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Research Article

Kinematic Analysis and MATLAB Applications of 2-3RRR Mechanism Chain

Bahar Kalkan[1*], Şenay Baydaş[1] and Bülent Karakaş[2]

[1] *Department of Mathematics, Faculty of Science, Van Yuzuncu Yil University, Van, 65080, Turkey*
[2] *Department of Business Administration, Faculty of Economics and Administrative Science, Van Yuzuncu Yil University, Van, 65080, Turkey*
[*] **Corresponding author:** baharkalkan1@gmail.com

Abstract. The Stewart platform is a mechanism design based on movement of two planes relative to each other by the motion of links. These type of mechanism can be structurally designed in many different ways. In this paper, the joints of connection points of fixed platform to moving platform is determined as RRR with three legs. A software has been developed in the MATLAB environment, which makes calculations using this Stewart platfom mechanism's transformation matrix by applying Denavit Hartenberg convention.

Keywords. Denavit-Hartenberg representation; mechanism chain; Stewart platform

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1. Introduction

Robot mechanisms are usually serial, parallel, and hybrid. Parallel mechanisms have closed-chains with fixed and moving bases. Their advantages are high rigidity and high loading capacity etc., but their working space is limited. Forward kinematic analysis proposes to calculate the position and orientation of mechanism while its joint parameters are given. Inverse kinematic analysis proposes to calculate the joint parameters of mechanism which are the rotation angle for the rotary joint and the translation distance for the prismatic joint for the desired position and orientation of its end-effector.

One of the first documented parallel mechanism named as Oxymoron is a spherical parallel mechanism and it is used as a cinema motion simulator [1]. After this invention, a spray painting machine, a simple five-bar parallel mechanism, is designed [2]. A paper on the design of an eight-bar linkage is published, pistons on limbs or revolute joint on the middle of a limb cause linear motion of its limbs [3]. Delta robot has an idea to use parallelograms to constrain rotational motion of moving base [4].

A kinematic analysis of a new type of hybrid (parallel-serial) robot manipulator is presented. Closed-form solutions for the direct and inverse position problems have been obtained [5]. The kinematics of a hybrid manipulator with a particular topology is approached by means of the theory of screws. Hybrid manipulators are parallel-serial connection robots that give rise to a multitude of highly articulate robotic manipulators. These manipulators are modular and can be extended by additional modules over large distances [6].

Expanding the working space of the traditional Stewart platform is one of the important research subjects. Adding revolute joints to links is one of these works. The type of Stewart mechanism considered in this study is the mechanism in which the revolute joints are added. The mechanism discussed in this paper is a chain of Stewart mechanisms of this type. In the following sections a special form of Stewart platform is considered and the kinematic equations of the chain formed by using Denavit-Hartenberg parameters, forward and inverse kinematic analysis are calculated. Then, in the next section, Matlab program is written in an algorithm that determines the kinematic structure created in the previous section and Matlab output is added. In the last section, conclusion and discussion are given.

2. Preliminaries

2.1 Rigid transformations

The position of one link relative to another in a kinematic chain is defined mathematically by a coordinate transformation between reference frames attached to each body. The link is rigid so this transformation must preserve the distance measured between points, and it is called *rigid transformation.* Rigid transformations will consist simply of rotations and translations. For planar mechanisms we need only consider rotations and translations in two dimensions. For spherical mechanisms we need only consider rotations in space. For general spatial mechanisms we consider rotations and translations in three dimensions. In this chapter rigid transformations are presented in general, and then specialized for use in the study of planar, spherical, and spatial mechanisms.

The same equation that defines the position of a body by a coordinate transformation can be interpreted as an operation that moves the body from an initial to a final position. This latter view is reflected in the term *displacement.* The points and lines in the body that do not move while the body is displaced, that is the pole of a planar displacement, the axis of a spatial rotation, and the screw axis of a spatial displacement, characterize the displacement. Because the two interpretations of the position of a body are equivalent, we find that these objects are the invariant subspaces, or eigenvalues, of the homogenous transforms that define the coordinate transformation [7].

2.2 Coordinate transformations

To study the position of one body relative to another, we attach coordinate frames to each. One is chosen as the ground with coordinate frame F, and the other, the moving body, has the coordinate frame M. We use the coordinate transformation $D : F \leftarrow M$, which transforms coordinates measured in M to those measured in F, to represent the position of M relative to F. This transformation is given by

$$X = [A]x + d, \tag{1}$$

where x is the coordinate vector of a point in M and X is the coordinate vector of the same point but measured in F. If the moving body is of dimension n (usually $n = 2$ or 3), then $[A]$ is an $n \times n$ matrix and d is an n-dimensional vector.

This transformation must preserve the rigidity of the body M which means that distances computed using either set of coordinates must be the same. The distance between to points P and Q in M is defined by the usual Euclidean distance formula, which is the magnitude of the difference of their coordinate vectors. Using coordinates measured in F we have

$$\begin{aligned} d(P,Q) &= |P - Q| \\ &= \sqrt{(P - Q)^T (P - Q)}\,. \end{aligned} \tag{2}$$

Now compute this distance using coordinates p and q measured in M; from (1) we obtain:

$$\begin{aligned} |P - Q| &= |([A]p + d) - ([A]q + d)| \\ &= |[A](p - q)| \\ &= \sqrt{(p - q)^T [A^T][A](p - q)}\,. \end{aligned} \tag{3}$$

This last equation equals $|p - q|$ only if the matrix $[A]$ satisfies the relation

$$[A^T][A] = [I]. \tag{4}$$

This is the constraint that guarantees that (1) is a rigid transformation.

