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Excitation of Electromagnetic Surface Waves at A Conductor-Plasma Interface by An Electron Beam

Research Article

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Abstract. Electromagnetic surface waves are driven to instability on a conductor plasma interface via Cerenkov and fast cyclotron interaction by an electron beam. A dispersion relation and the growth rate of the instability for this process has been derived. Numerical calculations of the growth rate and unstable mode frequencies have been carried out for the typical parameters of the surface plasma waves. The plasma and beam responses are obtained using fluid treatment and the growth rate is obtained using the first-order perturbation theory. The growth rate increases with the beam density and scales as one-third power of the beam density in Cerenkov interaction and is proportional to the square root of beam density in fast cyclotron interaction. In addition, the real frequency of the unstable wave increases with the beam energy and scales as almost one half power of the beam energy. The effect of the plasma parameters and the strength of the external magnetic field on unstable frequencies and growth rates are analyzed.

Keywords. Surface plasma wave; Electron beam; Instability; Dispersion relation; Growth rate

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1. Introduction

Propagation of surface waves on the interface between two media derives attention due to its technological applications as well as its relation to astrophysical problems [1]. A surface plasma wave is an electromagnetic wave that propagates at the boundary between two media with different conductivities and dielectric properties such as a conductor-plasma boundary, only if the permittivity of one of the media is negative or has a nonzero negative part. Trivelpiece and Gould [2] first reported the experimental observations of surface plasma waves using a cylindrical plasma column enclosed in a glass tube that was coaxial with a circular metallic waveguide. These waves in plasmas have already been the subject of many theoretical, numerical and experimental investigations due to their spatial frequency spectrum [3, 4]. The problem of transferring energy from a beam of particles into electromagnetic wave energy has been given considerable attention in various fields of physics. Beam-energy extraction requires that phase matching between waves and particles is maintained for as long as possible. It is well known that Cerenkov and cyclotron resonances have this desirable property of maintaining synchronization. The SPW amplitude falls off rapidly as one move away from the interface. Therefore, SPW excitation by laser or electron beam injection has been observed and studied extensively by several investigators [5–11]. Denton et al. [5] have studied the process of SPW excitation over a metal surface by charged particles. Liu and Tripathi [6] have developed a theory of the excitation, reflection and scattering of a SPW over a metal surface by modeling a localized surface ripple as an electron density perturbation. Khankina et al. [7] studied the excitation of surface waves in magnetoactive plasma by a moving charged particle along the spiral line relative to the constant magnetic field. SPW can also be excited by an ion beam [12], by *attenuated total reflection* (ATR) configuration [13], by ripples of suitable wave number over the metallic interface [14], by light via prism coupling [15] or by laser [16–18]. The SPW excitation by charged particle on the metal vacuum interface can provide important information on the structure of the energy spectrum of the electron Fermi fluid in metals. Later, Liu and Tripathi [14] observed that a laser incident on a metal film excites a surface plasma wave at the metal-free space interface or it can also be excited by a relativistic electron beam. Macchi et al. [19] studied the parametric excitation of electron surface wave in the interaction of intense laser pulse with an over-dense plasma. Shokri and Jazi [20] showed that non reciprocal electromagnetic surface waves can be excited in semi bounded magnetized plasmas by an electron beam flowing on the plasma surface. Borisov and Nielsen [21] studied the excitation of plasma waves by unstable photoelectron and thermal electron populations on closed magnetic field in the Martian ionosphere. Kumar and Tripathi [22] studied the excitation of a surface plasma wave over a plasma cylinder by a relativistic electron beam propagating in a plasma cylinder and an annular beam propagating outside the plasma cylinder.

In the present paper, we study the excitation of SPWs by an electron beam propagating across an external magnetic field parallel to the conductor-plasma interface. In Section 2, we study the plasma and beam electron responses to the SPW perturbation. In Section 3, we have derived the dispersion relation and growth rate of SPWs. The variation of the growth rate of the unstable mode as a function of electron plasma density n_{e0} for different propagating distances from the interface has been discussed in Cerenkov and fast cyclotron interactions. In Section 4, we discuss our results.

