Journal of Atomic, Molecular, Condensate & Nano Physics
Vol. 3, No. 1, pp. 17–25, 2016
ISSN 2349-2716 (online); 2349-6088 (print)
Published by RGN Publications
http://www.rgnpublications.com

Generation of Atomic-Squeezed States via Pondermotively Squeezed Light

Research Article

Neha Aggarwal[1,*], Aranya B. Bhattacherjee[2] and Man Mohan[1]

[1] *Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India*
[2] *School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India*
***Corresponding author:** ag.neha88@gmail.com

Abstract. We study the generation of atomic-squeezed states for a Bose-Einstein Condensate confined within the lossless optomechanical cavity using pondermotively squeezed light. We show that the radiation pressure coupling between the optical cavity field and mechanical motion of the cavity end mirror generates squeezing of light. This radiation pressure induced light squeezing gets transferred to the condensate atoms via Tavis-Cummings type interaction and results in squeezed-spin states. We further discuss the effect of optomechanical coupling on squeezed atomic states.

Keywords. Bose-Eintsein condensates; Optomechanical cavity; Spin-squeezing; Pondermotive squeezing

PACS. 03.75.Mn; 03.75.Kk; 37.30.+i; 42.65.Pc

Received: May 26, 2015 **Accepted:** December 31, 2015

1. Introduction

Both theoretically and experimentally, squeezing in spins has always attracted lots of attention for over a decade [1–3] due to its numerous applications. Such squeezed spin states have been proved to be a useful quantum resources in improving the measurements precision in experiments [2–5], in studying the correlations and entanglement among the particles [6–8]. The spin squeezing phenomenon in collective atomic system has also attracted considerable interest for its application in atomic clocks which is useful in both the quantum noise reduction [2,3,9–11] and quantum information [6, 12–14]. Several definitions of spin squeezing have been proposed but the most widely used spin squeezing parameters were given by Kitagawa and Ueda in [1] and by Wineland et al. in [2, 3]. The spin squeezing parameter ζ_S [1] was inspired by the well-known squeezing in photons, whereas, the squeezing parameter ζ_R [2, 3] was introduced in experiments on standard Ramsey spectroscopy.

Squeezing of electromagnetic field has also attracted very much attention for decades [15–17]. Squeezed states of light can be produced by exploiting the radiation pressure coupling between the light field and the movable mirror. This radiation pressure induced light squeezing is basically termed as ponderomotive squeezing [18–20]. The generation of pondermotively squeezed light has not only been proposed [20] but has also been demonstrated [21–23] experimentally.

Further note that the generation of spin squeezing in atomic ensembles via light-atom interaction has also been demonstrated in several papers which basically involves the transferring of squeezing from light to atoms [3,24–26]. The experimental realizations of atomic-squeezed states in BEC have also been reported [27–30]. Thus, motivated by these interesting developments in this field, we propose a hybrid optomechanical system consisting of an elongated cigar-shaped gas of two-level BEC atoms coupled to a single mode of a lossless optical cavity with a movable cavity boundary. We study this model to generate the correlated-particle states or atomic-squeezed states in both the x- and y-directions in the small-angle approximation by considering the cavity field to be initially prepared in the squeezed vacuum state. We further discuss how the pondermotive force acting on movable mirror helps in generating atomic-squeezed states and also how the variation in mirror-photon coupling controls this squeezing in spins.

2. Model Hamiltonian

The basic model that we are considering here involves a lossless Fabry-Perot optical cavity of length L and frequency ω_c with one of the mirrors movable. The movable mirror can be considered as a single oscillator with mechanical frequency ω_m and mass m', which can be experimentally realized by using a bandpass filter in the detection loop [31]. This optomechanical cavity also involves an elongated cigar-shaped gas of N BEC atoms of ^{87}Rb having two different hyperfine levels $|F=1, m_f=-1\rangle$ and $|F=2, m_f=1\rangle$ [32]. Here, the atomic transition frequency is denoted by ω_a and mass by m. The schematic representation of our system is shown in Figure 2. Note that each of the atomic modes is characterized by an annihilation operator c_j ($j=1,2$) in the two-mode approximation. Also, the atomic ensemble interacts strongly with a single quantized cavity mode that is equivalent to a single quantum-mechanical harmonic oscillator having unit mass.