An $n \times n$ matrix that satisfies (4) is called an *orthogonal matrix* because its columns are orthogonal unit vectors. The left inverse of this matrix is also its right inverse, so we have

$$[A][A^T] = [I], \tag{5}$$

which show that the rows of $[A]$ are also orthogonal unit vectors. From (4) or (5), we obtain a constraint on the determinant of $[A]$:

$$\det(I) = \det([A^T][A]) = \det^2([A]) = 1, \tag{6}$$

which implies that the $\det(A) = \pm 1$. Orthogonal matrices with $\det(A) = 1$ are called *rotations* and those with $\det(A) = -1$ are *reflections*. While reflections satisfy the rigid transformation constraint, only rotations are used to define the position of a rigid body, so in addition to satisfying (4) the matrix $[A]$ is restricted to have a determinant equal to 1 [7].

2.3 Displacements

It is convenient to view the coordinate transformation (1) as an operation that displaces a point from an original position to its present position. This can lead to confusion about the coordinate frame in which the vector x is measured. Our convention will be to view this transformation as a displacement of the entire body M from an initial position coinciding with F to its present position, in which case x is always measured in M. We symbolize this transformation by $D : F \to M$, and call it a *displacement*. It is important to remember that the mathematical operations given in (1) are not changed whether it is viewed as a displacement or a coordinate transformation.

A displacement in n-dimensional space is defined by the matrix-vector pair $D = (A,d)$, where $[A]$ is an $n \times n$ rotation matrix and d is an n-dimensional vector. There are two special cases: $R = (A,0)$, called a *pure rotation;* and $T = (I,d)$, called a *pure translation*.

One displacement can operate on another to yield a composite displacement. If we have $D_1 : F \to M_1$ and $D_2 : M_1 \to M_2$, then the composite displacement $D = D_1D_2 : F \to M_2$ exists. The formula for this composite displacement is obtained by substituting the coordinates obtained from the transformation $D_2 = (A_2,d_2)$ into (1) written for the transformation $D_1 = (A_1,d_1)$. The result is

$$X = [A_1A_2]x + [A_1]d_2 + d_1\,. \tag{7}$$

Thus the composite displacement $D = D_1D_2 : F \to M_2$ is defined to be

$$D = D_1D_2 = (A_1,d_1)(A_2,d_2) = (A_1A_2,[A_1]d_2 + d_1). \tag{8}$$

The inverse, D^{-1} of a displacement $D = (A,d)$ is defined by inverting (1) to obtain:

$$x = [A^T]X - [A^T]d, \tag{9}$$

thus $D^{-1} = (A^T,-A^Td)$. Notice that $DD^{-1} = D^{-1}D = I$, where $I = (I,0)$ is the identity displacement.

The set of displacements of an n-dimensional space $\mathbb{R}^n$ has the following important properties which identify it as an *algebraic group*.

(1) A product operation exists such that if D_1 and D_2 are displacements, then $D = D_1D_2$ is also a displacement;

(2) The displacement $I = (I,0)$ is the identity under the composite operation;

(3) Every displacement $D = (A,d)$ has an inverse $D^{-1} = (A^T,-A^Td)$ which is also a displacement.

This set is called the *Euclidean group* of n-dimensional space, often denoted $SE(n)$ [7].

2.4 Denavit-Hartenberg convention

The transformation has been constructed, which is usually a function of four-link parameters, defining the n frame according to the $n+1$ frame. By defining a frame for each link, the kinematic problem becomes a subproblem. To solve each of these subproblems; each subproblem is examined in four subproblems. Each of these four transformations is a function of a link parameter [8].

- Rotate about the z_n-axis an angle of θ_{n+1}. This will make x_n and x_{n+1} parallel to each other. This is true because a_n and a_{n+1} are both perpendicular to z_n and rotating z_n and angle of θ_{n+1} will make them parallel (and thus coplanar).

- Translate along the z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} collinear. Since x_n and x_{n+1} were already parallel and normal to z_n, moving along z_n will lay them over each other.
- Translate along the x_n-axis a distance of a_{n+1} to bring the origins of x_n and x_{n+1} together. At this point, the two origins of the two reference frames will be at the same location.
- Rotate z_n-axis about x_{n+1}-axis