2. Plasma and Electron Beam Response

Consider a conductor-plasma interface at $x = 0$, with conductor in region $x < 0$ characterized by effective relative permittivity ε_c and dielectric constant ε_L, while plasma in region $x > 0$ with dielectric constant ε_p (cf. Figure 1).

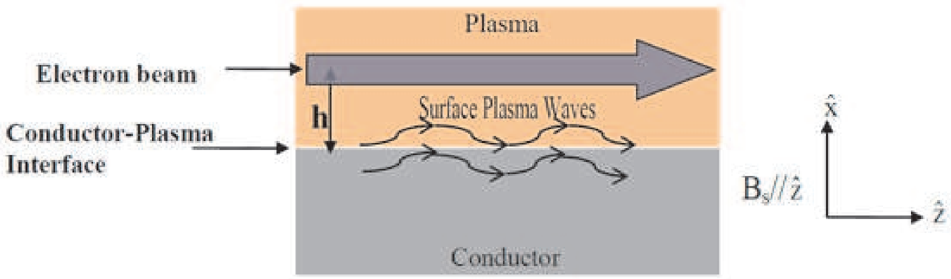

Figure 1. Schematic of a conductor-plasma interface with an electron beam propagating above the interface at a distance h.

The equilibrium electron and ion densities are $\boldsymbol{n}_{e0}$ and $\boldsymbol{n}_{i0}$, respectively, immersed in a static magnetic field B in the z-direction. We assume the t, z variations of fields as $\boldsymbol{E},\boldsymbol{B} \sim \exp[-i(\omega t - k_z z)]$ and consider E field to be polarized in the x-z plane. An electron beam is considered propagating along z-axis at a height 'h' above the conductor-plasma interface, with density n_{b0} and equilibrium beam velocity $v_{b0}\hat{z}$. To obtain the response of plasma electrons to the fields of the surface plasma waves, we solve the equation of motion

$$\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{t}} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\boldsymbol{e}}{\boldsymbol{m}}\left(\boldsymbol{E} + \frac{1}{\boldsymbol{c}}\boldsymbol{v}\times\boldsymbol{B}\right), \tag{1}$$

where $\boldsymbol{v} = v_{b0}\hat{z} + \boldsymbol{v}_{b1}$, $\boldsymbol{v}_{b1}$ refers to perturbed velocity.

The magnetic field of the wave is

$$B_w = \left(\frac{c}{\omega}\right)(\boldsymbol{k}\times\boldsymbol{E}) = \left(\frac{c}{\omega}\right)(k_z E_x - k_x E_z)\hat{y}.$$

Here $E_x = \left(\dfrac{ik_z}{k_2}\right)E_z$ for $\nabla\cdot E = 0$ SPWs.

The decay rate of wave amplitude in plasma is given by $k_1 = \dfrac{\frac{\omega\varepsilon_p}{c}}{[-(\varepsilon_p+\varepsilon_c)]^{\frac{1}{2}}}$, and the decay rate of wave amplitude in conductor is $k_2 = \dfrac{\frac{\omega}{c}}{\left[-\dfrac{\varepsilon_p+\varepsilon_c}{\varepsilon_c^2}\right]^{\frac{1}{2}}}$, where $\varepsilon_c = \varepsilon_L - \dfrac{\omega_{pe}^2}{\omega^2}$, $\omega_{pe}^2 = \dfrac{4\pi n_{e0}e^2}{m}$.

For k_1 and k_2 to be real, we must have $\varepsilon_p + \varepsilon_c < 0$, which gives

$$\boldsymbol{\omega} < \omega_{sc}, \tag{2}$$

where $\omega_{\rm sc} = \dfrac{\omega_p}{(\varepsilon_p + \varepsilon_L)^{\frac{1}{2}}}$ is the surface wave cut-off frequency.

Eq. (2) is essential for the existence of surface plasma wave.

