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Research Article

Incorporating Random Contractions in Decision Support Systems for Facilitating Synergies of Proactivity and Extremity

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Abstract. The paper formulates a stochastic model by introducing the present value of a continuous uniform cash flow, with rate of payment and duration being random and force of interest being constant, in the maximum of a random number of positive random variables. The paper also provides an interpretation and practical applications of the stochastic model. Moreover, the paper establishes sufficient conditions for the representation of the stochastic model as the maximum of a random number of random contractions. In addition, the paper makes clear that the structural elements, the principal components, and the mathematical form of the stochastic model facilitate the development of synergies between the concepts of extremity, proactivity, and information for supporting intelligent systems.

Keywords. Stochastic model; Random contraction; Random maximum; Continuous uniform cash flow; Synergy; Extremity; Proactivity; Information; Intelligent system

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1. Introduction

General and very thorough recognition is correctly attributed to the extremely important fact that a particularly wide variety of fundamental research activities in many theoretical and practical scientific disciplines make extensive and frequent employment of models for the representation, interpretation, prediction, and appraisal of various points of view of the complex real world [13]. It is very well known that the theoretical and practical contribution of models to the particularly significant and the remarkably fast advancement of a plethora of scientific disciplines is quite readily considered as exceptionally valuable. It is also very well known that models exhibit several exceedingly strong and profoundly useful interrelations with the main human mental processes [19]. Moreover, it can directly be said that models to the greatest extent constitute stimulating structures which are quite easily adopted as excessively appropriate providers of substantial support to the explicit description, the thorough clarification, and consequently the proper direction of the human ideas [18]. Furthermore, it is quite obvious that models are certainly identified as very effective instruments incorporating the extraordinary ability for the implementation, in a successful way, the inhibition of the nature of human understanding. In addition, models are generally recognized as the principal and the particularly significant constituent components of the human cognitive processes [18]. The development of a model or equivalently the process of modeling consists of three completely distinct and very crucial steps. More precisely, these three extremely important and fundamental steps are the model formulation, the model validation, and the model application [17]. In consequence, the human mental process of learning is generally recognized as an extremely valuable activity absolutely and strongly concentrating on the effective formulation of a very wide variety of models in order to make clearly possible the successful implementation of a completely thorough understanding of the complex real world [31].

Direct and thorough understanding of the casual procedures of real world systems is mainly achieved by incorporating the application of useful models. The implementation of that kind of understanding provides very strong support to the facilitation of the selection of suitable inflows for the significant propensity of outflows in advantageous orientation. It is particularly important to mention the outstanding contribution of models to making predictions concerning the future behavior and evolution of real world systems. The useful selection of actions, providing substantial practical contribution to the implementation of the various goals of a complex real world system, is very strongly supported by predictions of that kind. In consequence, models are unquestionably considered as really suitable tools for the satisfactory development and the successful application of diverse optimal control plans and the proper instruction of decision makers [25]. Many times, it is absolutely necessary the inclusion of a large number of models into a compact configuration in order to provide analysts, modelers, and other experts with extremely valuable information for obtaining distinct aspects of a complex real world system [27].