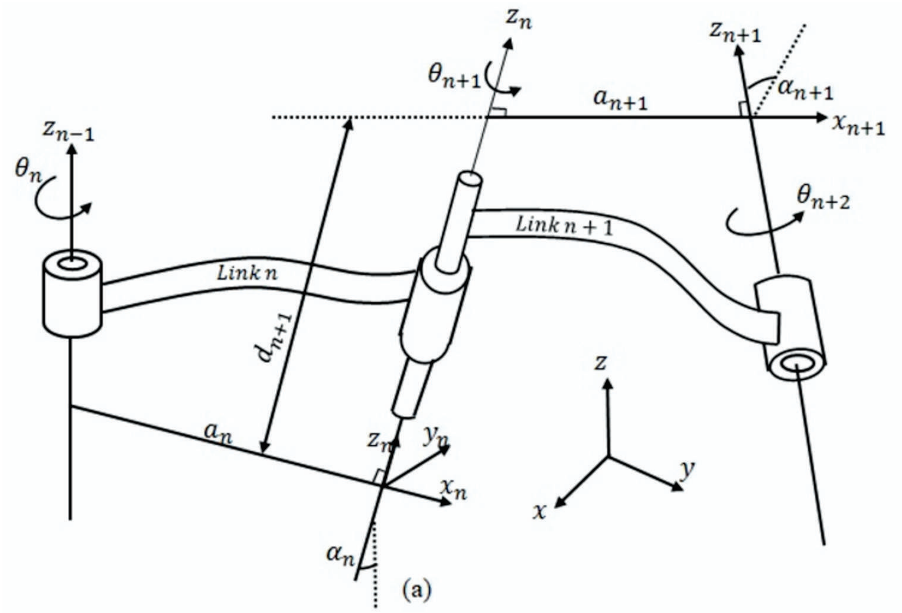

Figure 1. Denavit-Hartenberg Convention

3. Structure of 3-RRR Parallel Mechanism

3.1 Kinematics of 3-RRR parallel mechanism

The designed mechanism includes one fixed and two moving platforms. Planar representations of the platforms will be given at three points. The triangle $A_1A_2A_3$ is fixed platform, $C_1C_2C_3$ and $E_1E_2E_3$ represent moving platforms. Figure 2 is designed according to this information and it represents initial position of the platform.

The motion contains the of motion C with respect to A, and E with respect to C and A. First, the motion of the (C,A) pair will be examined. Figure 3 represents any moment of the motion of (C,A). The center of the equilateral triangle $A_1A_2A_3$ is connected to the fixed platform by an $F\{O-xyz\}$ coordinate system with the origin O and the x-axis is in the direction of the A_1 point. The center of the equilateral triangle $C_1C_2C_3$ is connected to the fixed platform by an $M'\{O'-x'y'z'\}$ coordinate system with the origin O' and the x'-axis is in the direction of the C_1 point.

Coordinates of A_i $(i=1,2,3)$ on $F\{O-xyz\}$ coordinate system is

$$P_{A_1}=[r \quad 0 \quad 0]^T, \tag{10}$$

$$P_{A_2}=R(z,\theta_2)P_{A_1}=\begin{bmatrix}-\frac{r}{2} & \frac{r\sqrt{3}}{2} & 0\end{bmatrix}, \tag{11}$$

$$P_{A_3} = R(z,\theta_3)P_{A_1} = \begin{bmatrix} -\frac{r}{2} & -\frac{r\sqrt{3}}{2} & 0 \end{bmatrix}. \tag{12}$$

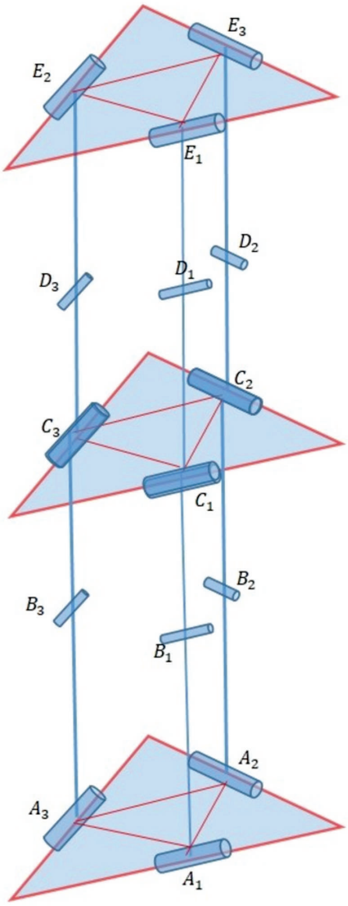

Figure 2. Initial position of 2-3RRR mechanism chain

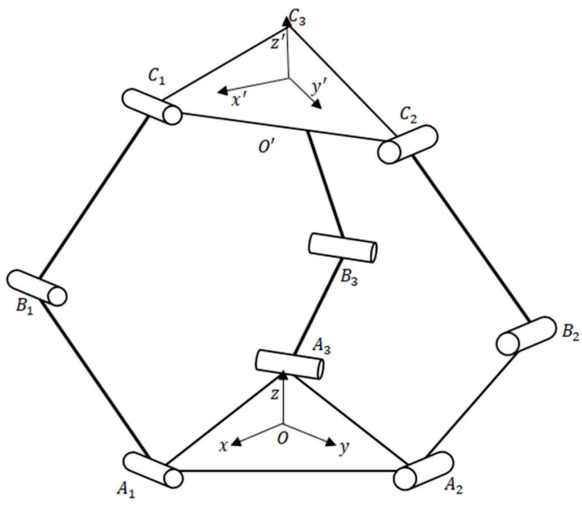

Figure 3. Moving position of 3-RRR parallel mechanism

The upper moving platform $C_1C_2C_3$ can be represented by an equilateral triangle; $|O'C_1| = |O'C_2| = |O'C_3| = r$. The position vectors of spherical coordinate frames of C_i points can be written as following:

