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Research Article

k-Odd Edge Mean Labeling of Some Basic Graphs

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Abstract. A (p,q) graph G is said to have a k-odd edge mean labeling $(k \geq 1)$, if there exists an injection f from the edges of G to $\{0,1,2,3,\dots,2k+2p-3\}$ such that the induced map f^* defined on V by $f^*(v) = \left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1, 2k+1, 2k+3, \dots, 2k+2p-3\}$. A graph that admits a k-odd edge mean labeling is called a k-odd edge mean graph. In this paper, we have introduced k-odd edge mean labeling and we have investigated the same labeling for basic graphs like path and star. Also we have examined the existence and non existence of cycles.

Keywords. k-Odd edge mean labeling; k-Odd edge mean graph

MSC. 05C78

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1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [5]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges). Then the labeling is called a vertex labeling (or an edge labeling). Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [7]. Mean labeling of graphs was discussed [8]. Manickam and Marudai [6] introduced the concept of odd mean graphs. Gayathri and Amuthavalli [1] extended this concept to k-odd mean labeling and (k,d)-odd mean labeling graphs.

In this paper, we have introduced the concept of k-odd edge mean labeling.

***For brevity,** we use k-**OEML** for k-**odd edge mean labeling** and k-**OEMG** for k-**odd edge mean graph**.*

2. Definition

Definition 2.1. *A (p,q) graph G is said to have a k-odd edge mean labeling $(k \geq 1)$, if there exists an injection f from the edges of G to $\{0,\dots 1,2,3,\dots,2k+2p-3\}$ such that the induced map f^* defined on V by $f^*(v) = \left\lceil \frac{\sum f(vu)}{\deg(v)} \right\rceil$ is a bijection from V to $\{2k-1,2k+1,2k+3,\dots,2k+2p-3\}$. A graph that admits a k-odd edge mean labeling is called a k-odd edge mean graph.*

3. Main Results

Theorem 3.1. *The path P_n is a k-odd edge mean graph for any k and $n \neq 4$.*

Proof. Let $V(P_n) = \{v_i,\ 1 \leq i \leq n\}$ and $E(P_n) = \{e_i,\ 1 \leq i \leq n-1\}$.

First, we label the edges as follows:

Define $f : E \to \{0,1,2,3,\dots,2k+2p-3\}$ by

$$f(e_1) = 2k-1,$$
$$f(e_i) = 2k+2i-2, \quad \text{for } 1 \leq i \leq n-3,$$
$$f(e_{n-2}) = 2k+2n-7,$$
$$f(e_{n-1}) = 2k+2n-3.$$

Then the induced vertex labels are

$$f^*(v_i) = 2k+2i-3, \quad \text{for } 1 \leq i \leq n,$$
$$f^*(v) = \{2k-1,2k+1,2k+3,\dots,2k+2p-3\}.$$

The above-defined function f provides k-odd edge mean labeling of P_n $(n \neq 4)$.
So, the path P_n is a k-odd edge mean graph for any k and $n \neq 4$. □

Theorem 3.2. *The Star S_{2n} $(n \geq 2$ is a k-odd edge mean graph for any k.*

Proof. Let $V(S_{2n}) = \{u, v_1, v_2, v_3, \dots, v_{2n}\}$ and $E(S_{2n}) = \{e_i = uv_i, 1 \le i \le 2n\}$.

First, we label the edges as follows:

Define $f : E \to \{0, 1, 2, 3, \dots, 2k + 2p - 3\}$ by

$$f(e_i) = \begin{cases} 2k + 2i - 3, & \text{for } 1 \le i \le n, \\ 2k + 2i - 1, & \text{for } n + 1 \le i \le 2n. \end{cases}$$

Then the induced vertex labels are

$$f^*(u) = 2k + 2n - 1,$$

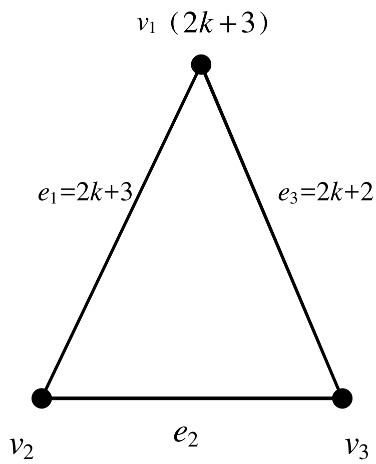
$$f^*(v_i) = \begin{cases} 2k + 2i - 3, & \text{for } 1 \le i \le n, \\ 2k + 2i - 1, & \text{for } n + 1 \le i \le 2n, \end{cases}$$

$$f^*(v) = \{2k - 1, 2k + 1, 2k + 3, \dots, 2k + 2p - 3\}.$$

The above-defined function f provides k-odd edge mean labeling of the graph.
So, the star S_{2n} $(n \ge 2)$ is a k-odd edge mean graph for any k. □

Theorem 3.3. *C_3 is not a k-odd edge mean graph for any k.*

Proof. On the contrary, suppose C_3 is a k-odd edge mean graph for some k.
Then the labels of the vertices take the values in the set $\{2k - 1, 2k + 1, 2k + 3\}$ and the labels of the edges take the values in the set $\{0, 1, 2, 3, \dots, 2k + 3\}$.