The simplest model of this hybrid optomechanical system can be described by the following Hamiltonian within the dipole approximation ($\hbar=1$ throughout the paper) [33, 34]:

$$H = \omega_a J_z + \omega_c a^\dagger a + \omega_m b^\dagger b + \frac{g_0}{\sqrt{N}}(a+a^\dagger)J_x + \epsilon a^\dagger a(b+b^\dagger). \tag{1}$$

Here, the ensemble of N BEC atoms can be described using the picture of dimensionless collective spin operators as [35]: $J_x = \frac{(c_1^\dagger c_2 + c_2^\dagger c_1)}{2}$, $J_y = \frac{(c_1^\dagger c_2 - c_2^\dagger c_1)}{2i}$ and $J_z = \frac{(c_1^\dagger c_1 - c_2^\dagger c_2)}{2}$, where $c_j^\dagger$

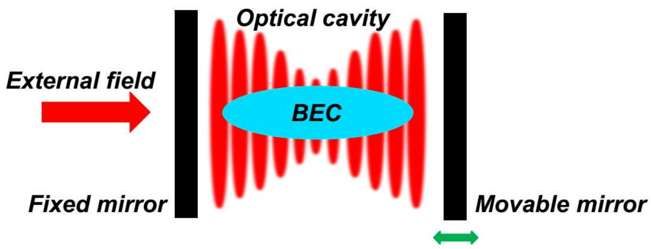

Figure 1. (color online) Schematic representation of the hybrid optomechanical system involving Bose-Einstein Condensate embedded within a lossless optical cavity that is driven by an external field. Here, one of the mirrors of the cavity is movable.

and c_j ($j = 1,2$) are the creation and annihilation operators for the two modes such that $J_+ = c_1^\dagger c_2$ and $J_- = c_2^\dagger c_1$. These atomic o perators obey the commutation relations given as $[J_+, J_-] = 2J_z$ and $[J_\pm, J_z] = \mp J_\pm$. Further note that the Hilbert space of this algebra is spanned by symmetric Dicke states [36] $|J,M\rangle$ with $M = -J, -J+1 \ldots J-1, J$ and $J = \frac{N}{2}$. The Dicke states are the eigenstates of J^2 and J_z in such a way that $J_z|J,M\rangle = M|J,M\rangle$ and $J^2|J,M\rangle = J(J+1)|J,M\rangle$. The lowering and raising operators act on the Dicke states as $J_\pm|J,M\rangle = \sqrt{J(J+1) - M(M\pm 1)}|J,M\pm 1\rangle$.

The annihilation and creation operators for the cavity photons are represented by a and $a^\dagger$ respectively which follow the commutation relation given as $[a, a^\dagger] = 1$. For the mechanical mode of the movable cavity boundary, the annihilation and creation operators are denoted by b and $b^\dagger$ respectively with $[b, b^\dagger] = 1$. Furthermore, parameters g_0 and ϵ denote the atom-photon coupling strength via dipole interaction and nonlinear dispersive mirror-photon coupling respectively.

Now, in order to study this system for the generation of atomic-squeezed states, we first rewrite the above Hamiltonian given by eqn. (1) in the interaction picture as [37]

$$H_{int} = \frac{g_0}{2\sqrt{N}}[aJ_+ e^{i(\omega_a-\omega_c)t} + aJ_- e^{-i(\omega_a+\omega_c)t}] + \frac{g_0}{2\sqrt{N}}[a^\dagger J_+ e^{i(\omega_a+\omega_c)t} + a^\dagger J_- e^{-i(\omega_a-\omega_c)t}] + \epsilon a^\dagger a(b e^{-i\omega_m t} + b^\dagger e^{i\omega_m t}). \tag{2}$$