$$\begin{aligned}
\vec{OC}_i &= \vec{r}+\vec{L}_{i1}+\vec{L}_{i2} \\
&= (c\theta_{i1}(r+L_{i1}c\theta_{i2}+L_{i2}c(\theta_{i2}+\theta_{i3})), s\theta_{i1}(r+L_{i1}c\theta_{i2}+L_{i2}c(\theta_{i2}+\theta_{i3})), L_{i1}s\theta_{i2}+L_{i2}s(\theta_{i2}+\theta_{i3})), \\
\vec{OC}_1 &= \vec{r}+\vec{L}_{11}+\vec{L}_{12} = (r+L_{11}c\theta_{12}+L_{12}c\varphi_1, 0, -L_{11}s\theta_{12}-L_{12}s\varphi_1), \\
\vec{OC}_2 &= \vec{r}+\vec{L}_{21}+\vec{L}_{22} = \left(-\frac{1}{2}(r+L_{21}c\theta_{22}+L_{22}c\varphi_2), \frac{\sqrt{3}}{2}(r+L_{21}c\theta_{22}+L_{22}c\varphi_2), L_{21}s\theta_{22}+L_{22}s\varphi_2\right), \\
\vec{OC}_3 &= \vec{r}+\vec{L}_{31}+\vec{L}_{32} = \left(-\frac{1}{2}(r+L_{31}c\theta_{32}+L_{i2}c\varphi_3), -\frac{\sqrt{3}}{2}(r+L_{31}c\theta_{32}+L_{32}c\varphi_3), L_{31}s\theta_{32}+L_{32}s\varphi_3\right).
\end{aligned} \tag{13}$$

Here $\varphi_i = \theta_{i2}+\theta_{i3}$ and s and c denotes cosine and sine functions, respectively.

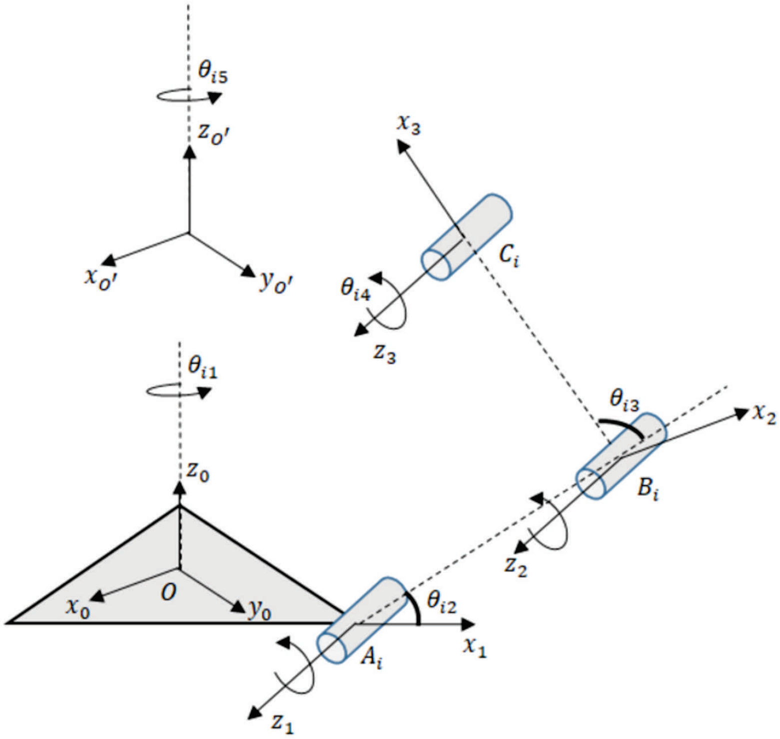

Figure 4. Reference frames placement of RRR manipulator

The first thing to do to model the manipulator with the D-H notation is to specify the reference frames for each joint. Thus it is necessary to assign the z-axes and the x-axes for each joint. y-axis is perpendicular to both the x-axis and the z-axis. The D-H parameters for the corresponding manipulator according to the reference frames given in Figure 4 are given in Table 1:

Table 1. Denavit-Hartenberg parameters of the mechanism

Links	θ_i	d_i	a_i	α_i
1	θ_{i1}	0	r	$\pi/2$
2	θ_{i2}	L_{i1}	15	0
3	θ_{i3}	0	L_{i2}	0
4	θ_{i4}	0	r	$\pi/2$
5	θ_{i5}	0	0	0

θ_{i1} and θ_{i5} are constants for the corresponding mechanism and their values are given for every leg:

$$\theta_{i1} = \begin{cases} 0, & i = 1; \\ \frac{2\pi}{3}, & i = 2; \\ \frac{4\pi}{3}, & i = 3, \end{cases} \qquad \theta_{i5} = \begin{cases} \pi, & i = 1; \\ \frac{\pi}{3}, & i = 2; \\ \frac{5\pi}{3}, & i = 3. \end{cases}$$

The rotation and displacement in the x-axis and the rotation and displacement matrices in the z-axis in homogeneous coordinates are as follows:

$$Rot\,X(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Rot\,Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Trans\,X(a) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad Trans\,Z(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