It is an absolutely acceptable fact that stochastic models constitute an extremely important category of models having an exceptional contribution to the precise description, the thorough investigation, and the good performance of many complex real world systems arising in a wide variety of fast growing disciplines with particular practical importance [23, 24, 26, 28, 30]. It is also generally recognized that maxima of a random number of positive, independent, and identically distributed random variables constitute a well known class of stochastic models having very useful applications in many practical disciplines [3–10, 12]. The structural elements of such a stochastic model are the concept of a discrete random variable and the concept of a sequence of positive, independent, and identically distributed random variables. The powerful probabilistic role of these structural elements and the mathematical form of the model of maximum of a random number of positive, independent, and identically distributed random variables constitute the main advantages for considering that model as a stochastic model of particular applicability in various important theoretical and practical disciplines [15, 16, 20]. In the particular case of a maximum of a random number of positive, independent, and identically distributed random variables which incorporates independent structural elements it is easily shown that such a random maximum constitutes a strong analytical tool providing analysts, modelers, decision makers, and other experts with quite explicit and exceedingly valuable probabilistic information for effectively treating practical situations. It is also approved that present values of continuous uniform cash flows with random rate of payment, random duration, and constant force of interest are widely considered as an interesting class of stochastic models with applications in many practical disciplines [1, 2]. This kind of approval is completely based on the fact that sufficient conditions can be readily established for the direct and simple representation of such present values as random contractions which are considered as powerful stochastic multiplicative models of exceptional theoretical and practical applicability in many diverse and leading disciplines [22]. A basic part of the main research intention of the present paper is the formulation of a stochastic model by the incorporation of the model of the present value of a continuous uniform cash flow with random rate of payment, random duration, and constant force of interest into the model of maximum of a random number of positive, independent, and identically random variables. The present paper also makes use of the formulated stochastic model in order to establish some essential interconnections between the fundamental concepts of extremity, proactivity, information, and intelligence of decision support systems [14].

The present paper mainly concentrates on the implementation of the following seven purposes. The first purpose is the formulation of a sequence of present values of continuous uniform cash flows by the incorporation of a sequence of positive, independent, and identically distributed random variables, another sequence of positive, independent, and identically distributed random variables, and a positive real number. The second purpose is the formulation of a stochastic model being the maximum of a random number of the above mentioned present values of continuous uniform cash flows. It is obvious that two sequences of random variables, a discrete random variable, and a positive real number constitute the structural elements of the formulated stochastic model. The interpretation in practice of the formulated stochastic model constitutes the third purpose of the paper. The fourth purpose is the establishment of sufficient conditions for the representation of the formulated stochastic model as the maximum of a random number of positive, independent, and identically distributed random variables. The use of such sufficient conditions for evaluating the distribution function corresponding to the formulated stochastic model constitutes the fifth purpose of the paper. In addition, the sixth purpose of the paper is the use of the formulated stochastic model for enhancing the role of extremity, proactivity, information and intelligence in decision support systems. It can be said that the implementation of the sixth purpose constitutes the most important result of this paper. The importance of that result is based on the ability of the formulated stochastic model to include probabilistic information suitable for improving the intelligent behavior of decision support systems. The seventh purpose is the embedment of the formulated stochastic model in some decision support systems with applications in electronic commerce and transportation. In consequence, it seems to be of some interest the undertaking of further research activities for the explicit formulation, the thorough investigation, and the embedment of stochastic models in decision support systems. More precisely, such research activities can provide very strong support for making sufficiently clear the use of various powerful results of probability theory in understanding the very crucial role of fundamental concepts in the development and applications of decision support systems arising in various leading practical disciplines. In conclusion, it can be said that the present paper extends the theoretical and practical applicability of the maximum of a random number of discounted random class flows [9].

2. Formulation and Interpretation of a Stochastic Model

The present section makes use of a discrete random variable, two sequences of positive random variables, and a positive real number as structural elements for the formulation of a stochastic model. In addition, this section establishes an interpretation in practice of the formulated stochastic model.

We suppose that N is a discrete random variable with values in the set $N = \{1,2,\dots,\}$, $\{X_n, n = 1,2,\dots\}$ is a sequence of positive random variables, $\{T_n, n = 1,2,\dots\}$ is also a sequence of positive random variables, and r is a positive real number. We consider the sequence

$$\left\{S_n = X_n \frac{1-e^{-rT_n}}{r}, n = 1,2,\dots\right\}$$

of positive random variables. Moreover, we consider the stochastic model

$$T = \max(S_1, S_2, \dots, S_N)$$

the formulation, investigation, and application of the above stochastic model constitute the main research intention of the present paper.

