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Study of Three Level Cascade System: A Complete Analytical Approach

Research Article

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Abstract. A density matrix based complete analytical solution for the probe transmission through a three-level cascade type Doppler free atomic system is obtained. The probe beam couples the ground level to the energy level lying at the middle whereas the control field is acting between the level at the middle and the upper most energy level. The analysis is valid for comparable values of probe and control Rabi frequencies too. The probe response shows EIT window when Raman condition is satisfied. The variation of the EIT width with control Rabi frequency under Doppler free condition is studied using the parameters of $5S_{1/2} \rightarrow 5P_{3/2} \rightarrow 5D_{5/2}$ transitions of ^{87}Rb. At high value of the probe field intensity compared to that of the control field, enhancement in probe absorption is noticed. The increase of the EIT width with control Rabi frequency under Doppler free situation is examined.

Keywords. Three-level; Cascade; Density matrix; Probe response; EIT

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1. Introduction

The study of *electromagnetically induced transparency* (EIT) under various level schemes is being conducted for over two decades now. Apart from academic point of view, important practical applications of EIT in manipulating the velocity of probe pulse through an otherwise absorptive medium and thereby the idea of optical delay generators, stopping and storage of light [1–4], optical switching [5], lasing without inversion [6, 7] and many more have attracted the attention of researchers worldwide. The physical mechanism producing EIT window under Raman resonance condition has been explained by several researchers and some excellent review articles too are available [8, 11]. Instead of explaining the mechanism once again, we shall present here some analytic results obtained for a three level cascade type system studied by using density matrix formalism under Doppler free condition. The analysis is complete in the sense that all terms in any power of control and probe Rabi frequencies have been retained in the analysis and thereby the final result is valid for arbitrary control and probe power ratio. In fact the transmission of a coherent laser field through an absorbing atomic vapour medium in presence of another coherent field can be studied by using this model without putting any restriction on the ratio of the intensities of the laser fields.

2. Theoretical Model and Simulation

In this section we shall discuss the level scheme and then present the analytical model used to solve for the probe transmission through an atomic vapour cell. For simplicity we shall consider a three-level system where we have one ground level $|1\rangle$, one intermediate energy level $|2\rangle$ and an uppermost energy level $|3\rangle$ as shown in the Figure 1.

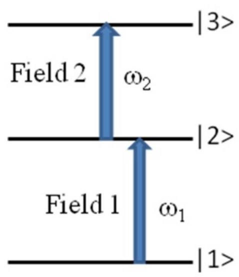

Figure 1. (Colour online) Schematic diagram of a three-level cascade type system. Field 1 is acting between the ground level $|1\rangle$ and the intermediate level $|2\rangle$. Field 2 is coupling the intermediate level $|2\rangle$ with the uppermost level $|3\rangle$.

The transitions $|1\rangle \to |2\rangle$ and $|2\rangle \to |3\rangle$ are dipole allowed but the transition $|1\rangle \to |3\rangle$ is dipole forbidden. Hence the population from the level $|2\rangle$ can decay spontaneously (radiative decay) to the ground state $|1\rangle$ and that from level $|3\rangle$ can come to level $|2\rangle$ through spontaneous emission. But the population from $|3\rangle$ cannot come directly to $|1\rangle$ through radiative decay (dipole forbidden). As an example we shall consider the $5^2S_{1/2} \to 5^2P_{3/2} \to 5^2D_{5/2}$ transition of ^{87}Rb and the corresponding spontaneous decay rates. A coherent radiation field of frequency ω_1 and Rabi frequency Ω_1 is acting between the levels $|1\rangle$ and $|2\rangle$ whereas another coherent radiation field of frequency ω_2 and Rabi frequency Ω_2 is coupling level $|2\rangle$ with level $|3\rangle$. The interaction Hamiltonian is given by

$$H_I = \frac{\hbar}{2}[(|2\rangle\langle 1|\Omega_1 \exp(-i\omega_1 t) + c.c.) + (|3\rangle\langle 2|\Omega_2 \exp(-i\omega_2 t) + c.c.)] \tag{2.1}$$