The perturbed electron velocity in x-, y- and z-directions, after linearization are obtained as

$$v_{b1x} = \frac{\left[e\left(\frac{ik_z}{k_2}\right) - e\left(\frac{v_{b0}}{\omega}\right)\left\{\left(\frac{ik_z^2}{k_2}\right) - k_x\right\}\right](\omega - k_z v_{b0})E_z}{im\left[(\omega - k_z v_{b0})^2 - \omega_{ce}^2\right]}, \tag{3}$$

$$v_{b1y} = \frac{\left[e\left(\frac{ik_z}{k_2}\right) - e\left(\frac{v_{b0}}{\omega}\right)\left\{\left(\frac{ik_z^2}{k_2}\right) - k_x\right\}\right]\omega_{ce}E_z}{m\left[(\omega - k_z v_{b0})^2 - \omega_{ce}^2\right]} \tag{4}$$

and

$$v_{b1z} = \frac{eE_z}{im(\omega - k_z v_{b0})} \tag{5}$$

where $\omega_{ce} = e\boldsymbol{B}/mc$ is the electron cyclotron frequency.

From equation of continuity $\dfrac{\partial n}{\partial t} + \nabla\cdot(nv) = 0$, where $n = n_{b0} + n_{b1}$, we obtain the perturbed beam density n_{b1} and perturbed electron density n_{e1} as

$$n_{b1} = \frac{en_{b0}\left[ik_z - \left(\frac{v_{b0}}{\omega}\right)(ik_z^2 - k_2k_x)\right]E_z}{m\left[(\omega - k_z v_{b0})^2 - \omega_{ce}^2\right]} + \frac{en_{b0}k_zE_z}{im(\omega - k_z v_{b0})^2} \tag{6}$$

and

$$n_{e1} = -\frac{n_{e0}ek_zE_z}{im}\left[\frac{\omega_{ce}^2}{\omega^2(\omega^2 - \omega_{ce}^2)}\right]. \tag{7}$$

3. Dispersion Relation and Growth Rate

The perturbed current density will be

$$\boldsymbol{J}_1 = -\boldsymbol{e}(\boldsymbol{n}_{b0}\boldsymbol{v}_{b1} + \boldsymbol{n}_{b1}v_{b0}\hat{z})\delta(x-h).$$

By retaining only those terms which go as $(\omega - k_z v_{b0})^{-2}$, the z-component of current density is obtained as

$$J_{1z} = \left[-\frac{e^2v_{b0}\left[ik_z - \left(\frac{v_{b0}}{\omega}\right)(ik_z^2 - k_2k_x)\right]E_zn_{b0}}{m\left[(\omega - k_z v_{b0})^2 - \omega_{ce}^2\right]} - \frac{e^2v_{b0}k_zE_zn_{b0}}{im(\omega - k_z v_{b0})^2}\right]\delta(x-h)$$

or

$$J_{1z} = -\frac{e^2v_{b0}n_{b0}E_z}{m}\left[\frac{\left[ik_z - \left(\frac{v_{b0}}{\omega}\right)(ik_z^2 - k_2k_x)\right]}{\left[(\omega - k_z v_{b0})^2 - \omega_{ce}^2\right]} + \frac{k_z}{i(\omega - k_z v_{b0})^2}\right]\delta(x-h). \tag{8}$$

Using Eq. (8) in the z-component of wave equation $\nabla^2 E - \nabla(\nabla\cdot E) + \frac{\omega^2}{c^2}\varepsilon_p E = -\frac{4\pi i\omega}{c^2}J$ we get

$$\left(k_2^2 - k_z^2 + \frac{\omega^2}{c^2}\varepsilon_p\right)E_z = -\frac{\omega v_{b0}}{c^2}\omega_{pb}^2\left[\frac{\left[k_z - \left(\frac{v_{b0}}{\omega}\right)\left(k_z^2 + ik_2k_x\right)\right]}{[(\omega - k_z v_{b0})^2 - \omega_{ce}^2]} - \frac{k_z}{(\omega - k_z v_{b0})^2}\right]E_z\delta(x-h) \tag{9}$$

where $\omega_{pb}^2 = \frac{4\pi n_{b0}e^2}{m}$.