Below, we establish an interpretation in practice of the formulated stochastic model. We suppose that the discrete random variable N denotes the number of continuous uniform cash flows arising at the time point 0. We also suppose that the positive random variable X_n denotes the rate of payment, the positive random variable T_n denotes the duration, and the positive real number r denotes the force of interest of the nth continuous uniform cash flow. In consequence, the positive random variable

$$S_n, \quad n = 1, 2, \ldots$$

denotes the present value, as viewed from the time point 0, of the nth continuous uniform cash flow. It is readily recognized that the stochastic model

$$Y = \max(S_1, S_2, \ldots, S_N)$$

denotes the maximum of the present values, as viewed from the time point 0, of the N continuous uniform cash flows starting at the time point 0.

Making a comment on the mathematical form of the formulated stochastic model seems to be of particular theoretical and practical importance. It is quite obvious that the formulated stochastic model constitutes an extension of the stochastic model of the maximum of a random number of positive random variables. More precisely, it is easily seen that the extension is completely based on the introduction of the fundamental stochastic model of the present value of a continuous uniform cash flow, having random rate of payment and duration and constant force of interest, to the fundamental stochastic model of the maximum of a random number of positive random variables. The consequence of such an introduction is the suitability of the formulated stochastic model for implementing operations of decision support systems. In particular, the formulated stochastic model becomes a useful analytical tool for the combinations of powerful concepts in the implementation of proactive operations of decision support systems. The fourth section of the present paper mainly concentrates on considering the role of a combination of some such concepts in the proactive selection of operations for treatment of extreme events.

3. Random Contractions in a Stochastic Model

The contribution of the previous section is the introduction of the stochastic model of the present value of the continuous uniform cash flow to the stochastic model of the maximum of a random number of positive random variables. Moreover, the present section concentrates on the introduction of the model of random contraction to the stochastic model of the maximum of a random number of present values of continuous uniform cash flows.

We suppose that L is a positive random variable and W is a random variable with values in the interval $(0,1)$. Moreover, we suppose that the random variables L, W are independent and we consider the random variable $C = LW$. The random variable C is said a random contraction of the random variable Lvia the random variable W [22]. Random contractions have significant theoretical applications in areas of probability theory and mathematical statistics such as unimodal distributions, scale mixtures of distributions, characterizations of distributions, limiting distributions, preservation of infinite divisibility, integral equations for characteristic functions, order statistics, and statistical underreporting processes [21, 29]. In addition, random contractions have useful practical applications in disciplines such as income distribution analysis, cindynics, continuous discounting, reliability theory, inventory control, operations research, proactive risk management, engineering, systemics, and informatics [11]. In short, the main research intention of the present paper consists of combining the stochastic model of the maximum of a random number of positive random variables, the stochastic model of the present value of a continuous uniform cash flow, and the model of random contraction for supporting proactive decision making. More precisely, the present paper makes clear that such a combination contribute to implementing operations of decision support systems. The present section concentrates on the establishment of sufficient conditions for the representation of the formulated stochastic model as the maximum of a random number of independent and identically distributed random contractions, which are the present values of continuous uniform cash flows, and the explicit analytical evaluation of the distribution function corresponding to that stochastic model.

Theorem. *We suppose that the discrete random variable N has probability generating function $P_N(z$, the positive random variables of the sequence $\{X_n, n = 1,2,\dots\}$ are independent and distributed as the random variable $X$with distribution function $F_X(x)$, and the positive random variables of the sequence $\{T_n, n = 1,2,\dots\}$ are independent and distributed as the random variable T with distribution function $F_T(t)$. If N, $\{X_n, n = 1,2,\dots\}$, $\{T_n, n = 1,2,\dots\}$ are independent then the stochastic model*

$$Y = \max(S_1, S_2, \dots, S_N)$$

is identified as the maximum of a random number of independent and identically distributed random contractions and the distribution function corresponding to that stochastic model has the following form

$$F_Y(y) = P_N\left[\int_0^1 F_X\left(\frac{ry}{w}\right) dF_T\left(-\frac{1}{r}\log(1-w)\right)\right].$$