For this manipulator by using Denavit-Hartenberg convention displacement matrix ${}^{o}_{o'}T$ is calculated as following:

$${}^{o}_{o'}T = A_1 A_2 A_3 A_4 A_5$$

where the motion matrices are

$$A_1 = Rot\,Z(\theta_{t1}) TX(r) Rot\,X\left(\frac{\pi}{2}\right)$$

$$A_2 = Rot\,Z(\theta_{i2}) TX(L_{i1})$$

$$A_3 = Rot\,Z(\theta_{i3}) TX(L_{i2})$$

$$A_4 = Rot\,Z(\theta_{i4}) TX(r) Rot\,X\left(\frac{\pi}{2}\right)$$

$$A_5 = Rot\,Z(\theta_{i5})$$

By using MATHEMATICA 7.0 indices of ${}^{o}_{o'}T$ matrix are

$$T_{11} = \cos\theta_{i1}\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5} + \sin\theta_{i1}\sin\theta_{i5}$$

$$T_{21} = \sin\theta_{i1}\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5} - \cos\theta_{i1}\sin\theta_{i5}$$

$$T_{31} = \sin(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5}$$

$$T_{41} = 0$$

$$T_{12} = -\cos\theta_{i1}\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\sin\theta_{i5}+\sin\theta_{i1}\sin\theta_{i5}$$
$$T_{22} = -\sin\theta_{i1}\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\sin\theta_{i5}-\cos\theta_{i1}\sin\theta_{i5}$$
$$T_{32} = -\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})\sin\theta_{i5}$$
$$T_{42} = 0$$
$$T_{13} = \cos\theta_{i1}\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$T_{23} = \sin\theta_{i1}\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$T_{33} = -\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$T_{43} = 0$$
$$T_{14} = \cos\theta_{i1}(r+L_{i1}\cos\theta_{i2}+L_{i2}\cos(\theta_{i2}+\theta_{i3})+r\cos(\theta_{i2}+\theta_{i3}+\theta_{i4}))$$
$$T_{24} = \sin\theta_{i1}(r+L_{i1}\cos\theta_{i2}+L_{i2}\cos(\theta_{i2}+\theta_{i3})+r\cos(\theta_{i2}+\theta_{i3}+\theta_{i4}))$$
$$T_{34} = L_{i1}\sin\theta_{i2}+L_{i2}\sin(\theta_{i2}+\theta_{i3})+r\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$T_{44} = 1$$

3.2 Inverse kinematics of 3-RRR parallel mechanism

To solve inverse kinematics problem, displacement matrix which carries the origin of fixed platform to the origin of moving platform, is multiplied from left by inverse joint displacement matrices, A_n^{-1}. Multiplying displacement matrix ${}^{o}_{o'}T$ with A_O^{-1}:

$$A_1^{-1}\,{}^{o}_{o'}T = A_1A_2A_3A_4A_5 = K.$$

The indices of K are as following:

$$k_{11} = \cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5}$$
$$k_{21} = \sin(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5}$$
$$k_{31} = \sin\theta_{i5}$$
$$k_{41} = 0$$
$$k_{12} = -\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5}$$
$$k_{22} = -\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})\cos\theta_{i5}$$
$$k_{32} = \cos\theta_{i5}$$
$$k_{42} = 0$$
$$k_{13} = \sin(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$k_{23} = \cos(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$k_{33} = 0$$
$$k_{43} = 0$$
$$k_{14} = L_{i1}\cos\theta_{i2}+L_{i2}\cos(\theta_{i2}+\theta_{i3})+r\cos(\theta_{i2}+\theta_{i3}+\theta_{i4})$$
$$k_{24} = L_{i1}\sin\theta_{i2}+L_{i2}\cos(\theta_{i2}+\theta_{i3})+r\sin(\theta_{i2}+\theta_{i3}+\theta_{i4})$$

$$k_{34} = 0$$

$$k_{44} = 1$$

By the help of the matrix K, the angles θ_{i2} and θ_{i2} are obtained as:

$$k_{14} + rk_{23} = L_{i1}\cos\theta_{i2} + L_{i2}\cos(\theta_{i2}+\theta_{i3}) = K_1\,, \tag{14}$$

$$k_{24} - rk_{13} = L_{i1}\sin\theta_{i2} + L_{i2}\sin(\theta_{i2}+\theta_{i3}) = K_2\,. \tag{15}$$

Multiplying equation (14) with $\cos\theta_{i2}$ and equation (15) with $\sin\theta_{i2}$ then equation (14) with $-\sin\theta_{i2}$ and equation (15) with $\cos\theta_{i2}$ following equations are obtained.

$$\cos\theta_{i2}K_1 + \sin\theta_{i2}K_2 = L_{i1}^2 + L_{i2}\cos\theta_{i3}\,, \tag{16}$$

$$-\sin\theta_{i2}K_1 + \cos\theta_{i2}K_2 = L_{i2}\sin\theta_{i3}\,. \tag{17}$$

Equations (16) and (17) can be written in matrix form:

$$\begin{bmatrix} L_{i1}^2 + L_{i2}\cos\theta_{i3} \\ L_{i2}\sin\theta_{i3} \end{bmatrix} = \begin{bmatrix} K_1 & K_2 \\ K_2 & -K_1 \end{bmatrix} \begin{bmatrix} \cos\theta_{i2} \\ \sin\theta_{i2} \end{bmatrix}. \tag{18}$$

By taking squares of equation (14) and (15) then taking their sum following equation is obtained:

$$(K_1)^2 + (K_2)^2 = L_{i1}^2 + L_{i2}^2 + 2L_{i1}L_{i2}\cos\theta_{i3}\,,$$

$$\cos\theta_{i3} = \frac{(K_1)^2 + (K_2)^2 - L_{i1}^2 - L_{i2}^2}{2L_{i1}L_{i2}} = \xi_{i3}\,,$$

$$\theta_{i3} = \tan^{-1}\left(\frac{\pm\sqrt{1-\xi_{i3}^2}}{\xi_{i3}}\right). \tag{19}$$

