**Journal of Informatics and Mathematical Sciences**
Vol. 9, No. 3, pp. 985–997, 2017
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

**Proceedings of the Conference**
**Current Scenario in Pure and Applied Mathematics**
**December 22-23, 2016**
**Kongunadu Arts and Science College (Autonomous)**
**Coimbatore, Tamil Nadu, India**

Research Article

# Nonlinear Radiative Effects on MHD Flow Past a Nonlinearly Stretching Surface Embedded in a Porous Medium

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**Abstract.** An analysis has been carried out to investigate the steady, laminar, two dimensional hydromagnetic flows with heat transfer of an incompressible, viscous and electrically conducting fluid over a surface stretching with a power-law velocity distribution and embedded in a porous medium in the presence of a variable magnetic field. The radiative heat flux term is taken to be nonlinear using Rosseland diffusion approximation. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. Then the resulting nonlinear ordinary differential equations are solved numerically using Fourth-Order Runge-Kutta based shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions and the numerical results for velocity and temperature distribution are obtained for different values of radiation parameter, permeability, velocity exponent parameter, surface temperature parameter, magnetic interaction parameter and Prandtl number. The dimensionless rates of heat transfer and skin friction coefficient are also obtained for different physical parameters and are presented graphically.

**Keywords.** Nonlinear Radaitive heat flux; Rosseland diffusion approximation; porous medium; Nachtsheim-Swigert iteration scheme and nonlinearly stretching surface

**MSC.** 80A20

**Received:** January 5, 2017 **Accepted:** March 16, 2017

## 1. Introduction

A study of flow problem with heat and mass transfer over stretching porous surfaces has generated considerable interest because of its numerous industrial applications. Flow through porous medium is very much prevalent in nature and therefore the study of flows through porous medium has become of principle interest in many scientific and engineering applications. These types of flows have shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoir; in chemical engineering for filtration and water purification processes. Further to study the underground water resources, seepage of water in river beds also one needs to analyse the flow of fluid through porous medium and the present paper deals with a problem of such type of flow.

The boundary layer analogies for convection and steady state heat transfer within porous medium with temperature dependent heat generation in a porous medium was investigated by various researchers [1, 2]. Studies were conducted on steady flow and heat transfer of a viscous incompressible fluid through porous medium over a stretching porous sheet with internal heat generation and suction or injection [3–5].

The era witnessed the investigation of exact solutions of the porous media equations which usually occur in nonlinear problems of heat and mass transfer and in biological systems were obtained using Adomian's decomposition method and the flow and heat transfer of viscous fluids saturated in porous media over a permeable non-isothermal stretching sheet [6, 7].

Modeling of viscous dissipation in a saturated porous medium and the effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour effects with joule dissipation were analyzed by many researchers [8–10].

Studies on forced convection with thermal radiation have increased greatly during the last decade due to its importance in many real world applications. With rising temperature levels, radiation heat transfer becomes more important and may be totally dominant over conduction and convection. Thus, thermal radiation is important in combustion applications (furnaces, rocket, nozzles, engines, etc.), in nuclear reactors and during atmospheric re-entry of space vehicles. In special, a radiation effect on heat transfer over a stretching surface finds numerous newer applications in recent times due to its applications in space technology.

Many excellent theoretical models have been developed for radiative-convection flows and radiative-conductive transport. Plumb et al. [11] analysed the effect of horizontal cross-flow and radiation on natural convection from vertical heated surface in saturated porous media. The thermal radiation of a gray fluid which is emitting and absorbing in laminar, steady boundary layer flow over an isothermal horizontal flat plate in a non-stretching medium has been examined by Chen et al. [12]. Chamkha [13] studied the thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source and sink. In their work they have considered the vertical plate problem and used the Taylor series expansion and considered only the linear terms in the temperature. Effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet has been analysed analytically by Ouaf [14]. Mukhopadhyay and Layek [15] investigated the effects of thermal radiation and variable fluid viscosity on free convection flow and heat transfer past a porous stretching surface. Investigation were carried to study the effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium, non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and Ohmic dissipation and steady boundary layer slip flow and heat transfer over a flat porous plate embedded in a porous media [16–18]. Hydromagnetic Boundary Layer Flow over Stretching Surface with Thermal Radiation was investigated by Siti Khuzaimah Soid et al. [19].

The study of non-linear radiation effects gained momentum as Mukhopadhyay [20] investigated the effect of boundary layer flow and heat transfer over a porous moving plate in the presence of thermal radiation, taking into account of the full form of radiation term. Effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power-law velocity was analysed by Anjali Devi and David Maxim Gururaj [21]. David Maxim Gururaj and Pavithra [22] studied the Nonlinear MHD boundary layer flow of a liquid metal with heat transfer over a porous stretching surface with nonlinear radiation effects.