Proof. It is quite obvious that a direct consequence constitutes the independence of the random variables

$$X_n/r, \quad 1-e^{-rT_n}.$$

Moreover, from the fact that the random variable X_n/r is positive and the random variable $1-e^{-rT_n}$ takes values in the interval $(0,1)$ we conclude that the random variable

$$S_n = X_n \frac{1-e^{-rT_n}}{r}$$

is considered as a random contraction of the random variable X_n/r via the random variable $1-e^{-rT_n}$. It is readily proved that the random contractions of the sequence

$$\{S_n, n = 1,2,\dots\}$$

are independent and distributed as the random variable

$$[S = X \frac{1-e^{-rT}}{r}.$$

In addition, it is readily shown that the discrete random variable N is independent of the sequence $\{S_n, n = 1,2,\dots\}$ of independent and identically distributed random contractions. Hence the stochastic model

$$Y = \max(S_1, S_2, \dots, S_N)$$

is the maximum of a random number of independent and identically distributed random contractions. Furthermore, it is easily shown that the distribution function corresponding to Y has the form

$$F_Y(y) = P_N\left[\int_0^1 F_X\left(\frac{ry}{w}\right) dF_T\left(-\frac{1}{r}\log(1-w)\right)\right]. \qquad \square$$

The explicit analytical evaluation of the distribution function corresponding to the formulated stochastic model constitutes the main theoretical result of the present section. It is very clear that the practical contribution of such a theoretical result consists of providing various experts with useful probabilistic information for the treatment of significant situations. More precisely, the mathematical form of the distribution function $F_Y(y)$ substantially supports analysts, modelers, and decision makers for thinking, planning, and acting in a proactive way. In conclusion, the explicit analytical evaluation of $F_Y(y)$ facilitates the investigation of systems evolving in an uncertain environment. The following section is partially devoted to investigating the role of the model of random contraction in the theoretical and practical applicability of the formulated stochastic model. It is shown that the model of random contraction enhances the performance of decision support systems in implementing fundamental activities of modern complex organizations.

4. Applications of a Stochastic Model

The present section concentrates on the clarification of the role of the formulated stochastic model in revealing interconnections between four concepts of notable theoretical and practical interest. More precisely, such a clarification supports the significance of the fundamental principles and goals of extreme value theory for the evolution of the leading discipline of proactive decision making. The present section also establishes applications of the formulated stochastic model in electronic commerce and transportation. In particular, the present section makes quite clear that the theoretical extension of the stochastic model of the maximum of a random number of positive random variables can contribute to the description, analysis and treatment of problems arising in a variety of practical operations. The powerful interdisciplinary contribution of stochastic modeling is supported by the establishment of the results of the present section.