Multiplying by E_z^* and integrating from $x = 0$ to ∞, taking $\int_0^\infty E_z^* E_z dx = 1$ and $\int_0^\infty E_z^* E_z \delta(x-h)dx = \exp(-2k_2h)$, Eq. (9) can be rewritten as

$$\omega^2 - k_z^2c^2\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right) = -\omega v_{b0}\omega_{pb}^2\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right)\left[\frac{\left[k_z - \left(\frac{v_{b0}}{\omega}\right)\left(k_z^2 + ik_2k_x\right)\right]}{[(\omega - k_z v_{b0})^2 - \omega_{ce}^2]} - \frac{k_z}{(\omega - k_z v_{b0})^2}\right]e^{-2k_2h} \tag{10}$$

or

$$(\omega - \omega_{spw})(\omega + \omega_{spw}) = -\omega v_{b0}\omega_{pb}^2\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right)\left[\frac{\left[k_z - \left(\frac{v_{b0}}{\omega}\right)\left(k_z^2 + ik_2k_x\right)\right]}{[(\omega - k_z v_{b0})^2 - \omega_{ce}^2]} - \frac{k_z}{(\omega - k_z v_{b0})^2}\right]e^{-2k_2h} \tag{11}$$

where ω_{spw} is the root of ω, given as

$$\omega_{spw}^2 = k_z^2c^2\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right). \tag{12}$$

Eq. (12) represents the standard dispersion relation of SPW [23].

In Cerenkov interaction, $(\omega - k_z v_{b0})^2 \approx 0 \Rightarrow \omega \sim k_z v_{b0}$, therefore neglect the first term on RHS in Eq. (11) and assume perturbed quantities $\omega = \omega_{spw} + \delta$ and $\omega = k_z v_{b0} + \delta$, where δ is the small frequency mismatch.

The growth rate is obtained as

$$\gamma = \mathrm{Im}(\delta) = \frac{\sqrt{3}}{2}\left[\frac{\omega_{pb}^2 k_z v_{b0}}{2}\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right)e^{-2k_2h}\right]^{\frac{1}{3}}. \tag{13}$$

The real part of frequency ω_r is obtained from the real part of δ as

$$\omega_r = k_z\left(\frac{2\mathrm{eV_b}}{\mathrm{m}}\right)^{\frac{1}{2}} - \frac{1}{2}\left[\frac{\omega_{pb}^2 k_z v_{b0}}{2}\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right)e^{-2k_2h}\right]^{\frac{1}{3}} \tag{14}$$

where V_b is the beam potential.

In cyclotron interaction $(\omega - k_z v_{b0})^2\gamma_o^2 \approx \omega_{ce}^2$, therefore neglecting the second term on RHS in Eq. (11), we get

$$(\omega - \omega_{spw})(\omega + \omega_{spw}) = -\omega v_{b0}\omega_{pb}^2\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p\varepsilon_c}\right)\left[\frac{\left[k_z - \left(\frac{v_{b0}}{\omega}\right)\left(k_z^2 + ik_2k_x\right)\right]}{[(\omega - k_z v_{b0})^2 - \omega_{ce}^2]}\right]e^{-2k_2h} \tag{15}$$

where $(\omega - k_z v_{b0})\gamma_o + \omega_{ce}$ corresponds to slow cyclotron interaction, and $(\omega - k_z v_{b0})\gamma_o - \omega_{ce}$ corresponds to fast cyclotron interaction.

Considering slow cyclotron interaction, and assuming perturbed quantities $\omega = \omega_{spw} + \delta$ and $\omega = k_z v_{b0} - \omega_{ce} + \delta$, the growth rate is obtained as

$$\gamma = 0. \tag{16}$$

The phase velocity of the unstable mode is obtained from the real part of ω as

$$\gamma = \left[\frac{\omega_{pb}^2}{4}\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p \varepsilon_c}\right)\left(1 - \frac{\omega_{ce}}{\omega}\right)e^{-2k_2 h}\right]^{\frac{1}{2}}. \tag{17}$$

That is, in case of slow cyclotron interaction there is no growing mode as the phase velocity exceeds the beam velocity.

Now, considering fast cyclotron interaction and assuming perturbed quantities $\omega = \omega_{spw} + \delta$ and $\omega = k_z v_{b0} + \omega_{ce} + \delta$, the growth rate is obtained as

$$\gamma = \left[\frac{\omega_{pb}^2}{4}\left(\frac{\varepsilon_p + \varepsilon_c}{\varepsilon_p \varepsilon_c}\right)\left(1 - \frac{\omega_{ce}}{\omega}\right)e^{-2k_2 h}\right]^{\frac{1}{2}}. \tag{18}$$

