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Resonant Ion Beam Interaction with Whistler Waves in A Magnetized Dusty Plasma

Research Article

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Abstract. The theory of whistler wave interaction with an ion beam injected parallel to the magnetic field in an unbounded plasma is considered. The excited whistler waves propagate parallel to the beam direction and their phase velocity is a characteristic of beam-whistler resonant cyclotron coupling. The frequency and the growth rate of the unstable wave increase with the relative density of negatively charged dust grains. The ion beam velocity responsible for maximum growth rate increases as the charge density carried by dust increases. The maximum value of growth rate increases with the beam density and is proportional to the square root of beam density. These results should shed light on mechanisms of whistler wave excitation in space plasmas by artificial beams injected from spacecraft in the ionosphere and the magnetosphere.

Keywords. Whistler wave; Ion beam; Dispersion relation; Threshold velocity; Growth rate.

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1. Introduction

A whistler wave is generated by a thunderstorm or lightning, and is rich in low frequency components. It propagates through the ionosphere along the Earth's magnetic field, and gets dispersed in such a way that the higher frequencies move faster than the lower ones. At various locations on the Earth there are stations that continuously record sonograms of whistler activity and provide a spectrum of the wave frequency versus time of arrival. These sonograms are used as an effective diagnostic tool for studying the ionospheric conditions. The whistler waves can be driven by electron temperature anisotropy or by charged particle beams [1–10]. A large number of papers have appeared regarding the theory of whistler excitation by a pulsed or modulated thin beam injected parallel to the magnetic field in an unbounded homogeneous magnetoplasma. Experiments performed with charged particle beams, modulated or unmodulated, have exhibited Cerenkov and cyclotron emission of whistler waves [11–13]. The whistler instability driven by an ion beam or by a ring beam is a consequence of the interaction between the solar wind and newborn ions of planetary, interstellar or cometary origin [14]. Krafft et al. [4] have studied emission of whistler waves by a density-modulated electron beam in a laboratory plasma and results have been compared to the excitation by loop antenna. Krafft et al. [12] have studied whistler wave excitation in a magnetized laboratory plasma by a density-modulated electron beam for frequency modulation below but in the range of electron cyclotron frequency. In this case, the maximum emission of the whistler waves occurred when the phase velocity of the whistler wave was equal to the beam velocity.

There has been a great deal of interest in studying waves and instabilities in dusty plasma. Dusty plasmas are found in space environments such as the lower ionosphere of the Earth, planetary atmospheres, asteroid zones, nebulae, and tails as well as in a variety of low temperature plasma devices. The presence of dust grains modifies the properties of waves in plasma [15–20]. Barkan et al. [15] have reported experimental results on the current driven electrostatic ion-cyclotron instability in a dusty plasma, where they found that the presence of negatively charged dust grains enhanced the growth rate of the instability. D'Angelo [16] has investigated the dispersion relation for low-frequency electrostatic waves in a magnetized dusty plasma. In the presence of negatively charged dust grains, he has found that the mode frequencies increased as the density ratio of negatively charged dust grains to positive ions is increased. Merlino et al. [17] have presented theoretical and experimental results on low frequency electrostatic waves in a plasma containing negatively charged dust grains and found that the presence of negatively charged dust grains modifies the properties of current driven electrostatic ion cyclotron instability through the quasineutrality condition even though the dust grains do not participate in the wave dynamics.

In this paper, we present the results of whistler wave interaction with an ion beam injected parallel to the external static magnetic field in a dusty magnetized plasma. In Section 2, we study the plasma electron, beam ion and dust grain response to whistler wave perturbation. We obtain the dispersion relation and growth rate of excited whistlers using first order perturbation theory. The variation of the growth rate of the unstable mode as a function of relative density $\delta\left(=\frac{n_{i0}}{n_{e0}}\right)$, (where n_{i0} is the ion plasma density and n_{e0} is the electron plasma density) of negatively charged dust grains has been discussed in slow cyclotron interaction. Results and discussions are given in Section 3 and conclusions are given in Section 4.