MHD boundary layer flow with forced convection past a nonlinearly stretching surface with variable conductivity and nonlinear radiation effects and hydromagnetic flow with forced convection past a porous nonlinearly stretching surface with variable magnetic field and nonlinear radiation effects were investigated by David Maxim Gururaj and Anjali Devi [23, 24]. Adding a new feather to the cap, non-linear radiation effects are taken in to consideration of Nanofluid flows. Non-linear Radiation Effects on MHD Flow of Copper and Alumina Nanofluids over a Porous Stretching Surface with Heat Transfer and Suction/Injection was analysed by David Maxim Gururaj and Haseena [25].

But so far, no contribution is made on MHD flow with nonlinear radiation over a stretching surface embedded in a porous medium and hence the present work is carried out due to its abundant applications.

## 2. Formulation of the Problem

Two-dimensional, nonlinear, steady, MHD laminar boundary layer flow of a viscous, incompressible gray fluid with heat transfer through a porous medium of permeability $k_p$ over a stretching surface in the presence of nonlinear radiation is considered Under the usual assumptions in the literature, the continuity, momentum, and energy equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\left(\frac{\sigma B^2(x)}{\rho}\right)u-\frac{v}{k_p}u\,,\quad \text{where } B(x)=B_0x^{\frac{(m-1)}{2}} \tag{2.2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 u}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{2.3}$$

with the associated boundary conditions

$$\left.\begin{aligned} &u=u_ox^m,\ v=0,\ T=T_w; &&\text{at } y=0 \quad (u_o>0)\\ &u=0,\ T=T_\infty; &&\text{at } y\to\infty \end{aligned}\right\} \tag{2.4}$$

The radiative heat flux term is simplified by using the Rosseland approximation. In literature, most of the problems with radiation effects have considered the Rosseland approximation in a linear form which is valid only for small temperature difference between the plate and the ambient fluid [11–19]. In the present work we consider the nonlinear Rosseland thermal radiation effects which holds good for small and large temperature difference between the plate and the ambient fluid. This assumption leads to a new physical parameter $\theta_w$, namely the surface temperature parameter. Thus the radiative heat flux is given by [26]

$$q_r=-\frac{16\sigma^* T^3}{3\alpha^*}\frac{\partial T}{\partial y} \tag{2.5}$$

where $\sigma^*$ is the Stefan-Boltzmann constant, $\alpha^*$ is the Rosseland mean absorption coefficient. Introducing the usual similarity transformation and defining $T_w(x)=T_\infty+bx^n$ where $b$ is a dimensional constant and $n$ is the index of power-law variation of wall temperature which is a constant, equations (2.2) and (2.3) can be written as

$$f'''+ff''-\frac{2m}{m+1}f'^2-M^2f'-\left(\frac{1}{R_1}\right)f'=0 \tag{2.6}$$

$$\left\{1+\frac{4}{3R^*}(1+(\theta_w-1)\theta)^3\right\}\theta''+\frac{4}{R^*}(1+(\theta_w-1)\theta)^2(\theta_w-1)\theta'^2+\Pr\left[f\theta'-\frac{(2n)}{(m+1)}f'\theta\right]=0 \tag{2.7}$$

where $M=\sqrt{\frac{2\sigma B_o^2}{\rho u_o(m+1)}}$ is the magnetic interaction parameter,

$R^*=\frac{\kappa\alpha^*}{4\sigma^* T_\infty^3}$ is the radiation parameter and $R_1=\frac{k_p u_w(m+1)}{2vx}$ is the local permeability parameter with boundary conditions

$$\left.\begin{aligned} &f(0)=0,\ f'(0)=1, &&\theta(0)=1;\\ &f'(\infty)=0, &&\theta(\infty)=0 \end{aligned}\right\} \tag{2.8}$$

## 3. Numerical Solution of the Problem

Equations (2.6) and (2.7) are nonlinear ordinary differential equations which constitute the nonlinear boundary value problem. The above boundary value problem is converted into an initial value problem by shooting method. Equations (2.6) and (2.7) are solved numerically subject to (2.8) using Fourth-Order Runge-Kutta based shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions by making an initial guess for the values of $f''(0)$ and $\theta'(0)$ to initiate the shooting process. Numerical values are depicted graphically by means of figures for velocity $f'(\eta)$, temperature distribution $\theta(\eta)$ for several sets of values of magnetic interaction parameter $M$, radiation parameter $R^*$, surface temperature parameter $\theta_w$, index of power-law variation of wall temperature $n$, velocity exponent parameter $m$ and Prandtl number Pr.