Here we have used subscripts 1 and 2 instead of the usual procedure of designating the probe and control field frequencies by subscripts 'p' and 'c' respectively since the two fields coupling the level $|1\rangle$ to the level $|2\rangle$ and the level $|2\rangle$ to the level $|3\rangle$ can be assigned comparable intensities according to the non-perturbative solution obtained at the end of the analysis, making the names such as probe and control fields meaningless. As per standard practice the probe field should have much smaller intensity (by a factor of ten at least) compared to the control field, but we shall discuss the case when the coherent field coupling the levels $|1\rangle$ and $|2\rangle$ will have comparable or greater intensity than the other coherent field coupling the levels $|2\rangle$ and $|3\rangle$ at the later part of this article. Hence we shall denote the radiation field acting between the levels $|1\rangle$ and $|2\rangle$ as field 1 and the other field connecting the level $|2\rangle$ with level $|3\rangle$ will be known as field 2. A set of nine density matrix based equations for the population terms (ρ_{ii}; $i = 1,2,3$) and the coherence terms (ρ_{ij}; $i, j = 1,2,3$; $i \neq j$) are derived and then solved analytically under the steady state condition ($\frac{\partial \rho}{\partial t} = 0$) following the rotating wave approximation [10] by using the boundary condition $\rho_{11} + \rho_{22} + \rho_{33} = 1$. The expression for the coherence between the levels $|2\rangle$ and $|1\rangle$ is given by:

$$\rho_{21} = \Omega_2^2 \Omega_1 \frac{(\alpha_2 - \imath\beta_2)(\rho_{22} - \rho_{33})}{2(\alpha_2^2 + \beta_2^2)} - \Omega_1 \frac{[(\alpha_1\alpha_2 + \beta_1\beta_2) + \imath(\beta_1\alpha_2 - \beta_2\alpha_1)](\rho_{11} - \rho_{22})}{2(\alpha_2^2 + \beta_2^2)} \tag{2.2}$$

where α_is and β_is are complicated functions involving the detunings and Rabi frequencies of the field 1 and field 2, decay rates of the excited states etc. The real and imaginary parts of the above expression for ρ_{21} are required to determine the probe dispersion (η_1) and probe absorption (η_2) as given below:

$$\eta_1 = \frac{4\pi\omega_1}{c} \mathrm{Re}(\rho_{21}), \tag{2.3}$$

$$\eta_2 = \frac{4\pi\omega_1}{c} \mathrm{Im}(\rho_{21}). \tag{2.4}$$

In the simulation of the theoretical expression we have not considered thermal averaging. The decay rate (γ_{21}) from level $|2\rangle$ to level $|1\rangle$ has been taken to be 6 MHz [11] whereas the decay rate (γ_{32}) of the atoms from level $|3\rangle$ to level $|2\rangle$ has been assumed to be 4.19 MHz [12]. Analytical expressions for the level populations have been obtained in terms of the detunings, Rabi frequencies and decay rates, thereby eliminating the need of providing approximate numerical values for the population terms in the simulation. This removes error related to the population terms in the final result.

We shall first present the plot of the transmission of the field 1 through atomic vapour medium versus its frequency detuning (Δ_1) at different values of the Rabi frequency (Ω_2) of the field 2. We have kept the value of the Rabi frequency (Ω_1) of the field 1 fixed at 1 MHz in the entire simulation process. The so called control field i.e. the field 2 in our case is kept on-resonance with the $|2\rangle \to |3\rangle$ transition. At and around the zero detuning of the field 1 ($\Delta_1 = 0$), an *electromagnetically induced transparency* (EIT) window appears. It is seen that the width of the EIT window increases with the increase in the value of the Rabi frequency (Ω_2) of the field 2.

It is clear from Figure 2 that the separation between the two AT (Autler-Townes) like peaks [13] appearing on either side of the transparency window is equal to the corresponding

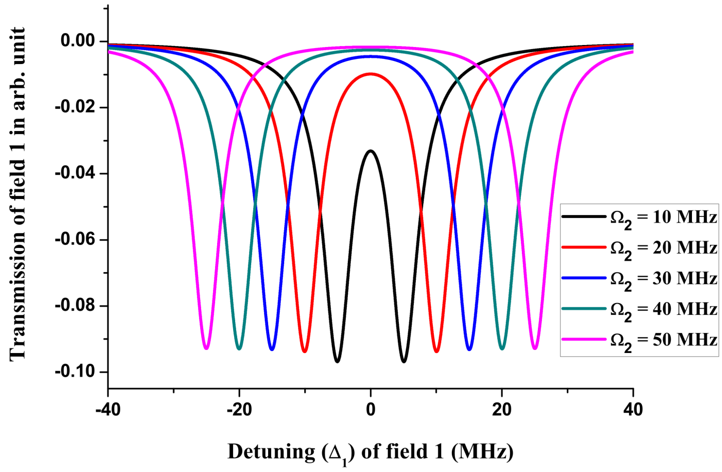

Figure 2. (Colour online) Plot of transmission of field 1 versus its detuning (Δ_1) at different values of the Rabi frequency (Ω_2) of field 2 as shown in the figure. The Rabi frequency (Ω_1) of the field 1 is kept constant at 1 MHz.