2. Instability Analysis

Consider a plasma with equilibrium electron, ion and dust particle densities as n_{e0}, n_{i0} and n_{d0}, respectively, immersed in a static magnetic field B_s in the z-direction. An ion beam with velocity $v_{b0}\hat{z}$ and density n_{b0} propagates through the plasma. In equilibrium, there is overall charge neutrality, i.e.,

$$en_{i0} + en_{b0} = en_{e0} + Q_{d0}n_{d0}.$$

We assume the t, z variations of whistler fields as $\boldsymbol{E},\boldsymbol{B} \sim \exp[-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})]$ and consider E field to be polarized in x-z plane,

where $\boldsymbol{k} = k_x\hat{x} + k_z\hat{z}$ and $\omega_{ci} \ll \omega \ll \omega_{ce}$, ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies respectively.

The magnetic field of the wave is $\boldsymbol{B} = \frac{c}{\omega}\boldsymbol{k}\times\boldsymbol{E}$.

The equation of motion for the perturbed beam electrons is

$$\frac{\partial v}{\partial t} + (v\cdot\nabla)v = \frac{e}{m}\left(E + \frac{1}{c}v\times B_s + \frac{1}{c}v\times B\right), \tag{2.1}$$

where $v = v_{b0}\hat{z} + v_{b1}$, v_{b1} refers to perturbed velocity.

On linearizing Eq. (2.1), we obtain

$$\frac{\partial v_{b1}}{\partial t} = \frac{eE}{m} + v_{b1}\times\hat{z}\omega_{ci}. \tag{2.2}$$

Writing x, y, z-components of Eq. (2.2), we obtain the perturbed beam electron velocities as

$$v_{b1x} = \frac{ie}{m}\frac{\bar{\omega}^2}{\omega\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_x + \frac{e}{m}\frac{\bar{\omega}\omega_{ci}}{\omega\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_y, \tag{2.3}$$

$$v_{b1y} = -\frac{e}{m}\frac{\bar{\omega}\omega_{ci}}{\omega\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_x + \frac{ie}{m}\frac{\bar{\omega}^2}{\omega\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_y, \tag{2.4}$$

$$v_{b1z} = \frac{ie}{m\bar{\omega}}E_z, \tag{2.5}$$

where $\bar{\omega} = \omega - k_z v_{bo}$.

Substituting the perturbed beam velocities from Eqs. (2.3)-(2.5) in the equation of continuity, we obtain the perturbed ion beam density as

$$n_{b1} = -\frac{n_{bo}ek_z}{im\bar{\omega}^2}E_z. \tag{2.6}$$

The response of the plasma electrons can be obtained from Eqs. (2.3)-(2.6) by replacing e, m, ω_{ci}, n_{b0} by $-e$, m_e, $-\omega_{ce}$, n_{e0} respectively and putting $v_{bo} = 0$:

$$v_{e1x} = -\frac{e\left(i\omega E_x + \omega_{ce}E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2.7}$$

$$v_{e1y} = \frac{e\left(\omega_{ce}E_x - i\omega E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2.8}$$

$$v_{e1z} = \frac{eE_z}{m_e i\omega}. \tag{2.9}$$

The perturbed electron current density

$$J_{e1} = -n_{e0} e v_{e1}. \tag{2.10}$$

Substituting Eqs. (2.7), (2.8) and (2.9) in Eq. (2.10), we get

$$J_{e1x} = n_{eo}\frac{e^2}{m_e}\frac{\left(i\omega E_x + \omega_{ce}E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2.11}$$

$$J_{e1y} = -n_{eo}\frac{e^2}{m_e}\frac{\left(\omega_{ce}E_x - i\omega E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2.12}$$

$$J_{e1z} = -n_{eo}\frac{e^2 E_z}{m_e i\omega}. \tag{2.13}$$

The response of the dust grains can also be obtained from Eqs. (2.11)-(2.13) by replacing e, m_e, ω_{ce}, n_{e0} by Q_{d0}, m_d, ω_{cd}, n_{d0} respectively and putting $v_{bo} = 0$:

$$J_{d1x} = n_{do}\frac{Q_{d0}^2}{m_d}\frac{\left(i\omega E_x + \omega_{cd}E_y\right)}{\left(\omega^2 - \omega_{cd}^2\right)}, \tag{2.14}$$

$$J_{d1y} = -n_{do}\frac{Q_{d0}^2}{m_d}\frac{\left(\omega_{cd}E_x - i\omega E_y\right)}{\left(\omega^2 - \omega_{cd}^2\right)}, \tag{2.15}$$

$$J_{d1z} = -n_{do}\frac{Q_{d0}^2 E_z}{m_d i\omega}. \tag{2.16}$$

The perturbed beam current density is given as

$$J_{b1} = -en_{b0}v_{b1} - en_{b1}v_{b0}\hat{z}. \tag{2.17}$$

Writing the x, y and z components of Eq. (2.17) and using Eqs. (2.3)-(2.6), we obtain

$$J_{b1x} = \frac{ie^2 n_{bo}}{m\omega}\frac{\bar{\omega}^2}{\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_x + \frac{e^2 n_{bo}}{m\omega}\frac{\bar{\omega}\omega_{ci}}{\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_y, \tag{2.18}$$

$$J_{b1y} = -\frac{e^2 n_{bo}}{m\omega}\frac{\bar{\omega}\omega_{ci}}{\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_x + \frac{ie^2 n_{bo}}{m\omega}\frac{\bar{\omega}^2}{\left(\bar{\omega}^2 - \omega_{ci}^2\right)}E_y, \tag{2.19}$$

$$J_{b1z} = \frac{ie^2 n_{bo}}{m\bar{\omega}^2}\omega E_z. \tag{2.20}$$

The wave equation which governs the mode structure of low frequency whistler waves is given as

$$\nabla^2 E - \nabla(\nabla\cdot E) + \left(\frac{\omega^2}{c^2}\right)E = -\frac{4\pi i\omega}{c^2}J_1. \tag{2.21}$$

Writing the x, y and z components of Eq. (2.18), we get

$$AE_x + iBE_y = 0, \tag{2.22}$$

$$AE_y - iBE_x = 0, \tag{2.23}$$

$$CE_z = 0, \tag{2.24}$$

where

$$A = -k_z^2c^2 + \omega^2 - \frac{\omega_{pe}^2\omega^2}{(\omega^2-\omega_{ce}^2)} - \frac{\omega_{pd}^2\omega^2}{(\omega^2-\omega_{cd}^2)} - \frac{\omega_{pb}^2\bar{\omega}^2}{(\bar{\omega}^2-\omega_{ci}^2)},$$

$$B = \frac{\omega\omega_{ce}\omega_{pe}^2}{(\omega^2-\omega_{ce}^2)} + \frac{\omega\omega_{cd}\omega_{pd}^2}{(\omega^2-\omega_{cd}^2)} + \frac{\bar{\omega}\omega_{ce}\omega_{pb}^2}{(\bar{\omega}^2-\omega_{ci}^2)}, \qquad C = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2} - \frac{\omega_{pb}^2}{\bar{\omega}^2}$$

$\omega_{pb}^2 = \dfrac{4\pi n_{b0}e^2}{m}$, $\omega_{pe}^2 = \dfrac{4\pi n_{e0}e^2}{m_e}$ and $\omega_{pd} = \left(\dfrac{4\pi n_{do}Q_{do}^2}{m_d}\right)^{\frac{1}{2}}$.

A non-trivial solution of Eqs. (2.22)-(2.24) demands that the determinant of coefficients of E_x and E_y must vanish, i.e., $A^2 - B^2 = 0$.

It gives two distinct modes of wave propagation. We examine here right-hand polarized electromagnetic whistler waves where the ions' contributions have been neglected due to the range of frequencies considered, obtained from $A - B = 0$, or

$$k_z^2 - \frac{\omega^2}{c^2}\varepsilon = -\frac{\omega_{pb}^2\bar{\omega}}{c^2(\bar{\omega}+\omega_{ci})}, \tag{2.25}$$

where $\varepsilon = 1 - \dfrac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})} - \dfrac{\omega_{pd}^2}{\omega(\omega-\omega_{cd})}$.