Now, by treating the mechanical mode operator for the movable mirror b semiclassically, we can replace b by its scalar quantity β that is real such that $|\beta| \gg 1$. Thus, the above Hamiltonian given by eqn. (2) can now be rewritten as:

$$H_1 = \frac{g_0}{2\sqrt{N}}[aJ_+ + a^\dagger J_-] + g(t)a^\dagger a, \tag{3}$$

for $\omega_a = \omega_c$. The high-frequency terms are neglected in deriving the above Hamiltonian (rotating wave approximation). Also, parameter $g(t) = 2\epsilon\beta\cos(\omega_m t)$ is the modified mirror-photon coupling, which is sinusoidally modulated at frequency ω_m. Here, $2\epsilon\beta$ denotes the modulation amplitude. This modified optomechanical coupling is related to the time modulated frequency of the optical cavity that arising due to the motion of the mirror [38]. Furthermore, last term in this Hamiltonian is the well-known Tavis-Cummings interaction [39] which is valid for small atom-photon coupling values [40].

It is well known that the pondermotive interaction between the incident light field and the movable mirror in optomechanical systems can be exploited for producing the squeezed states of the light field. This radiation-pressure induced light squeezing is termed as pondermotive [41] squeezing. It has been demonstrated very recently by using a mechanical mode of an ultracold gas of atoms within an optical cavity [21] and through optomechanical interaction between light and membrane mechanical oscillator enclosed within an optical cavity [22]. Thus, we are now interested in studying the effect of optomechanical coupling on the atomic-squeezed states generated using pondermotively squeezed light. The squeezed-spin states prepared this way could then be used in Ramsey spectroscopy.

3. Atomic-Squeezing Generation

In this section, we study how the squeezing can be transferred to the spins from the harmonic oscillator (single-mode quantized cavity field) through the interaction H_1. Without making the problem mathematically complicated, we consider the following special case. The spin is considered to be initially prepared in the Dicke state $|J, M = -J\rangle$ with $J = \frac{N}{2}$ and $M = \langle J_z \rangle$ in such a way that $\langle J_x(0)\rangle = \langle J_y(0)\rangle = 0$, $\Delta J_x(0) = \Delta J_y(0) = \sqrt{\frac{J}{2}}$ and $\Delta J_z(0) = 0$, where ΔA $(= \sqrt{\langle A^2\rangle - \langle A\rangle^2})$ is the square root of the variance of any operator A. Also, the harmonic oscillator is assumed to be initially prepared in a squeezed vacuum state. The squeezing of harmonic oscillator can be characterized by the parameters $\chi_x(t) = \frac{\Delta x(t)}{x_0}$ or $\chi_p(t) = \frac{\Delta p(t)}{x_0}$. The harmonic oscillator initially prepared in an amplitude squeezed vacuum state is characterized by $\langle x(0) = \langle p(0)\rangle = 0$ and $\Delta x(0) < \Delta x(\text{coherent state}) = x_0$ such that $\chi_x(0) < 1$, where x_0 is its zero-point amplitude. However, harmonic oscillator initially prepared in a momentum (or velocity) squeezed vacuum state is characterized by $\langle x(0)\rangle = \langle p(0)\rangle = 0$ and $\Delta p(0) < \Delta p(\text{coherent state}) = x_0$ such that $\chi_p(0) < 1$. The Heisenberg equations of motion obtained by using the Hamiltonian H_1 are given as follows:

$$\dot{a}(t) = -\frac{ig_0}{2\sqrt{N}} J_-(t) - ig(t)a(t), \tag{4}$$

$$\dot{J}_-(t) = -\frac{i\gamma}{N} J_z(t)J_-(t) - \frac{i\gamma}{N} J_-(t)J_z(t) + \frac{ig_0}{\sqrt{N}} a(t)J_z(t). \tag{5}$$