The angle θ_{i2} is obtained from vector-matrix form in equation (18).

$$\begin{bmatrix} \cos\theta_{i2} \\ \sin\theta_{i2} \end{bmatrix} = \frac{1}{L_{i1}^2 + L_{i2}^2 + 2L_{i1}L_{i2}\cos\theta_{i3}} \begin{bmatrix} K_1(L_{i2}^2 + L_{i2}\cos\theta_{i2}) + K_2L_{i2}\sin\theta_{i2}) \\ -K_2(L_{i1}^2 + L_{i2}\cos\theta_{i2}) + K_1L_{i2}\sin\theta_{i2} \end{bmatrix}.$$

Hence, θ_{i2} is

$$\theta_{i2} = \tan^{-1}\left(\frac{-K_2(L_{i1}^2 + L_{i2}\cos\theta_{i3}) + K_1L_{i2}\sin\theta_{i3})}{K_1(L_{i2}^2 + L_{i2}\cos\theta_{i3}) + K_2L_{i2}\sin\theta_{i3}}\right). \tag{20}$$

By multiplying the matrix K with $A_3^{-1}A_2^{-1}$ following matrix is obtained: $A_3^{-1}A_2^{-1}A_1^{-1}{}^{o}_{o'}T = A_4A_5 = M$

$$m_{11} = \cos\theta_{i4}\cos\theta_{i5}$$

$$m_{21} = \sin\theta_{i4}\cos\theta_{i5}$$

$$m_{31} = \sin\theta_{i5}$$

$$m_{41} = 0$$

$$m_{12} = -\cos\theta_{i4}\sin\theta_{i5}$$

$$m_{22} = -\sin\theta_{i4}\sin\theta_{i5}$$

$$m_{32} = \cos\theta_{i5}$$

$$m_{42} = 0$$

$$m_{13} = \sin\theta_{i4}$$

$m_{23} = -\cos\theta_{i4}$

$m_{33} = 0$

$m_{43} = 0$

$m_{14} = r\cos\theta_{i4}$

$m_{24} = r\sin\theta_{i4}$

$m_{34} = 0$

$m_{44} = 1$

By using this matrix value of the angle θ_{i4} is obtained from the solution of following pair of equations:

$$\begin{cases} \frac{m_{14}}{r} = \cos\theta_{i4} \\ \frac{m_{24}}{r} = \sin\theta_{i4} \end{cases}$$

$$\theta_{i4} = \tan^{-1}\left(\frac{m_{24}}{m_{14}}\right) \tag{21}$$

Hence, inverse kinematics analysis of 3-RRR parallel mechanism is completed.

3.3 Matlab applications of 3-RRR mechanism

In this section, Matlab program will be given for the 3-RRR mechanism. In Figure 5 and 6 illustration of single arm of 3-RRR mechanism and coordinate frames of the first arm are given respectively, but the given program will be done on three arms for determined values of rotation angles Q_i's with some constants like link lengths L_i and the distance between O and A_i r and so on.