The explicit description, the thorough investigation, and the successful treatment of extreme practical processes are greatly facilitated by the use of the significant stochastic model of the maximum of a random number of positive random variables. Such a stochastic model provides researchers and practitioners with valuable probabilistic information for handling the worst or the best outcomes of systems in an intelligent way [32]. Moreover, the explicit description, the thorough investigation, and the successful treatment of crucial future processes are substantially supported by the significant stochastic model of the present value of a continuous uniform cash flow with random rate of payment, random duration, and constant force of interest. Such a stochastic model provides decision makers with the very strong advantage of making use of valuable probabilistic information, arising in a time interval starting at the time point 0 and having random length. In other words, the present value of such a continuous uniform cash flow incorporates the concept of proactivity. It is generally recognized that proactivity constitutes a structural element very strongly promoting the manifestation, preservation, and evolution of intelligent behavior. In consequence, it is particularly important to clarify the contribution of the advantages corresponding to the maximum of a random number of positive random variables and the advantages corresponding to the present value of a continuous uniform cash flow, with rate of payment and duration being random and force of interest being constant, to the extension of the practical applicability of the stochastic model formulated by the second section of the present paper. From the fact that, the formulated stochastic model is the maximum of a random number of present values of continuous uniform cash flows having random rate of payment and random duration but constant force of interest then it easily follows that such a stochastic model can be considered as a strong analytical tool for the explicit description, thorough investigation, and successful treatment of the extreme outcomes, corresponding to a random number of systems, in a proactive way. In short, it is clear that the structural elements, the principal component, and the mathematical form of the formulated stochastic model are considered as the main factors providing essential facilitation to decision support systems for the identification, development, adoption, and implementation of fundamental principles strongly stimulating the emergence of intelligent behavior. The establishment of sufficient conditions for representing the formulated stochastic model as the maximum of a random number of random contractions advocates the theoretical investigation of the distribution corresponding to that stochastic model. More clearly the incorporation of the model of random contraction in the formulated stochastic model reduces the difficulties for investigating the presence of unimodality, infinite divisibility, selfdecomposability, stability, and other fundamental properties in the explicit analytical form of the distribution function corresponding to that stochastic model. In particular, the presence of the property of unimodality in such a distribution function incorporates valuable probabilistic information for the most probable scenario of the maximum of a random number of present values of continuous uniform cash flows with random rate of payment, random duration, and constant force of interest. It can be said that the consideration of the principal component of the formulated stochastic model as a sequence of random contractions means the ability of decision support systems to obtain specific probabilistic information being very useful for the facilitation of the emergence and evolution of intelligent behavior. As a result, it is directly accepted that the first part of the present section makes very clear the useful role of the formulated stochastic model as an analytical tool for revealing some particularly interesting interconnections between the fundamental concepts of extremity, proactivity, information, and intelligence. The second part of the present section is devoted to the better understanding of such interconnections by providing applications in various practical processes of the formulated stochastic model as a fundamental factor of a decision support system. More precisely, the second part of the present section provides two such applications.

The first application of the formulated stochastic model is in some practical operations of electronic commerce. At the given future time point 0, an organization is going to sell an asset by auction. We suppose that a random number N of prospective buyers will show interest to obtain the asset at the time 0. We also suppose, that the nth prospective buyer will make an offer to obtain the asset by making use of a continuous uniform cash flow having as rate of payment the positive random variable X_n, as duration the positive random variable T_n, and as force of interest in continuous compounding the positive real number r. Hence, the positive random variable S_n denotes the present value, as viewed from time point 0, of such a continuous uniform cash flow. Equivalently, S_n is the offer of the nth prospective buyer to obtain the asset at the time point 0. Hence the stochastic model $Y = \max(S_1, S_2, \ldots, S_N)$ denotes the maximum of offers made by the N prospective buyers in order to obtain the asset at the time point 0. In other words, the stochastic model $Y = \max(S_1, S_2, \ldots, S_N)$ denotes the selling price of the asset at the time point 0. It is readily understood that the structural elements $\{X_n, n = 1, 2, \ldots\}$, $\{T_n, n = 1, 2, \ldots\}$, N and r, the assumptions on the structural elements, the principal component $\{S_n, n = 1, 2, \ldots\}$ and the mathematical form of the stochastic model $Y = \max(S_1, S_2, \ldots, S_N)$ substantially contribute to the development of strong synergies between fundamental concepts of proactivity and extremity. It is also obvious that the development of such synergies facilitates thinking, planning, deciding and acting in a strategic way. In particular, the application of the formulated stochastic model becomes particularly useful in electronic commerce where the sale of an asset can be connected with the purchase of another asset. In this case, it constitutes a common practice the use of the selling price of an asset for financing the purchasing price of another asset. It is well known that such a financing is advisable for the replacements of the main assets of modern complex organizations. In consequence, it is easily recognized as valuable the embedment of the formulated stochastic model in a decision support system for analyzing and treating situations of liquidity, cash flow stability, and investment strategy. More precisely, the presence of the concepts of proactivity and extremity in such an embedment strongly contributes to the manifestation of intelligence in the corresponding decision support system. It is of some particular practical interest to mention that the applicability in electronic commerce of the embedment of the formulated stochastic model in a decision support system is significantly extended by making the theoretical assumption that the discrete random variable N has the form of a sum incorporating a random number of discrete random variables.