## 4. Results and Discussions

To validate the numerical results obtained by the present work, the values obtained for rate of heat transfer are compared with those of SitiKhuzaimahSoid et al. [19]. It is found that the results are in good agreement which is shown in Table 1.

**Table 1.** Values of $\theta'(0)$ for various values of Pr and $n$ ($m = 1$ and $M = \lambda = 0$ and $R^* = 10^9$)

| | Results due to Siti Khuzaimah Soid et al. [19] | | | Present Results | | |
|---|---|---|---|---|---|---|
| | $n = 0$ | $n = 1$ | $n = 2$ | $n = 0$ | $n = 1$ | $n = 2$ |
| Pr = 1 | -0.5820 | -1.0000 | -1.3333 | -0.58242 | -1.00035 | -1.33345 |
| Pr = 3 | -1.1652 | -1.9237 | -2.5097 | -1.16532 | -1.92372 | -2.51020 |
| Pr = 10 | -2.3080 | -3.7207 | -4.7969 | -2.30830 | -3.72065 | -4.79688 |

Non-linear radiation effect on heat transfer for different Prandtl Number when compared to linear radiation effect (SitiKhuzaimahSoid et al. [19]) is tabulated below.

**Table 2**

| Pr | $R^*$ | Rate of Heat Transfer $\theta'(0)$ with $M = 1$: Under Linear Radiation Assumption ($\theta_w = 0$) | Under Non-linear Radiation Assumption |
|---|---|---|---|
| 1 | 1 | -0.6721 | -0.7264 |
| | 2 | -0.5345 | -0.5999 |
| | 3 | -0.4554 | -0.5319 |
| 3 | 1 | -1.2629 | -1.3385 |
| | 2 | -1.0000 | -1.0778 |
| | 3 | -0.8475 | -0.9231 |
| 5 | 1 | -1.6915 | -1.7593 |
| | 2 | -1.3417 | -1.4182 |
| | 3 | -1.1371 | -1.1938 |

It is evident from Table 2, that the values are dominant when the radiation assumptions are non-linear radiation, whereas the values are just simple rescaling of the Prandtl number by a factor involving the radiation parameter in the case of linear radiation assumptions.

Figure 1 illustrates the plot of dimensionless velocity $f'(\eta)$ for different values of velocity exponent parameter. It is seen that the effect of velocity exponent parameter is to reduce the velocity. The effect of magnetic field $M$ over the dimensionless transverse velocity field $f'(\eta)$ for different values of the magnetic interaction parameter is shown with help of Figure 2. It is seen from the graph that as magnetic interaction parameter $M$ increases, the transverse velocity decreases elucidating the fact that the effect of magnetic field is to decelerate the velocity. Figure 3 displays the plot of dimensionless temperature $\theta(\eta)$ for a nonlinearly stretching surface for different values of magnetic interaction parameter $M$. It is seen that the effect of $M$ is to increase the temperature. Prandtl number variation over the dimensionless temperature profile when the surface is nonlinearly stretching is given by Figure 4. It is seen that as Prandtl number Pr increase, temperature $\theta(\eta)$ decreases, ascertaining the fact that the effect of Prandtl number is to decrease the temperature in the presence of magnetic field. Figure 5 illustrates the effect of index of power-law variation of wall temperature $n$ over the temperature field. It is observed that the effect of $n$ is to decrease the temperature. The effect of radiation parameter $R^*$ over the dimensionless temperature $\theta(\eta)$, for a nonlinear stretching surface is illustrated by Figure 6. It is observed that the effect of radiation parameter is to reduce the temperature, elucidating the fact that the thermal boundary layer thickness decreases as $R^*$ increases. Figure 7 shows the effect of surface temperature parameter $\theta_w$ over the dimensionless temperature $\theta(\eta)$ for a nonlinearly stretching surface. It is observed that increasing surface temperature parameter $\theta_w$ is to increase the temperature. The magnetic interaction parameter $M$ on skin friction $f''(0)$ for different value of velocity exponent parameter for a nonlinearly stretching surface is shown through Figure 8. It is seen that skin friction coefficient decreases with the increase of velocity exponent parameter and it decreases for increasing magnetic interaction parameter. Figure 9 displays the dimensionless rate of heat transfer $\theta'(0)$ against magnetic interaction parameter $M$ for different $n$. It is seen the rate of heat transfer increases with a decrease in $n$ and magnetic interaction parameter. The effect of heat transfer $\theta'(0)$ against magnetic interaction parameter $M$ for different $R^*$ is shown in Figure 10. It is seen that the dimensionless rate of heat transfer $\theta'(0)$ increases with an increase in radiation parameter and magnetic interaction parameter. The surface temperature parameter effect on dimensionless rate of heat transfer $\theta'(0)$ for different values of magnetic interaction parameter $M$ is shown through Figure 11. It is noted that the rate of heat transfer $\theta'(0)$ decreases with increase of surface temperature parameter $\theta_w$ and magnetic interaction parameter $M$.