Similarly, we can find their adjoint expressions. Now, we make the small-angle approximation that $J_z(t) = -JI$, where I is an identity operator. This approximation is valid under the assumption that the number of atoms N is large enough such that the value of $\langle J_z(t)\rangle$ does not change appreciably during the time H_1 is applied. With this approximation and by rewriting the cavity field operators and the angular momentum operators in terms of their quadratures as $x(t) = \frac{(a(t)+a^\dagger(t))}{\sqrt{2}}$, $p(t) = \frac{(a(t)-a^\dagger(t))}{(i\sqrt{2})}$, $J_x(t) = \frac{(J_+(t)+J_-(t))}{2}$ and $J_y(t) = \frac{(J_+(t)-J_-(t))}{(2i)}$, the Heisenberg equations of motion becomes:

$$\dot{x}(t) = -\frac{g_0}{\sqrt{2N}} J_y(t) + g(t)p(t), \tag{6}$$

$$\dot{p}(t) = -\frac{g_0}{\sqrt{2N}} J_x(t) - g(t)x(t), \tag{7}$$

$$\dot{J}_x(t) = \frac{2\gamma J}{N} J_y(t) + \frac{g_0 J}{\sqrt{2N}} p(t), \tag{8}$$

$$\dot{J}_y(t) = -\frac{2\gamma J}{N} J_x(t) + \frac{g_0 J}{\sqrt{2N}} x(t). \tag{9}$$

Note that different definitions of spin-squeezing parameters can be used in various contexts. The squeezing parameter given by Kitagawa and Ueda [1] is denoted by ζ_S such that $\zeta_S = \sqrt{\frac{2}{J}}\Delta J_\perp$. The subscript $\perp$ denotes an axis that is perpendicular to the mean angular momentum $\langle J\rangle$ where the minimum value of ΔJ is attained. It is used to indicate the degree of quantum correlations among the elementary spins such that $\zeta_S < 1$ signifies atomic-squeezing condition. Another squeezing parameter has been given by Wineland [2, 3] and is defined as $\zeta_R = \sqrt{2J}\Delta J_\perp / |\langle J\rangle|$ which is intimately connected to the improvement of the sensitivity of angular-momentum states to rotations such that $\zeta_R < 1$ signifies atomic-squeezing. In the context of these definitions, the squeezing parameters that we are considering here are defined as follows:

$$\zeta_{S,x} = \sqrt{\frac{2}{J}}\Delta J_x, \tag{10}$$

$$\zeta_{S,y} = \sqrt{\frac{2}{J}}\Delta J_y, \tag{11}$$

$$\zeta_{R,x} = \sqrt{2J}\Delta J_x / |\langle J_z\rangle|, \tag{12}$$

$$\zeta_{R,y} = \sqrt{2J}\Delta J_y / |\langle J_z\rangle|. \tag{13}$$

Further note that $\zeta_{R,x} = \zeta_{S,x} = \zeta_x$ and $\zeta_{R,y} = \zeta_{S,y} = \zeta_y$ under the small-angle approximation $J_z(t) = -JI$. The averages $\langle J_x\rangle$, $\langle J_y\rangle$, $\langle x\rangle$ and $\langle p\rangle$ are assumed to be zero at all times. Now using the eqns. (6)-(9) and the definitions of squeezing parameter for spins and harmonic oscillator with $J_x \to J_x\sqrt{N}$ and $J_y \to J_y\sqrt{N}$, we obtain the following equations of motion:

$$\dot{\zeta_x^2}(t) = 2\sqrt{2}g_0\langle J_x(t)p(t)\rangle, \tag{14}$$

$$\dot{\zeta_y^2}(t) = 2\sqrt{2}g_0\langle J_y(t)x(t)\rangle, \tag{15}$$

$$\dot{\chi_x^2}(t) = -2\sqrt{2}g_0\langle J_y(t)x(t)\rangle + 4g(t)\langle x(t)p(t)\rangle, \tag{16}$$