Case (i): $v_2 = 2k - 1$.
By the definition of k-OEML, the label of v_3 must be $2k + 1$ and

$$\left\lceil \frac{e_1 + e_2}{2} \right\rceil = v_2.$$

Sub-case (i): $e_1 + e_2$ is even.

In this sub case the edge e_2 gets the label $2k-5$.

Therefore,

$$\left\lceil \frac{e_2+e_3}{2} \right\rceil = 2k-1.$$

Thus the vertex labels of v_2 and v_3 are identical which contradict the fact that f^* is a bijection.

Sub-case (ii): e_1+e_2 is odd.

In this sub case the edge e_2 gets the label $2k-6$.

Therefore,

$$\left\lceil \frac{e_2+e_3}{2} \right\rceil = 2k-2$$
$$\Rightarrow \quad f^*(v_3) \text{ is even for any } k$$

which is a contradiction to C_3 is a k-odd edge mean graph for some k.

Case (ii): $v_2 = 2k+1$.

By the definition of k-OEML, the label of v_3 must be $2k+1$ and

$$\left\lceil \frac{e_1+e_2}{2} \right\rceil = v_2.$$

Sub-case (i): e_1+e_2 is even.

In this sub case the edge e_2 gets the label $2k-1$.

Therefore,

$$\left\lceil \frac{e_2+e_3}{2} \right\rceil = 2k+1$$

Thus the vertex labels of v_2 and v_3 are identical which contradict the fact that f^* is a bijection.

Sub-case (ii): e_1+e_2 is odd.

In this sub case the edge e_2 gets the label $2k-2$.

Therefore,

$$\left\lceil \frac{e_2+e_3}{2} \right\rceil = 2k$$
$$\Rightarrow \quad f^*(v_3) \text{ is even for any } k$$

which is a contradiction to C_3 is a k-odd edge mean graph for some k.

So in all the cases, C_3 cannot be k-odd edge mean graph for any k. □

Theorem 3.4. *The cycle C_n $(n \neq 6,7)$ is k-odd edge mean graphs for any k.*

Proof. Let $V(C_n) = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E(C_n) = \{e_i = (v_i v_{i+1})$ for $1 \le i \le n-1$ and $e_n = (v_n v_1)\}$.

Case (i): $n = 4$.

Define $f : E \to \{0,1,2,3,\dots,2k+2p-3\}$ by

$$f(e_1) = 2k,$$
$$f(e_2) = 2k+5,$$
$$f(e_3) = 2k+4,$$
$$f(e_4) = 2k-2.$$

Then the induced vertex labels are

$$f^*(v_1) = 2k-1,$$
$$f^*(v_2) = 2k+3,$$
$$f^*(v_3) = 2k+1,$$
$$f^*(v_4) = 2k+5,$$
$$f^*(v) = \{2k-1, 2k+1, 2k+3, \dots, 2k+2p-3\}.$$

Case (ii): $n = 5$.

We label the edges for the cycle C_5 as follows:

$$f(e_1) = 2k,$$
$$f(e_2) = 2k+6,$$
$$f(e_3) = 2k+7,$$
$$f(e_4) = 2k+3,$$
$$f(e_5) = 2k-2.$$

Then the induced vertex labels are

$$f^*(v_i) = 2k = 4i-5, \quad 1 \le i \le 3,$$
$$f^*(v_4) = 2k+5,$$
$$f^*(v_5) = 2k+1.$$

Case (iii): $n \ge 4$ is even.

We label the edges for the cycle C_{2n} as follows:

$$f(e_1) = 2k,$$
$$f(e_i) = 2k+4i-3, \quad 2 \le i \le n,$$
$$f(e_{n+1}) = 2k+4n-4,$$
$$f(e_{n+2}) = 2k+4n-10,$$
$$f(e_i) = 2k+8n-4i-1, \quad n+3 \le i \le 2n-1,$$
$$f(e_{2n}) = 2k-2.$$

Then the induced vertex labels of C_{2n} are

$$f^*(v_1) = 2k-1,$$
$$f^*(v_i) = \begin{cases} 2k+4i-7, & 2 \le i \le n+1, \\ 2k+8n-4i+1, & n+2 \le i \le 2n-1, \end{cases}$$
$$f^*(v_{2n}) = 2k+1.$$

Case (iv): $n \ge 4$ is odd.

We label the edges for the cycle C_{2n+1} as follows:

$$f(e_1) = 2k-2,$$
$$f(e_i) = 2k+4i-5, \ \ 2 \le i \le 2n-6,$$
$$f(e_{2n-5}) = 2k+8n-30,$$
$$f(e_{2n-4}) = 2k+8n-36,$$
$$f(e_i) = 2k+16n-4i-51, \ \ 2n-3 \le i \le 2n,$$
$$f(e_{2n+1}) = 2k.$$

Then the induced vertex labels of C_{2n+1} are

$$f^*(v_1) = 2k-1,$$
$$f^*(v_i) = \begin{cases} 2k+4i-7, & 2 \le i \le n+1, \\ 2k+12n-4i-21, & n+2 \le i \le 2n+1. \end{cases}$$

Thus in all the cases

$$f^*(v) = \{2k-1, 2k+1, 2k+3, \dots, 2k+2p-3\}.$$

It follows that the vertex labels are all distinct and odd.

Hence, the cycle C_n $(n \ne 6,7)$ is k-odd edge mean graphs for any k. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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