4. Results and Discussion

Typical parameters of the SPWs used for numerical calculations are: electron plasma density $n_{e0} = 10^{10}\,\text{cm}^{-3}$ and $10^{11}\,\text{cm}^{-3}$, mass of electron $m_e = 9.1 \times 10^{-28}$ g, charge of electron $e = 4.8 \times 10^{-10}$ ergs, magnetic field $B = 30$ G, dielectric constant of plasma $\varepsilon_p = 1.2$, dielectric constant of conductor $\varepsilon_L = 4$ and beam velocity $v_{bo} = 2 \times 10^{10}$ cm/s. The electron beam is assumed to travel at a distance of 2 cm, 4 cm and 6 cm from the conductor-plasma interface with beam density bo $n_{bo} = 2 \times 10^9\,\text{cm}^{-3}$.

We have plotted the dispersion curves of surface plasma waves on a conductor-plasma interface for the two values of electron plasma densities using Eq. (12) in Figure 2.

We have also plotted the beam modes via Cerenkov interaction and fast cyclotron interaction with SPWs, where the beam is assumed to travel at a distance 2 cm from the interface. The frequencies and the corresponding wave numbers of the unstable mode are obtained by the points of intersection between the beam modes and the plasma modes. We can say that the unstable wave frequencies and the axial wave vector k_z (cm^{-1}) of the SPWs increase with an increase in plasma density, in Cerenkov as well as cyclotron interactions. The variation in the value of applied magnetic field does not affect the Cerenkov interaction, but the fast cyclotron beam mode with $B > 45$ G could not interact with SPW for plasma density of $10^{10}\,\text{cm}^{-3}$ and the beam mode with $B > 135$ G do not intersect with SPW dispersion curve. It implies that the value of magnetic field should be less for a less dense plasma for the existence of cyclotron interaction between electron beam and SPW mode.

Using Eq. (13), we have plotted in Figure 3 the growth rate γ (rad./sec) of the surface plasma waves as a function of unstable frequency ω in Cerenkov interaction with a beam travelling at different distances from the interface, and for two values of plasma electron densities.

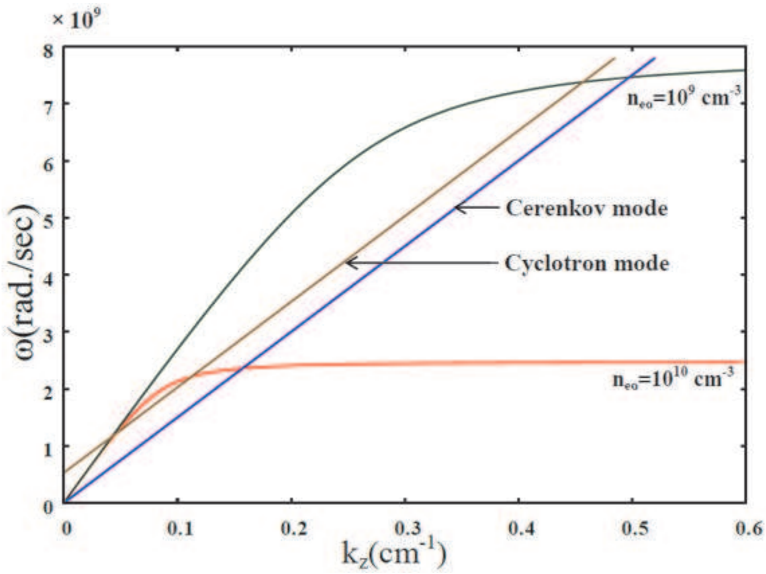

Figure 2. Dispersion curves of SPWs over a magnetized plasma for different values of n_{e0} and beam modes. The parameters are given in the text.