$$\dot{\chi_p^2}(t) = -2\sqrt{2}g_0\langle J_x(t)p(t)\rangle - 4g(t)\langle x(t)p(t)\rangle, \tag{17}$$

$$\langle J_x(t)\dot{J}_y(t)\rangle = \frac{g_0}{2\sqrt{2}}(\langle J_x(t)x(t)\rangle + \langle J_y(t)p(t)\rangle), \tag{18}$$

$$\langle J_x(t)\dot{p}(t)\rangle = \frac{g_0}{4\sqrt{2}}\chi_p^2 - \frac{g_0}{4\sqrt{2}}\zeta_x^2 - g(t)\langle J_x(t)x(t)\rangle, \tag{19}$$

$$\langle J_y(t)\dot{x}(t)\rangle = \frac{g_0}{4\sqrt{2}}\chi_x^2(t) - \frac{g_0}{4\sqrt{2}}\zeta_y^2(t) + g(t)\langle J_y(t)p(t)\rangle, \tag{20}$$

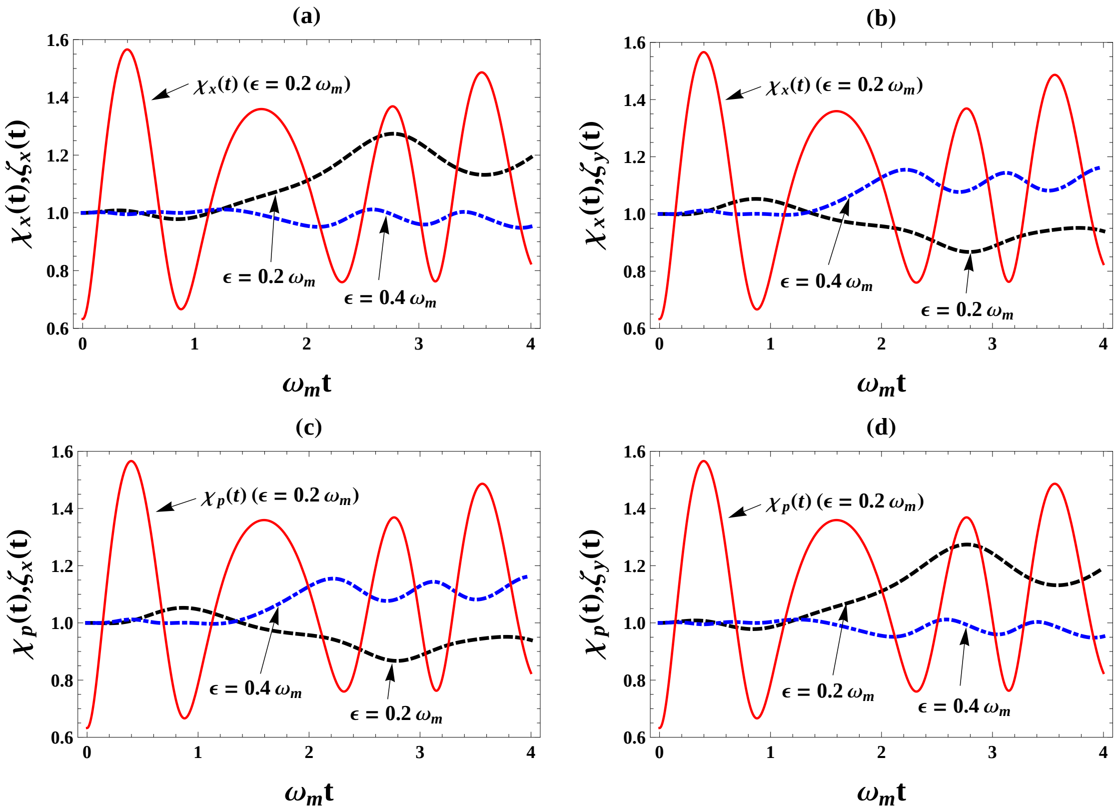

Figure 2. (color online) plots of squeezing parameters for harmonic oscillator $\chi_x(t)$ and $\chi_p(t)$ as a function of scaled time ($\omega_m t$) for $\epsilon = 0.2\omega_m$ (solid line) and the spin squeezing parameters $\zeta_x(t)$, $\zeta_y(t)$ versus scaled time ($\omega_m t$) for two different values of mirror-photon coupling with $\epsilon = 0.2\omega_m$ (dashed line) and $\epsilon = 0.4\omega_m$ (dot dashed line). (a): Plots of $\chi_x(t)$ and $\zeta_x(t)$ versus time. (b): Plots of $\chi_x(t)$ and $\zeta_y(t)$ versus time. Here, the harmonic oscillator is initially prepared in the amplitude squeezed vacuum state with $\chi_x(0) = 0.632$. (c): Plots of $\chi_p(t)$ and $\zeta_x(t)$ versus time. (d): Plots of $\chi_p(t)$ and $\zeta_y(t)$ versus time. Here, the harmonic oscillator is initially prepared in the momentum squeezed vacuum state with $\chi_p(0) = 0.632$. The other parameters used are $\gamma = 0$, $g_0 = \omega_m$ and $\beta = 10$.

$$\langle J_x(t)\dot{x}(t)\rangle = \frac{g_0}{2\sqrt{2}}\langle x(t)p(t)\rangle - \frac{g_0}{\sqrt{2}}\langle J_x(t)J_y(t)\rangle + g(t)\langle J_x(t)p(t)\rangle, \tag{21}$$

$$\langle J_y(t)\dot{p}(t)\rangle = \frac{g_0}{2\sqrt{2}}\langle x(t)p(t)\rangle - \frac{g_0}{\sqrt{2}}\langle J_x(t)J_y(t)\rangle - g(t)\langle J_y(t)x(t)\rangle, \tag{22}$$