Eq. (2.25) can be written as

$$\left(k_z - \frac{\omega}{c}\sqrt{\varepsilon}\right)\left(k_z + \frac{\omega}{c}\sqrt{\varepsilon}\right)(\bar{\omega}+\omega_{ci}) = -\frac{\omega_{pb}^2\bar{\omega}}{c^2}, \quad \text{or}$$

$$\left(k_z - \frac{\omega}{c}\sqrt{\varepsilon}\right)\left(k_z + \frac{\omega}{c}\sqrt{\varepsilon}\right)\left(k_z - \frac{\omega+\omega_{ci}}{v_{bo}}\right) = -\frac{\omega_{pb}^2}{c^2}\left(k_z - \frac{\omega}{v_{bo}}\right). \tag{2.26}$$

Eq. (2.26) gives the dispersion relation of parallel whistler waves in a dusty plasma in the absence of beam given as

$$k_z = \frac{\omega}{c}\left[1 - \frac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})} - \frac{\omega_{pd}^2}{\omega(\omega-\omega_{cd})}\right]^{\frac{1}{2}}. \tag{2.27}$$

Assuming perturbed quantities $k_z = \dfrac{\omega}{c}\sqrt{\varepsilon} + \Delta$ and $k_z = \dfrac{\omega+\omega_{ci}}{v_{bo}} + \Delta$ in Eq. (2.26), we get

$$\Delta^2 = -\frac{\omega_{pb}^2\omega_{ci}}{2c\omega\sqrt{\varepsilon}v_{bo}}.$$

Therefore, Growth rate $\gamma = \mathrm{Im}(\Delta)$ or

$$\gamma = \left[\frac{\omega_{pb}^2}{2c\omega\sqrt{\varepsilon}}\frac{\omega_{ci}}{v_{bo}}\right]^{\frac{1}{2}}. \tag{2.28}$$

It may be noted from Eq. (2.25), that Cerenkov or fast cyclotron interaction between parallel beam and parallel whistlers is not possible, as there is only slow cyclotron interaction term on right hand side of Eq. (2.25). The growth rate depends on the external static magnetic field, unstable whistler frequency, plasma electron density, dust grain density, beam velocity and beam density. It can be inferred from Eq. (2.26) that the whistlers exhibit a growth rate only if the beam velocity is greater than their phase velocity.

3. Results and Discussion

The parameters used in the present calculations are: ion plasma density $n_{i0} = 10^{12}\,\text{cm}^{-3}$, electron plasma density $n_{e0} = 10^{12} - 0.2 \times 10^{12}\,\text{cm}^{-3}$, magnetic field $B_S = 300\,\text{G}$, mass of ion $m_i = 39 \times 1836 m_e$ (potassium-plasma), mass of dust grain $m_d = 10^{12} m_i$, dust grain density $n_{d0} = 10^8\,\text{cm}^{-3}$, beam density $n_{b0} = 10^9\,\text{cm}^{-3}$ and beam velocity $v_{b0} = 1.2 \times 10^9$ cm/s. The relative density of negatively charged dust grains $\delta \left(= \dfrac{n_{i0}}{n_{e0}}\right)$ has been varied from 1 to 5. In Figure 1, we have plotted the dispersion curves of whistler waves in a dusty plasma, for different values of δ. We have also plotted the beam mode via slow cyclotron interaction for an ion beam travelling inside the dusty plasma. The velocity of the beam is chosen in such a way so that they intersect the dispersion curves of whistler waves in the required frequency range $\omega_{ci} \ll \omega \ll \omega_{ce}$. The frequencies and the corresponding wave numbers of the unstable mode obtained from the points of intersection between the beam mode and the electron whistler modes are given as Table 1. The unstable frequencies of the whistler waves in presence of dust grains and the parallel wave vector k_z decrease with increase in relative density of negatively charged dust grains δ.