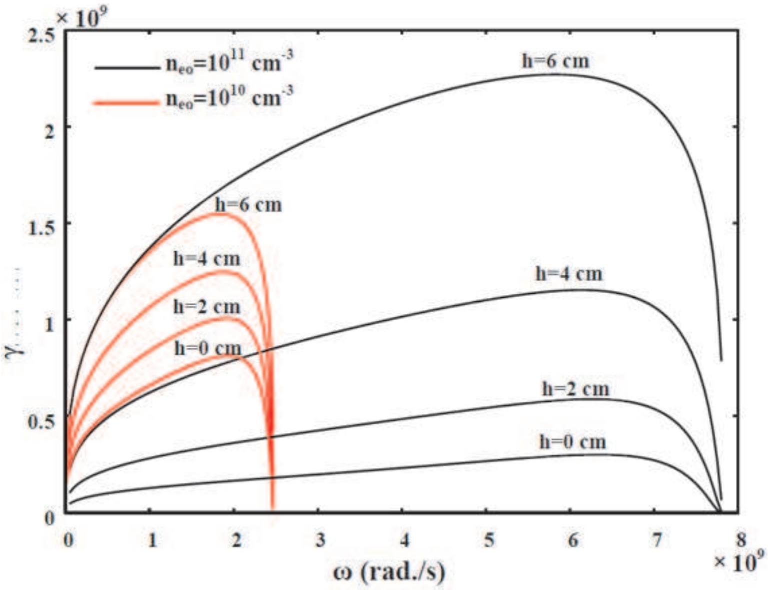

Figure 3. Growth rate γ of the unstable mode as a function of wave frequency for a beam propagating at different heights from the interface via Cerenkov interaction, with two different values of n_{e0}.

From Figure 3, it can be seen that the growth rate of the unstable mode first increases and then shows a maxima in all the curves. When $h = 0$ cm, i.e. when the beam is propagating along the interface, the growth rate is more for a denser plasma. However, as the value of h increases i.e. as the beam moves away from the interface, the growth rate decreases rapidly in case of a denser plasma as compared to the rate of decrease of growth rate in less dense plasma. The growth rate γ (in rad./sec) of the unstable wave decreases from 1.55 to 0.54 for plasma density of 10^{10} cm^{-3}, and from 2.27 to 0.09 for plasma density of 10^{11} cm^{-3}, when h increases from

0 cm to 6 cm. The growth rate of the unstable mode also increases with the beam density and scales as the one-third power of the beam density [cf. Eq. (13)].

In Figure 4, we have plotted the variation of the growth rate γ (in rad./sec) of SPW as a function of unstable frequency via fast cyclotron interaction and the beam mode for same beam velocity and static magnetic field, for the two values of plasma densities and for electron beam travelling at different distances from the interface.

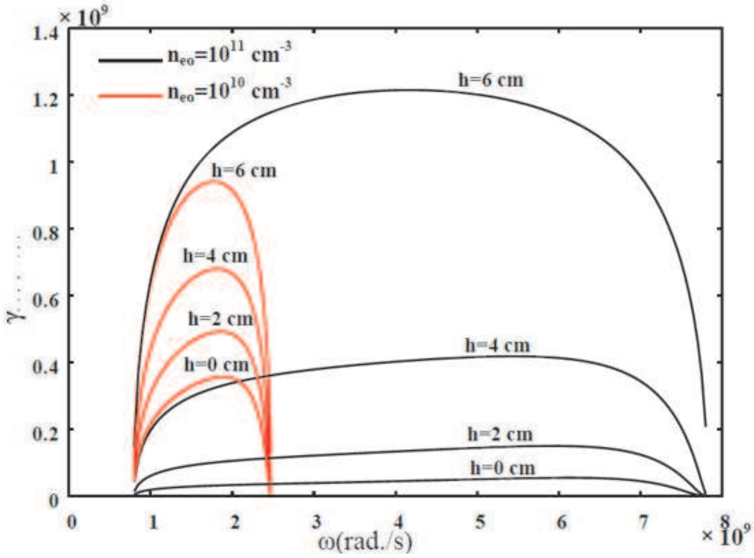

Figure 4. Growth rate γ of the unstable mode as a function of wave frequency for a beam propagating at different heights from the interface via Cerenkov interaction, with two different values of n_{e0}.

The behavior of the curves is almost same as in the case of Cerenkov interaction. The values of growth rate are comparatively smaller in cyclotron interaction, with a maximum of 0.95 for $n_{e0} = 10^{10}\ \text{cm}^{-3}$ and a maximum of 1.21 for $n_{e0} = 10^{11}\ \text{cm}^{-3}$. The growth rate also increases with the beam density and scales as the square root of the beam density. In case of slow cyclotron interaction there is no growing mode as the phase velocity exceeds the beam velocity [cf. Eq. (17)].

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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