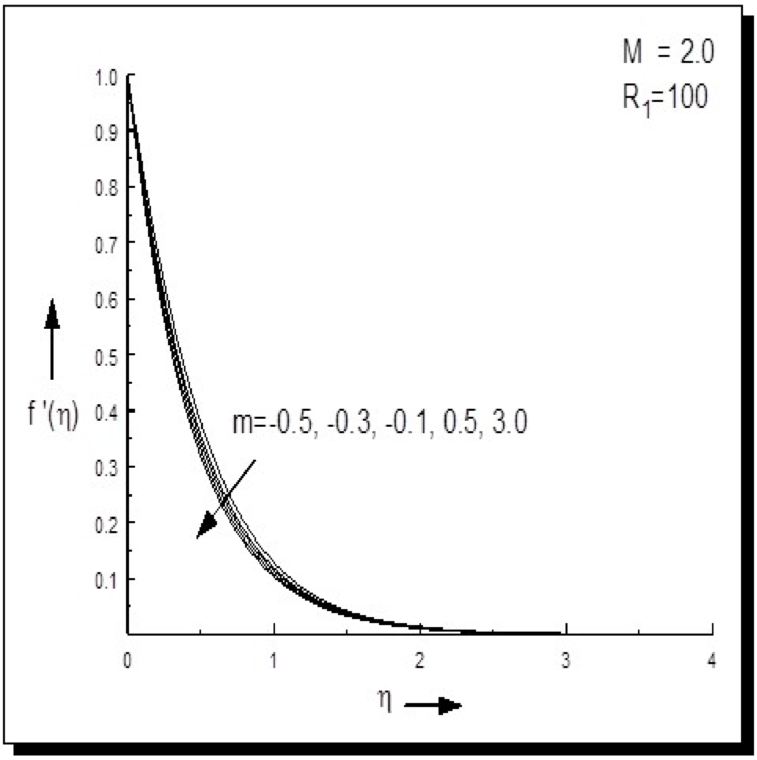


**Figure 1.** Velocity profiles for different $m$

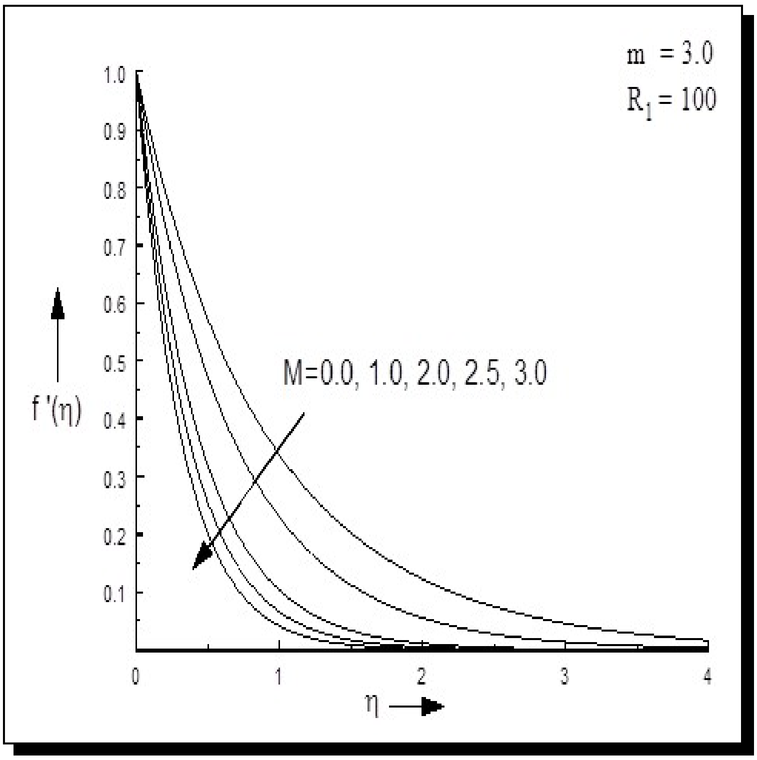


**Figure 2.** Velocity profiles for different $M$

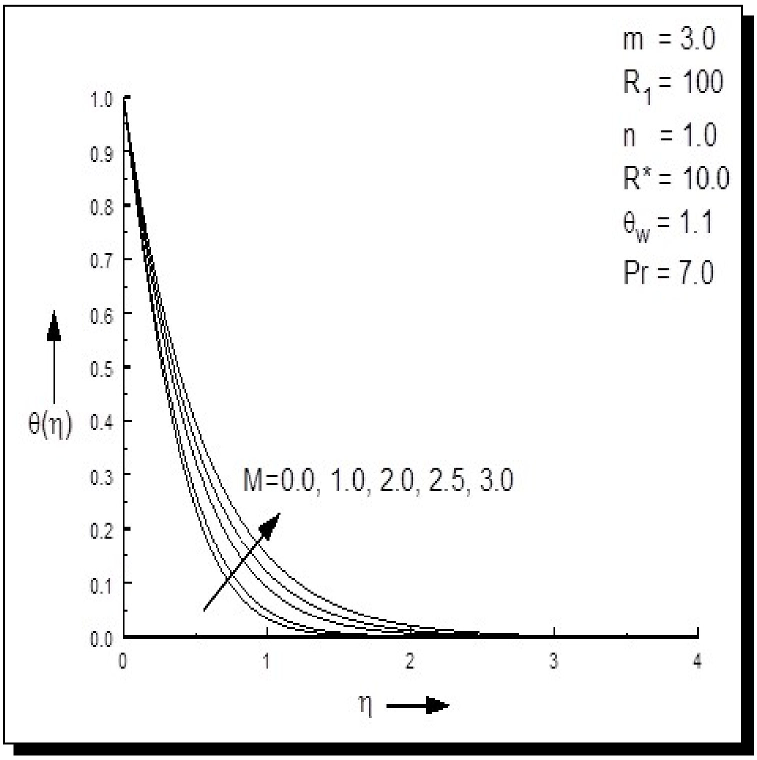


**Figure 3.** Temperature profiles for different $m$

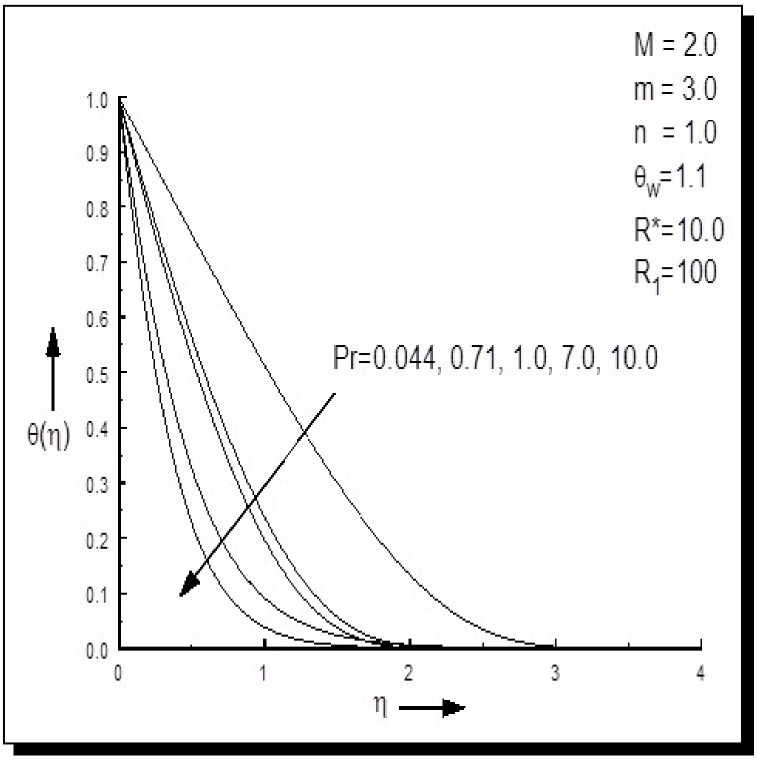


**Figure 4.** Temperature profiles for different Pr

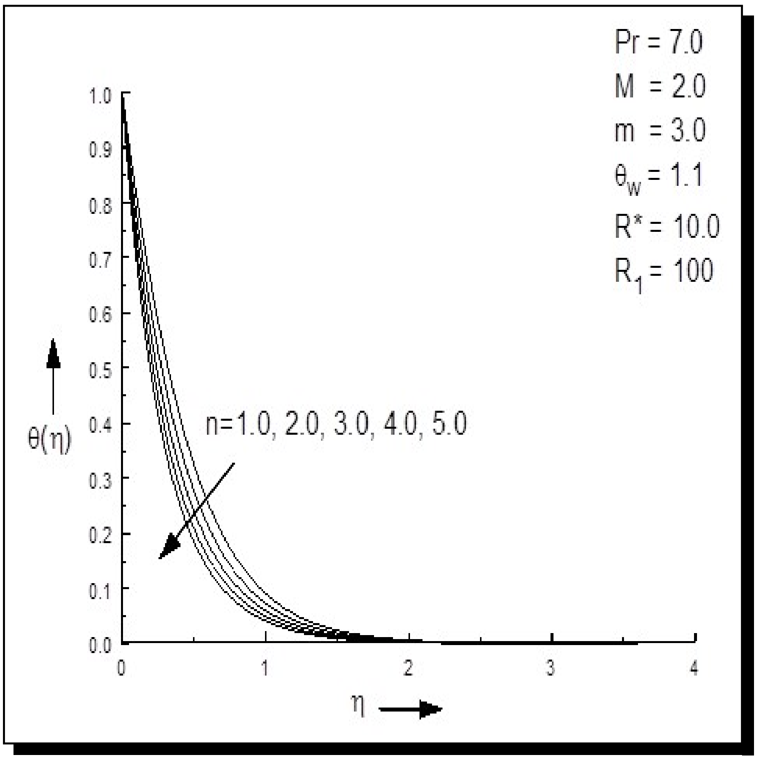