$$\langle x(t)\dot{p}(t)\rangle = -\frac{g_0}{\sqrt{2}}(\langle J_y(t)p(t)\rangle + \langle J_x(t)x(t)\rangle) + \frac{g(t)}{2}\chi_p^2 - \frac{g(t)}{2}\chi_x^2. \tag{23}$$

We now solve the coupled equtions of motion (14)-(23) numerically using MATHEMATICA 9.0. Figure 2 shows the plots of squeezing parameters for harmonic oscillator $\chi_x(t)$ and $\chi_p(t)$ versus scaled time ($\omega_m t$) for $\epsilon = 0.2\omega_m$ (solid line) and the spin squeezing parameters $\zeta_x(t)$, $\zeta_y(t)$ as a function of scaled time ($\omega_m t$) for two different values of mirror-photon coupling with $\epsilon = 0.2\omega_m$ (dashed line) and $\epsilon = 0.4\omega_m$ (dot dashed line). Figures 2(a) and 2(b) show the squeezing parameters plots for the harmonic oscillator to be initially prepared in an amplitude squeezed vacuum state with $\chi_x(0) = 0.632$. However, Figures 2(c) and 2(d) depict the squeezing parameters plots for the harmonic oscillator to be initially prepared in a momentum squeezed vacuum state with $\chi_p(0) = 0.632$. Squeezed-spin states can be created by transferring the squeezing from the harmonic oscillator ($\chi_x(0) < 1$ or $\chi_p(0) < 1$) to the spins through H_1, which is clearly depicted in Figures 2(b) and 2(c). The figures show back a nd forth transferring of squeezing between the spins and the harmonic oscillator as the modified mirror-photon coupling $g(t)$ changes with time. The transfer of squeezing to the spins in this small-angle approximation is like wave-function exchange between coupled harmonic oscillators [42]. Moreover, Figures 2(a) and 2(b) depict that when ζ_x increases with time, then the corresponding ζ_y decreases and vice-versa. Figures 2(c) and 2(d) also illustrate the similar kind of observations. This is due to the following uncertainty relation $\Delta J_x \Delta J_y \geq |< J_z > /2\,|$ satisfied by J_x and J_y.