Furthermore, the second application of the formulated stochastic model concentrates on some practical operations arising in transportation situations. At the given future time point 0, the fleet of a transportation company incorporates a random number N of vehicles. We suppose that the positive random variable X_n denotes the operating cost per unit of time generated by the nth vehicle and the positive random variable T_n denotes the residual time of use of the nth vehicle by the transportation company. We also suppose that the positive real number r denotes the force of interest in continuous compounding. Hence the positive random variable S_n denotes the present value, as viewed from the time point 0, of the operating cost generated by the nth vehicle during the random time interval $[0, T_n]$. In addition, the stochastic model $Y = \max(S_1, S_2, \ldots, S_N)$ denotes the maximum of the present values, as viewed from the time point 0, of the operating costs generated by the N vehicles in their corresponding residual times of use by the transportation company. Interpreting the term vehicle as truck, train, ship and airplane, it easily follows that the second application of the formulated stochastic model provides very significant information at the given future time point 0, concerning the maximum contingency reserve that a transportation company must retain for the operating cost of each vehicle and in extension for the fleet, consisting of a random number N of vehicles, that the transportation company uses. Moreover, particularly useful synergies, between proactivity and extremity, are created by the structural elements $\{X_n, n = 1,2,\ldots\}$, $\{T_n, n = 1,2\ldots\}$, N and r, the assumptions on the structural elements, the principal component $\{S_n, n = 1,2,\ldots\}$ and the mathematical form of the stochastic model $Y = \max(S_1,S_2,\ldots,S_N)$. Hence the main characteristics of the formulated stochastic model strongly direct a transportation company to strategic thinking, planning, deciding and acting. In addition, it constitutes a useful venture the embedment of the formulated stochastic model in a decision support system. Finally, the theoretical assumption that the discrete random variable N has the form of a sum of random number of discrete random variables makes the formulated stochastic model suitable for decision support systems having a global horizon.

In consequence, it is easily accepted that the implementations of the results of the present section sufficiently reveals the role of the formulated stochastic model in the incorporation of the concepts of extremity, proactivity, information, and intelligence for the thorough investigation and effective management of extreme behaviors related to projects arising in a wide variety of practical disciplines. Such incorporation is absolutely based on the interesting combination of the maximum of a random number of positive random variables with the present value of a continuous uniform cash flow having random rate of payment, random duration, and constant rate of interest. In addition, the establishment of such a combination has as a direct and valuable consequence the development of a very strong synergy between the concepts of extremity, proactivity and information. It is quite easily understood that the development of this kind of synergy provides substantial support to the emergence and evolution of intelligence in various decision support systems of particular practical and theoretical applicability. In conclusion, it is easily seen that the implementations of the purposes of the present section can significantly contribute to the undertaking of further research activities concentrating on the formulation, investigation and application of stochastic models facilitating the development of synergies between fundamental concepts, for providing substantial support to the emergence of intelligence in various decision support systems.

5. Conclusions

It can be said that the formulation, interpretation, investigation, and application of a stochastic model constitute the results of the present paper. It is shown that these results contribute to clarifying interconnections between the formulated model and the concepts of extremity, proactivity, information, and intelligence. More precisely, it is shown that the structural elements, the principal component, and the mathematical form of the formulated stochastic model facilitate the development of synergies between the concepts of extremity, proactivity, and information for supporting the manifestation and evolution of intelligent behavior in decision support systems arising in electronic commerce, transportation and other practical disciplines. It seems that the main research intention of the present paper can be extended by introducing the concept of sum of a random number of discrete random variables in the concept of maximum of a random number of present values of continuous uniform cash flows with random rate of payment, random duration, and constant force of interest.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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