Rabi frequency of the field 2. As for example, when the Rabi frequency of the field 2 is set at 40 MHz, the separation between the above mentioned peaks has been found from the figure to be equal to 40.0 MHz. The two peaks are identical in terms of width, shape and size for fixed set of Rabi frequencies of the two radiation fields although occurrence of asymmetric EIT peaks have been reported earlier [13]. The reason behind the asymmetry has also been discussed [13]. The transparency almost becomes 100% when the Rabi frequency of the field 2 is set equal to 50 MHz (Figure 2). Let us now compare this complete analytical result with the approximate result obtained for weak probe transmission through a vapour medium [13] using similar level scheme. In the Figure 3 below we have shown the transmission of the field 1 through the vapour medium at the Rabi frequency (Ω_2) of the field 2 of 40 MHz by keeping the Rabi frequency of the field 1 fixed at 1 MHz. The reason for the deviation in the peak size is the use of different normalization constant [13] although we have used identical values of the Rabi frequencies as well as the decay rates for both the expressions (eq. (2.1) of this article and eq. (5) of ref. [13]) in the simulation. The widths of the AT peaks (appearing on either sides of the transparency window and shown in the Figure 3) obtained from the two different treatments (ours and ref. [13]) are 5.228 MHz and 5.028 MHz respectively when a Lorentzian profile was used to fit them. This 3.825% deviation in these two results is due to the inclusion of all the terms involving the Rabi frequencies of the two coherent fields interacting with the atoms in our theoretical model. The omission of higher order terms of the probe Rabi frequency in the perturbative treatment [13] leads to some error.

Let us now concentrate into the situation when the Rabi frequency of the field 1 is increased from lower value to a value comparable to that of the field 2 and then increased further. The complete analytical expression used in this article gave us the unique advantage of comparing the transmission of the field 1 when its intensity is comparable or even greater than the intensity of the field 2. The approximate analytical models do not support such flexibility and requires

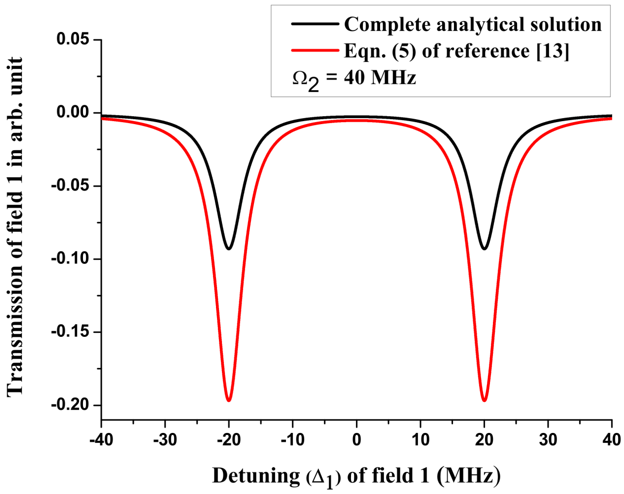

Figure 3. (Colour online) Plot of transmission of the filed 1 as a function of its frequency detuning (Δ_1) at $\Omega_2 = 40$ MHz. The Rabi frequency (Ω_1) of field 1 is kept constant at 1 MHz.

recalculation of the coherence terms using the opposite arguments regarding the strengths of the coupling fields. We observe that the transmission of the filed 1 is reduced appreciably (absorption is increased) when the Rabi frequencies of both the fields are made equal to 5 MHz but the EIT window still appears in the background, although reduced in depth as compared to the case when the Rabi frequency of the field 1 is set equal to 1 MHz. This is unique in the sense that on fulfilment of the Raman resonance condition [8] we still get EIT in the response curve of field 1 at equal values of the Rabi frequencies of the field 2 (control) and field 1 (probe), with the distinction that the EIT width is reduced with the increase in Rabi frequency of the field 1 at the cost of lower EIT depth. This is shown in the Figure 4. It also displays that with further increase in the Rabi frequency of the field 1 it starts assuming the role of the control field and no EIT could be observed when the value of the Rabi frequency of the field 1 is made equal to 10 MHz or more. This is what one can expect under such circumstances. The Rabi frequency of the field 2 is kept at 5 MHz during the entire simulation process.

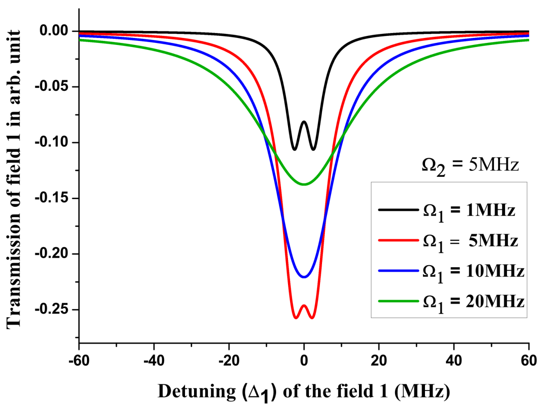

Figure 4. (Colour online) Plot of transmission versus detuning of the field 1 at different values of its Rabi frequency (Ω_1). The Rabi frequency (Ω_2) of the field 2 is set equal to 5 MHz in the simulation.