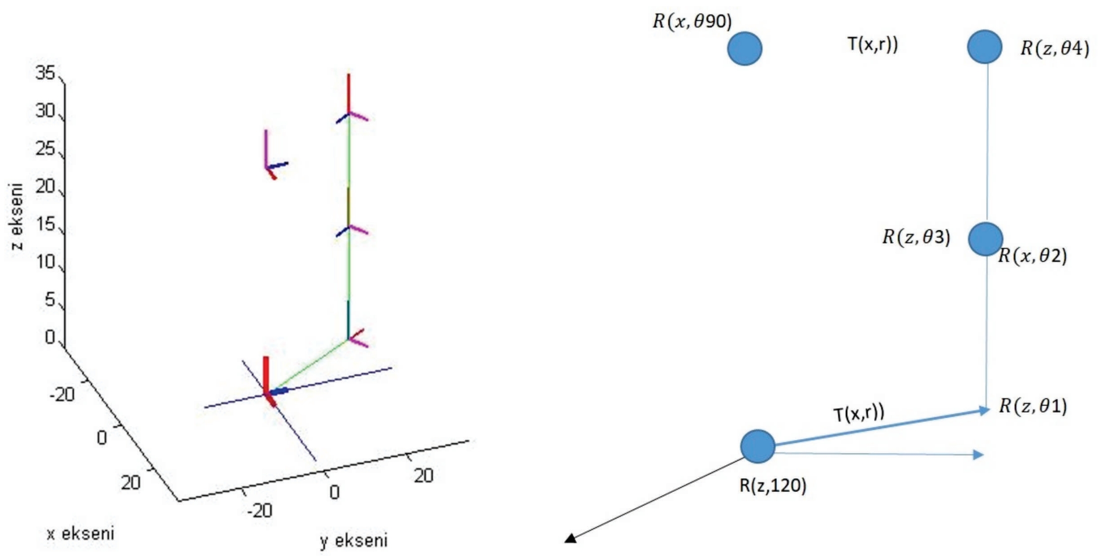

Figure 5. Algorithm of 3-RRR mechanism

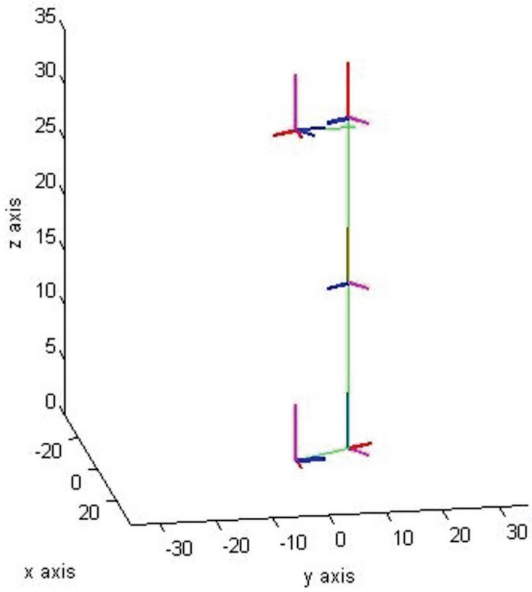

Figure 6. Coordinate frames of first arm

In the following, Matlab algorithm of 3-RRR mechanism is given then. Figure 7 shows the coordinate frames of 3-RRR mechanism and in Figure 8 this mechanism is plotted as a 2-3RRR mechanism chain. It can be extended as n-3RRR chain.