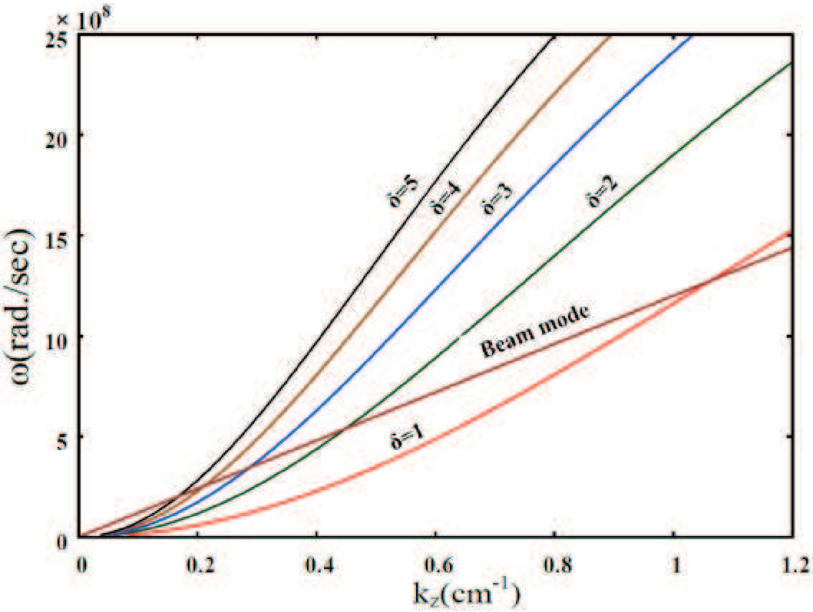

Figure 1. Dispersion curves of whistler waves for different values of $\delta \left(= \dfrac{n_{i0}}{n_{e0}}\right)$ and a beam mode via slow cyclotron interaction.

Table 1. Unstable wave frequencies ω (rad./s) and axial wave numbers k_z(cm^{-1}) for different values of δ from Figure 1.

δ	k_z (cm^{-1})	ω (rad./sec)$\times 10^8$
1	1.06	12.79
2	0.45	5.40
3	0.29	3.53
4	0.21	2.61
5	0.17	2.09

The phase velocity of the whistler waves has been plotted as a function of parallel whistler frequency in Figure 2. It can be seen that, as the whistler frequency increases, the phase velocity first increases then remains constant for certain frequency range and then decreases. The phase velocity of the whistlers is found to increase from 14.1×10^8 cm/s to 31.4×10^8 cm/s as δ increases from 1 to 5, indicating that the addition of dust in plasma effects the propagation of whistler waves.

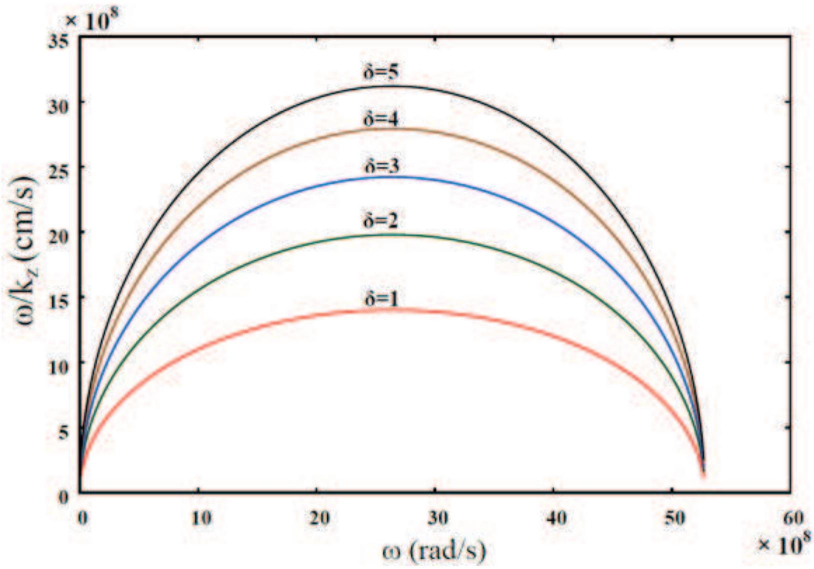

Figure 2. Phase velocity of whistler waves as a function of unstable whistler frequency for different values of δ.