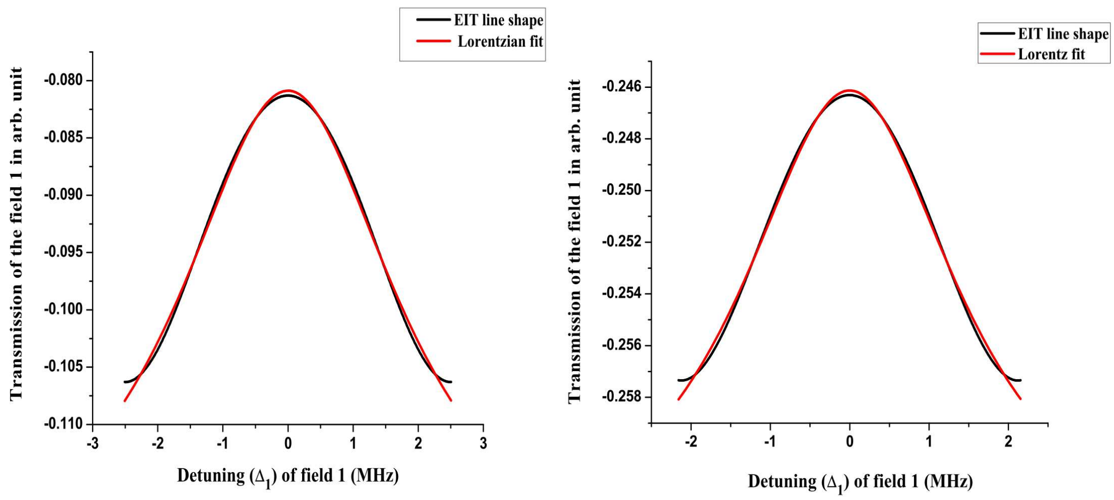

Fig.5a **Fig.5b**

Figure 5. (Colour online) Plot of the EIT windows at two different values of Rabi frequency of field 1 $\Omega_1 = 1$ MHz (Figure 5a) and $\Omega_1 = 5$ MHz (Figure 5b). The Rabi frequency Ω_2 of field 2 is kept constant at 5 MHz. The fitted curves appear in red colour in both the figures.

In Figures 5a,b we have presented the result we got after fitting the EIT window with Lorentzian line shape. It is seen that with increase in the Rabi frequency of the filed 1, the width as well as height of the EIT window get lowered. For two coherent radiation fields of equal intensities acting on a cascade type system, EIT window still appears prominently on the background of the enhanced absorption profile of the field acting from the ground level (field 1 in this case).

The fitted widths of the EIT windows, shown in Figure 5a and Figure 5b, are 4.179 MHz and 3.478 MHz corresponding to Rabi frequencies of 1 MHz and 5 MHz of field 1. The Rabi frequency of field 2 is held constant at 5 MHz. The reduction in the EIT depth is visible from the y-axis scaling of the two plots.

3. Conclusion

We have studied the conventional three-level cascade type system by using density matrix method in a Doppler-free medium as is the case with ultra cold atomic ensemble. The analytical solution for the transmission of the field 1 (usually the field that is kept at a lower power and termed as the probe field) presented here is unique in the sense that we have not neglected any term in our analysis, thereby no restriction on the ratio of the intensities of the two radiation fields acting on the system has been imposed. This allows us to study the case when the intensities of the fields are comparable as well as when the Rabi frequency of the conventional probe field (field 1 in this article) is increased above the value of the Rabi frequency of the so called pump or control field (field 2). The analysis also revealed quantitatively the difference in the widths of the EIT windows as compared to the result obtained by perturbation technique that is usually used. The occurrence of EIT under equally intense coherent fields has been confirmed and the width of the EIT window has been obtained through fit. It is an interesting observation that the so called control field causes enhanced absorption depth (of the probe field) under this condition but still EIT can be produced. The population decay rates used in the simulation are all practical values taken from the literature and no arbitrary or approximate values of populations of different levels have been assigned. The result of this study will be helpful to the researchers especially to the experimentalists who wish to explore the response of an atomic vapour medium to the transmission of two resonant coherent radiation fields of comparable intensities. We shall soon report the result of a similar study in a Doppler broadened medium.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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