**Figure 5.** Temperature profiles for different $n$

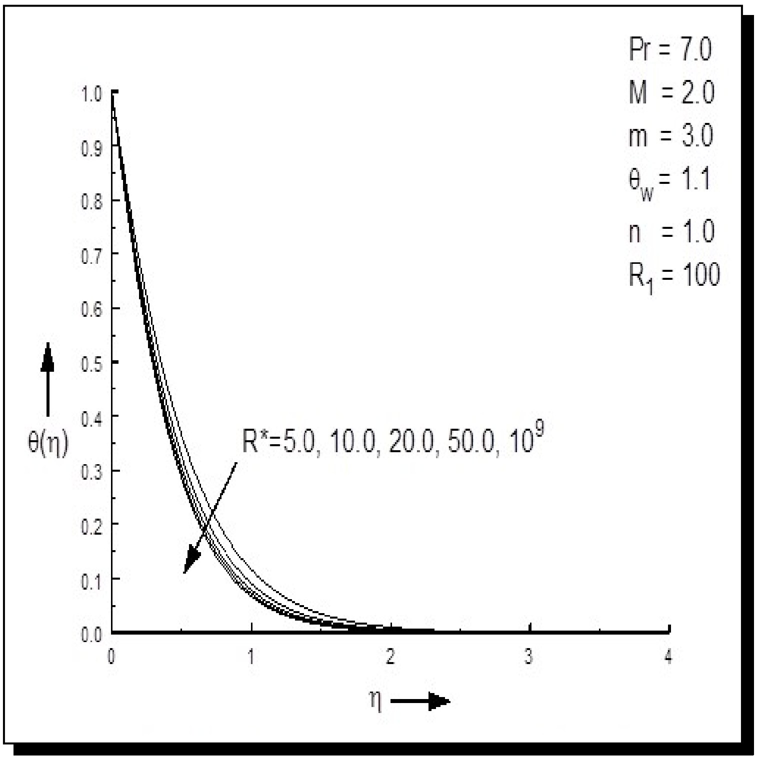


**Figure 6.** Temperature profiles for different $R^*$

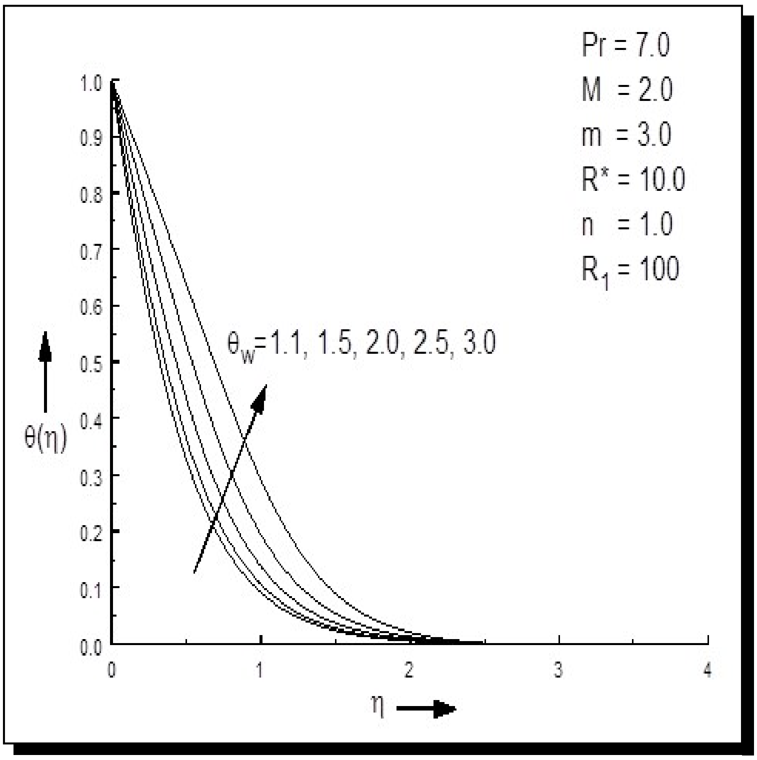


**Figure 7.** Temperature profiles for different $\theta_w$

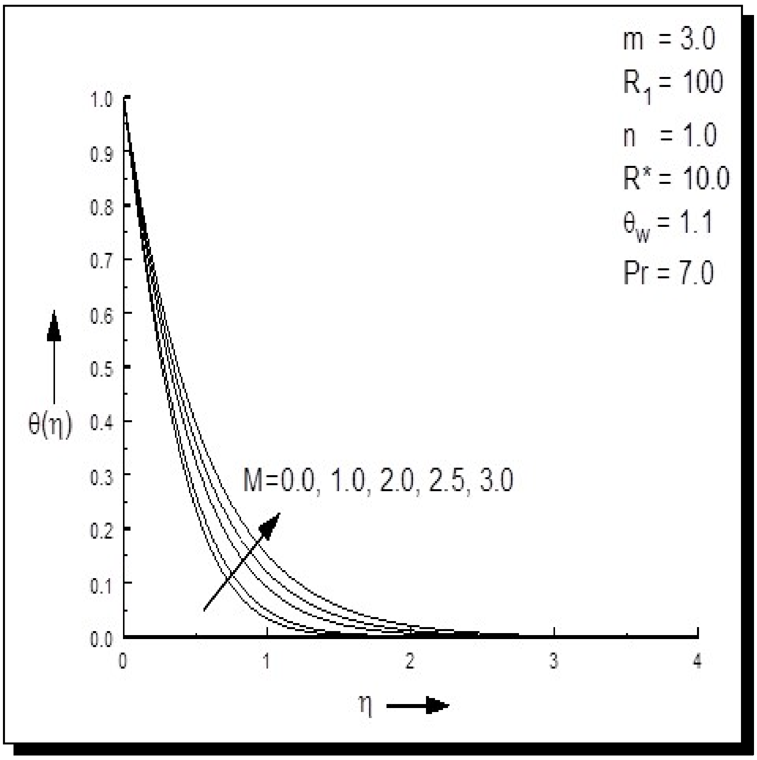


**Figure 8.** Skin friction coefficient for different $m$

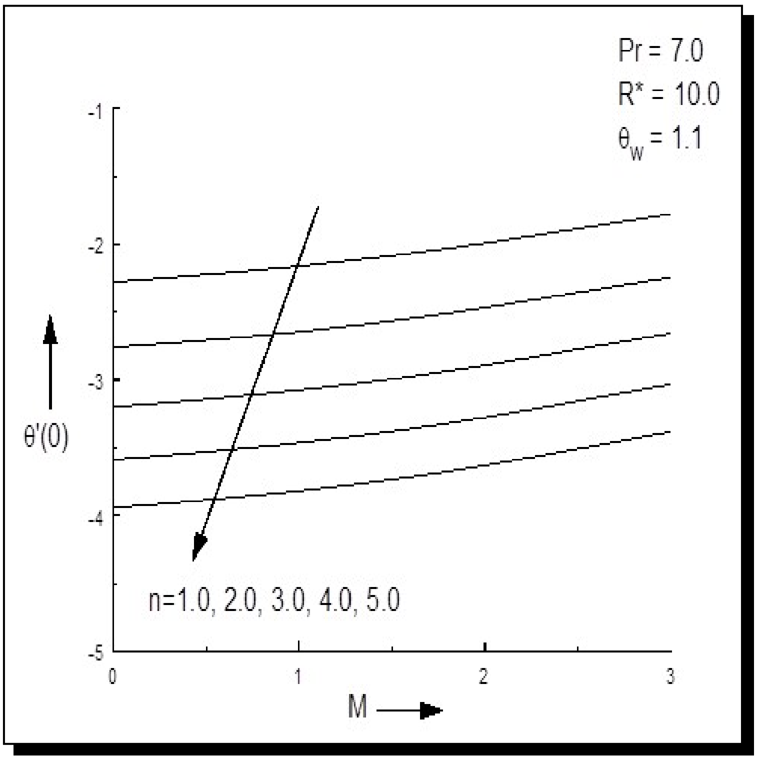


**Figure 9.** Dimensionless rate of heat transfer for different $n$

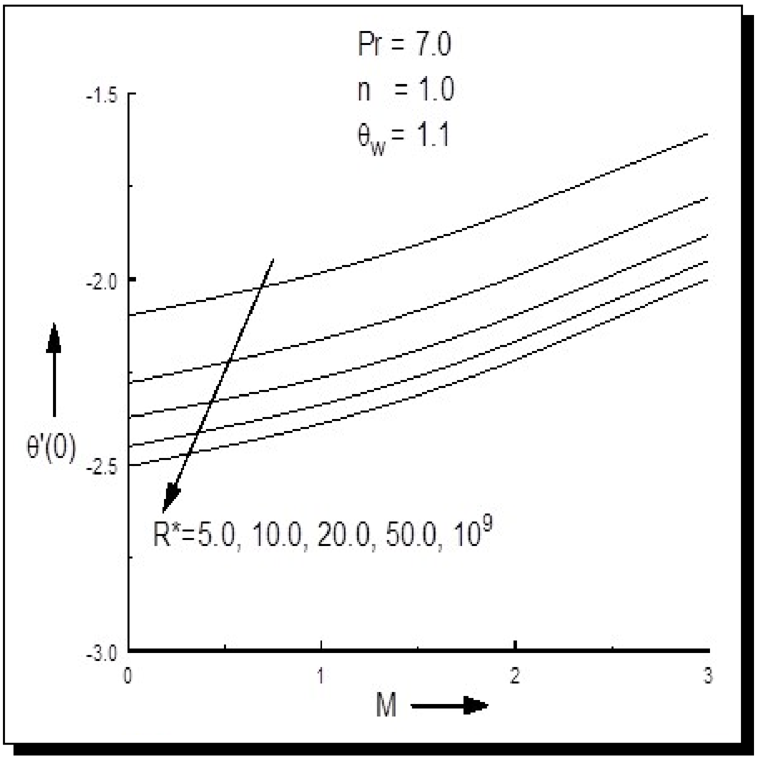