Further notice one important observation from the figure that, for the initially amplitude squeezed vacuum cavity field, the increase in mirror-photon coupling ϵ enhances the spin squeezing along x-direction (see Figure 2(a)). However, the squeezing parameter ζ_y reaches a minimum value for $\epsilon = 0.2\omega_m$ (see Figure 2(b)). Exactly opposite behaviors of spin-squeezing parameters are observed for the initially momentum squeezed vacuum cavity field (see Figures 2(c) and 2(d)). This is because of the fact that the radiation pressure force acting on the end mirror as a result of coupling between the light field and the movable mirror also helps in generating further squeezing in harmonic oscillator, known as pondermotive squeezing [18–20], which is then transferred to the spins via Tavis-Cummings type interaction. Hence, the optomechanical coupling acts as a new additional handle in coherently controlling the squeezing of spins along both the x- and y-directions.

We now discuss the experimental prospects of the various parameters used here in order to demonstrate the experimental feasibility of our system. The mechanical oscillator frequency may take the values from $2\pi \times 100$Hz [43], $2\pi \times 10$kHz [44] to $2\pi \times 73.5$MHz [45]. Also, the optomechanical coupling can have the values as low as $2\pi \times 2.7$Hz [46] to as high as $2\pi \times 4 \times 10^5$Hz [47]. The coherent coupling strength for the intracavity field interacting with the cloud of condensate atoms embedded within the optomechanical cavity may take the value $2\pi \times 5.86$kHz [48] ($2\pi \times 10.9$MHz [49]). Note that the loss of photons through the mirrors of the cavity can be reduced by using a high quality factor cavity.

4. Conclusion

In conclusion, we have shown the generation of correlated-particle states or squeezed-spin states for the condensate atoms confined in a lossless optomechanical cavity via pondermotively generated squeezed light. We have observed that the spin-squeezing along both the x- and y-directions can be coherently controlled by choosing the appropriate value of mirror-photon coupling. The radiation pressure or pondermotive force acting on the movable cavity boundary induces further squeezing in the light field with the increase in optomechanical coupling. This squeezing gets transferred to the BEC atoms via Tavis-Cummings type interaction and results in further squeezing of the atomic states. Such atomic-squeezed states have applications in spectroscopy, high precision metrology and entanglement detection.

Acknowledgements

Neha Aggarwal and Aranya B. Bhattacherjee acknowledge financial support from the Department of Science and Technology, New Delhi for financial assistance vide grant SR/S2/LOP-0034/2010.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed equally and significantly in writing this article. All the authors read and approved the final manuscript.

References

[1] M. Kitagawa and M. Ueda, *Phys. Rev. A* **47**, 5138 (1993).

[2] D.J. Wineland et al., *Phys. Rev. A* **46**, R6797 (1992).

[3] D.J. Wineland et al., *Phys. Rev. A* **50**, 67 (1994).

[4] E.S. Polzik, *Nature* **453**, 45 (2008).

[5] A.D. Cronin, J. Schmiedmayer and D.E. Pritchard, *Rev. Modern Phys.* **81**, 1051 (2009).

[6] A. Sorensen et al., *Nature* **409**, 63 (2001).

[7] N. Bigelow, *Nature* **409**, 27 (2001).

[8] O. Guehne and G. Toth, *Phys. Rep.* **474**, 1 (2009).

[9] Q.A. Turchette et al., *Phys. Rev. Lett.* **81**, 3631 (1998).

[10] V. Meyer et al., *Phys. Rev. Lett.* **86**, 5870 (2001).

[11] D. Leibfried et al., *Science* **304**, 1476 (2004).

[12] X. Wang, B.C. Sanders, *Phys. Rev. A* **68**, 012101 (2003).

[13] J.K. Korbicz, J.I. Cirac and M. Lewenstein, *Phys. Rev. Lett.* **95**, 120502 (2005); J.K. Korbicz et al., *Phys. Rev. A* **74**, 052319 (2006).