When the frequency is much smaller than the electron cyclotron frequency, the phase velocity increases i.e., the higher frequencies move faster than the lower ones and ascending frequency whistlers are observed, which produce an ascending pitch whistle. When the frequency is near (but smaller than) the electron cyclotron frequency, the phase velocity starts decreasing with whistler frequency and the low frequency whistlers reach the Earth's surface before the high frequency ones, giving out a descending pitch tone. The frequency regime in which the ascending tone turns into a descending tone decreases with an increase in the value of δ. The phase velocity for all the values of δ shows maxima at a whistler frequency of 26.3×10^8 rad/s.

The growth rate of the whistler waves has been plotted using Eq. (2.28), as a function of δ in Figure 3. The unstable wave frequencies and wave numbers of the whistler waves for plotting the growth rate of Figure 3, have been used from Table 1. From Figure 3, it can be seen that the growth rate of the unstable mode increases with δ. The growth rate of the unstable mode also increases with the beam density and is proportional to the square root of beam density. The beam velocity needed to excite the whistler instability increases with an increase in the value of external static magnetic field. It also depends on the plasma density, and a higher beam velocity is required for interaction with whistler waves in denser plasma.

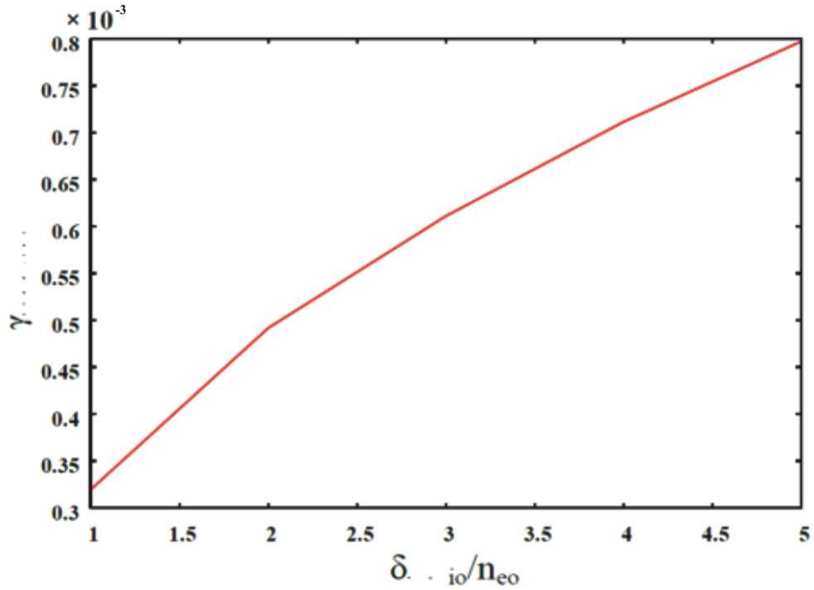

Figure 3. Growth rate δ (rad./sec) of the unstable whistler mode in a dusty plasma via slow cyclotron interaction as a function of δ $\left(=\frac{n_{i0}}{n_{e0}}\right)$.

4. Conclusion

In this paper, we study the excitation of whistler waves via slow cyclotron interaction with an ion beam in magnetized plasma. The dispersion relation of the whistler modes has been derived for beam-whistler interaction and the standard relation is retrieved in the absence of beam. Cerenkov and fast cyclotron interaction between ion beam and parallel whistlers has not been observed analytically. The waves show a spatiotemporal growth rate, which depends o n the beam number density, beam velocity, plasma number density and external static magnetic field.

The main physical process occurring during beam-wave interaction is that the beam ions bunch along the magnetic field, which are continuously accelerated or decelerated while keeping resonance with the emitted wave. The bunches are the main cause which support the wave emission whereas the nonresonant beam ions practically do not exchange energy with the wave. All the loss of the resonant beam particles' energy is transformed into emitted wave energy and the wave grows. The unstable frequency and the growth rate are sensitive to the beam velocity. The results presented here should be useful for the understanding of wave activities in space and laboratory plasmas.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed equally and significantly in writing this article. All the authors read and approved the final manuscript.

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