**Figure 10.** Dimensionless rate of heat transfer for different $R^*$

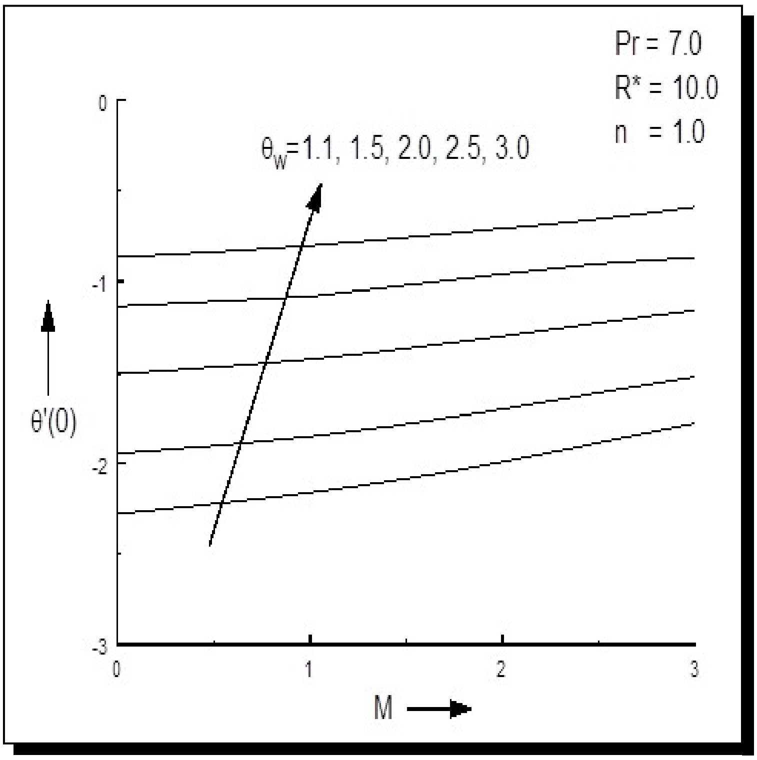


**Figure 11.** Dimensionless rate of heat transfer for different $\theta_w$

## 5. Conclusion

The following conclusions are drawn from the foregoing results and discussions:

- The effect of non-linear radiation contributes to the enhancement of thermal boundary layer drastically due to the presence of the surface temperature parameter $\theta_w$.
- It is found that the effect of magnetic field is to decelerate the velocity due to the Lorentz force and increase the thermal boundary layer.
- The effect of velocity exponent parameter is to decreases the velocity and reduces the skin friction.
- The effect of nonlinear radiation is to reduce the temperature and the rate of heat transfer is to increase for increasing nonlinear radiation.
- The thermal boundary layer thickness increases sharply with increasing surface temperature parameter and rate of heat transfer decreases sharply with increasing surface temperature parameter.
- It is found that the boundary layer thickness decreases for increasing Prandtl number.
- The rate of heat transfer increases with a decrease in $n$ and magnetic interaction parameter.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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