[14] S. Yi and H. Pu, *Phys. Rev. A* **73**, 023602 (2006).

[15] H.J. Kimble and D.F. Walls (eds.), *J. Opt. Soc. Am. B* **4**, 1450 (1987), special issue on Squeezed States of Electromagnetic Field.

[16] R. Loudon and P.L. Knight (eds.), *J. Mod. Opt.* **34**, 709 (1987), special issue on Squeezed Light.

[17] H.J. Kimble, in *Fundamental Systems in Quantum Optics*, edited by J. Dalibard, J.M. Raimond and J. Zinn-Justin (Elsevier, New York, 1991).

[18] C. Fabre et al., *Phys. Rev. A* **49**, 1337 (1994).

[19] S. Mancini and P. Tombesi, *Phys. Rev. A* **49**, 4055 (1994).

[20] T. Corbitt et al., *Phys. Rev. A* **73**, 023801 (2006).

[21] Daniel W.C. Brooks et al., *Nature* **488**, 476 (2012).

[22] T.P. Purdy et al., *Phys. Rev. X* **3**, 031012 (2013).

[23] A.H. Safavi-Naeini et al., *Nature* **500**, 185 (2013).

[24] B. Julsgaard, A. Kozhekin and E. Polzik, *Nature* **413**, 400 (2001).

[25] A. Banerjee, *Phys. Rev. A* **54**, 5327 (1996).

[26] L. Vernac, M. Pinard and E. Giacobino, *Eur. Phys. J. D* **17**, 125 (2001).

[27] C. Orzel et al., *Science* **291**, 2386 (2001).

[28] J. Esteve et al., *Nature* **455**, 1216 (2008).

[29] G.-B. Jo et al., *Phys. Rev. Lett.* **98**, 030407 (2007).

[30] W. Li et al., *Phys. Rev. Lett.* **98**, 040402 (2007).

[31] M. Pinard, Y. Hadjar and A. Heidmann, *Eur. Phys. J. D* **7**, 107 (1999).

[32] M.R. Matthews et al., *Phys. Rev. Lett.* **81**, 243 (1998).

[33] A. Bhattacherjee, *Phys. Rev. A* **80**, 043607 (2009).

[34] S.-C. Li, L.-B. Fu and J. Liu, *Phys. Rev. A* **84**, 053610 (2011).

[35] J.J. Sakurai, *Modern Quantum Mechanics*, New York : Addison-Wesley (1993).

[36] F.T. Arecchi, E. Courtens and R. Gilmore, *Phys. Rev. A* **6**, 2211 (1972).

[37] M.O. Scully and M.S. Zubairy, *Quantum Optics*, Cambridge University Press (1997).

[38] S. Mahajan, N. Aggarwal, A.B. Bhattacherjee and M. Mohan, *Chin. Phys. B* **23**, 020315 (2014).

[39] M. Tavis and F.W. Cummings, *Phys. Rev.* **170**, 379 (1968).

[40] C. Emary and T. Brandes, *Phys. Rev. E* **67**, 066203 (2003).

[41] V.B. Braginsky and A.B. Manukin, *Soviet Physics JETP* **25**, 653 (1967).

[42] D.J. Heinzen and D.J. Wineland, *Phys. Rev. A* **42**, 2977 (1990).

[43] G.D. Cole, *Proc. SPIE* **8458**, 845807 (2012).

[44] D. Hunger et al., *Phys. Rev. Lett.* **104**, 143002 (2010).

[45] A. Schliesser et al., *Nature Phys.* **4**, 415 (2008).

[46] S. Gigan et al., *Nature* **44**, 67 (2006).

[47] J. Chan et al., *Opt. Express* **17**, 3802 (2009).

[48] B. Nagorny et al., *Phys. Rev. A* **67**, 031401 (2003).

[49] F. Brennecke et al., *Science* **322**, 235 (2008).