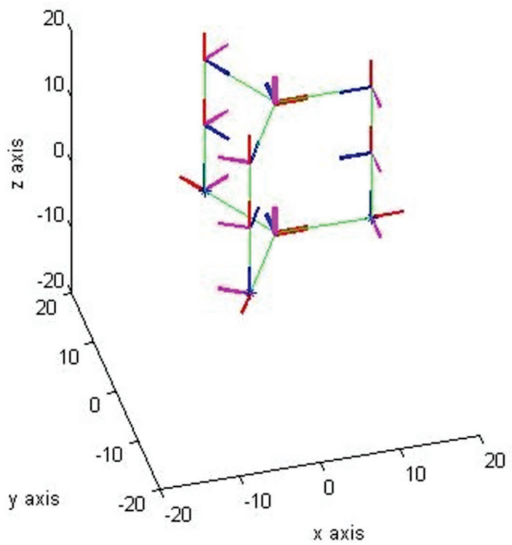

Figure 7. Coordinate frames of 3-RRR mechanism

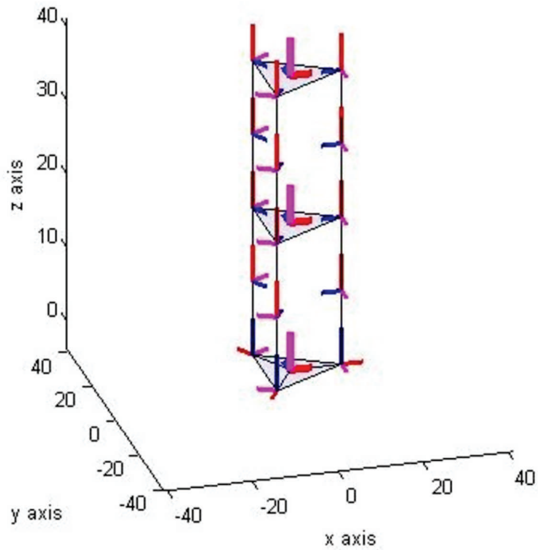

Figure 8. 2-3RRR mechanism chain

```
axis square
k=35;
xlabel('x axis'); ylabel('y axis'); zlabel('z axis')
axis([-k k -k k 0 k])
r=12;
for Q0=0:120:240;
```

```
Q1=90; Q2=90; Q3=0; Q4=90; Q5=90; Q6=180-Q0;
L1=15; L2=15;
AA=[5;0;0;1]; BB=[ 0;5;0;1]; CC=[0;0;5;1];
O=[0;0;0;1]; P=[1 0 0 r;0 1 0 0;0 0 1 0;0 0 0 1;
line([0,AA(1)],[0, AA(2)],[0,AA(3)],'LineWidth',2,'Color',[1 0 0])
line([0, BB(1)],[0, BB(2)],[0,BB(3)],'LineWidth',2,'Color',[0 0 1])
line([0, CC(1)],[0, CC(2)],[0,CC(3)],'LineWidth',2,'Color',[1 0 1])
hold on
RotzQ0=[cosd(Q0) -sind(Q0) 0 0; sind(Q0) cosd(Q0) 0 0; 0 0 1 0 ; 0 0 0 1];
RotzQ2=[cosd(Q2) -sind(Q2) 0 0; sind(Q2) cosd(Q2) 0 0; 0 0 1 0 ; 0 0 0 1];
RotxQ1 =[ 1 0 0 0; 0 cosd(Q1) -sind(Q1) 0; 0 sind(Q1) cosd(Q1) 0;0 0 0 1];
RotzQ3=[cosd(Q3) -sind(Q3) 0 0; sind(Q3) cosd(Q3) 0 0; 0 0 1 0 ; 0 0 0 1];
TxL1=[1 0 0 L1;0 1 0 0;0 0 1 0;0 0 0 1]; TxL2=[1 0 0 L2;0 1 0 0;0 0 1 0;0 0 0 1];
RotzQ4=[cosd(Q4) -sind(Q4) 0 0; sind(Q4) cosd(Q4) 0 0; 0 0 1 0 ; 0 0 0 1];
P=[1 0 0 r;0 1 0 0;0 0 1 0;0 0 0 1];
RotxQ5 =[ 1 0 0 0; 0 cosd(Q5) -sind(Q5) 0; 0 sind(Q5) cosd(Q5) 0;0 0 0 1];
RotzQ6=[cosd(Q6) -sind(Q6) 0 0; sind(Q6) cosd(Q6) 0 0; 0 0 1 0 ; 0 0 0 1];
a=(RotzQ0*P*RotxQ1)*AA   b=(RotzQ0*P*RotxQ1)*BB
c=(RotzQ0*P*RotxQ1)*CC   o=(RotzQ0*P*RotxQ1)*O
hold on
line([o(1),a(1)],[o(2), a(2)],[o(3),a(3)],'LineWidth',2,'Color',[1 0 0])
line([o(1), b(1)],[o(2), b(2)],[o(3),b(3)],'LineWidth',2,'Color',[0 0 1])
line([o(1), c(1)],[o(2), c(2)],[o(3),c(3)],'LineWidth',2,'Color',[1 0 1])
pause(1)
line([o(1), O(1)],[o(2), O(2)],[o(3),O(3)],'LineWidth',1,'Color',[0 1 0])
at=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*AA; bt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*BB;
ct=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*CC; ot=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*O;
hold on
line([ot(1),at(1)],[ot(2), at(2)],[ot(3),at(3)],'LineWidth',2,'Color',[1 0 0]);
line([ot(1), bt(1)],[ot(2), bt(2)],[ot(3),bt(3)],'LineWidth',2,'Color',[0 0 1]);
line([ot(1), ct(1)],[ot(2), ct(2)],[ot(3),ct(3)],'LineWidth',2,'Color',[1 0 1]);
line([o(1), ot(1)],[o(2), ot(2)],[o(3),ot(3)],'LineWidth',1,'Color',[0 1 0]);
pause(1)
att=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*AA;
btt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*BB;
ctt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*CC;
ott=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*O;
hold on
line([ott(1),att(1)],[ott(2), att(2)],[ott(3),att(3)],'LineWidth',2,'Color',[1 0 0]);
line([ott(1), btt(1)],[ott(2), btt(2)],[ott(3),btt(3)],'LineWidth',2,'Color',[0 0 1]);
line([ott(1), ctt(1)],[ott(2), ctt(2)],[ott(3),ctt(3)],'LineWidth',2,'Color',[1 0 1]);
```

```
line([ot(1), ott(1)],[ot(2), ott(2)],[ot(3),ott(3)],'LineWidth',1,'Color',[0 1 0]);
pause(1)
end
attt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*(RotzQ4*P*RotxQ5)*AA;
bttt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*(RotzQ4*P*RotxQ5)*BB;
cttt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*(RotzQ4*P*RotxQ5)*CC;
ottt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*(RotzQ4*P*RotxQ5)*O;
hold on
atttt=(RotzQ0*P*RotxQ1)*(RotzQ2*TxL1)*(RotzQ3*TxL2)*(RotzQ4*P*RotxQ5)*(RotzQ6*AA);
btttt=RotzQ0*P*RotxQ1*RotzQ2*TxL1*RotzQ3*TxL2*RotzQ4*P*RotxQ5*RotzQ6*BB;
ctttt=RotzQ0*P*RotxQ1*RotzQ2*TxL1*RotzQ3*TxL2*RotzQ4*P*RotxQ5*RotzQ6*CC;
ortttt=RotzQ0*P*RotxQ1*RotzQ2*TxL1*RotzQ3*TxL2*RotzQ4*P*RotxQ5*RotzQ6*O;
hold on
line([ortttt(1),atttt(1)],[ortttt(2),atttt(2)],[ortttt(3),atttt(3)],'LineWidth',2,'Color',[1 0 0]);
line([ortttt(1),btttt(1)],[ortttt(2),btttt(2)],[ortttt(3),btttt(3)],'LineWidth',2,'Color',[0 0 1]);
line([ortttt(1),ctttt(1)],[ortttt(2),ctttt(2)],[ortttt(3),ctttt(3)],'LineWidth',2,'Color',[1 0 1]);
pause(1)
```

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] J. E. Gwinnett, Amusement device, *US Patent No. 1,789,680* (1931), 1 – 5.

[2] W. L. V. Pollard, Position controlling apparatus, *US Patent No. 2,286,571* (1942), 1 – 5.

[3] D. Stewart, A platform with six degrees of freedom, *Proceedings of the Institution of Mechanical Engineers (Part I)* **180**(15) (1965), 371 – 386, DOI: 10.1243/PIME_PROC_1965_180_029_02.

[4] R. Clavel, Conception D'un Robot Parallele Rapide A 4 Degres De Liberte, *Presentee Au Departement De Microtechnique, Ecole Polytechnique Federale de Lausanne*, Lausanne (1991), DOI: 10.5075/epfl-thesis-925.

[5] T. K. Tanev, Kinematics of a hybrid (parallel-serial) robot manipulator, *Mechanism and Machine Theory* **35**(9) (2000), 1183 – 1196, DOI: 10.1016/S0094-114X(99)00073-7.

[6] M. Shahinpoor, Kinematics of a parallel-serial (hybrid) manipulator, *Journal of Robotic Systems* **9**(1) (1992), 17 – 36, DOI: 10.1002/rob.4620090103.

[7] J. M. McCarthy, *Introduction to Theoretical Kinematics*, The MIT Press, USA (1990), DOI: 10.1017/S0263574700015721.

[8] S. B. Niku, *Introduction to Robotics: Analysis, Systems, Applications*, Prentice Hall, Upper Saddle River, New Jersey, USA (2001).