji} = \alpha_j$, $\beta_{ji} = \beta_j$, $A_{ji} = A_j$, $B_{ji} = B_j$ in (7.2), we have Arjun Rathie [8]. □

8. Conclusion

By nature of Psi-function one can say, it is generalization of I-function [17] and H-function [8]. So this function can generate many interesting results in Physics, Statistics and Applied Sciences. Also, one can extend to two or more variables.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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